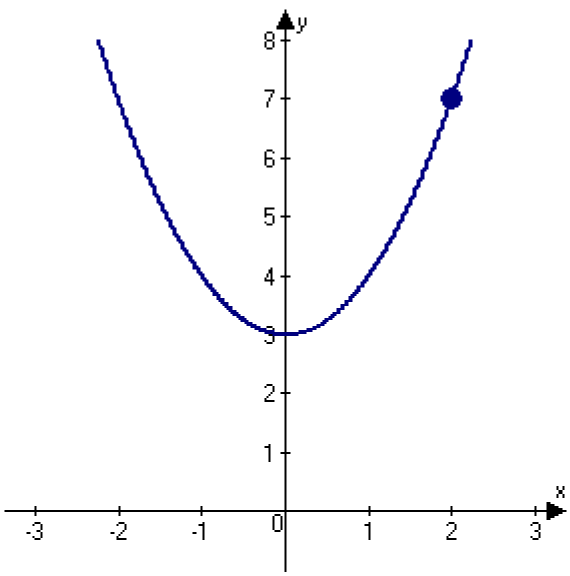
AP Calculus AB First Semester Final Exam Practice Test
Content covers chapters 1-3

Name: ____ Date: ______________ Period: _____________

This is a big tamale review for the final exam. Of the 69 questions on this review, 25 questions will be on the final exam. The exam will be administered on Thursday December 13 during the two hour block period. The exam will consist of 25 multiple choice questions. This is a closed notes and non-calculator exam.

1. Determine the following limit. (Hint: Use the graph of the function.)

$$\lim_{x\to 2}\left(x^2+3\right)$$

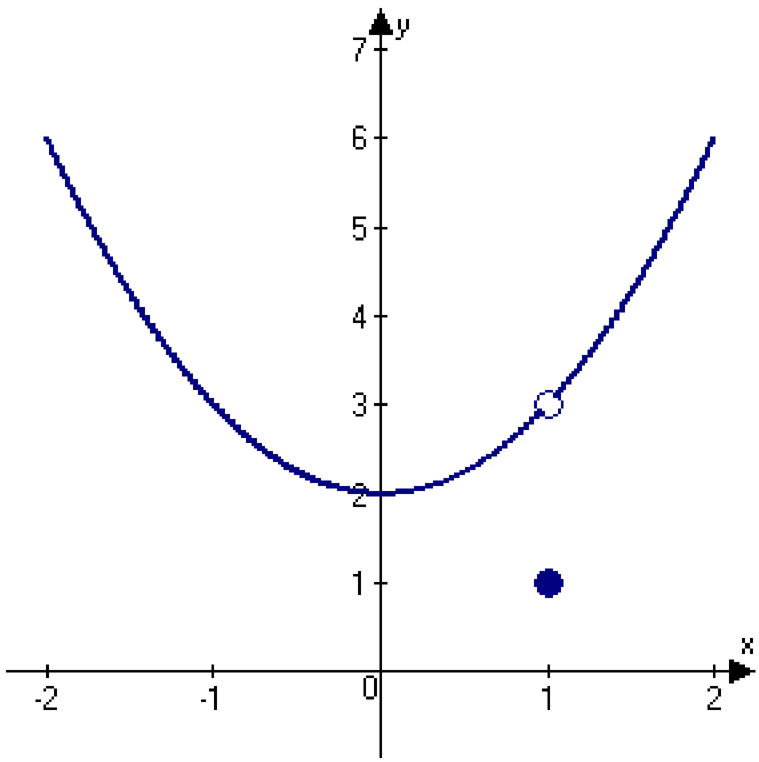


2. Let

$$f(x)=\begin{cases} x^2+2, & x\neq 1 \\ 1, & x=1 \end{cases}.$$

Determine the following limit. (Hint: Use the graph of the function.)

$\lim_{x\to 1} f(x)$



3. Let $f(x)=-5x-1$ and $g(x)=x^3$. Find the limits:

(a) $\lim_{x\to -2} f(x)$ (b) $\lim_{x\to -1} g(x)$ (c) $\lim_{x\to -4} g(f(x))$

4. Let $f(x)=-x^2+4$ and $g(x)=3x$. Find the limits:

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to -1} g(x)$ (c) $\lim_{x\to 3} g(f(x))$

5. Find the limit:

$$\lim_{x \to \frac{2\pi}{3}} \sin x$$

6. Find the limit:

$$\lim_{x \to 1} \cos\left(\frac{\pi x}{6}\right)$$

7. Find the limit:

$$\lim_{x \to \pi} \tan\left(\frac{3x}{4}\right)$$

8. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \to 5} \frac{x^3 - 125}{x - 5}$$

9. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \to 9} \frac{-x^2 + 23x - 126}{x - 9}$$

10. Determine the limit (if it exists):

$$\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{-2x^8}$$

11. Determine the limit (if it exists):

$$\lim_{x \to 0} \frac{-4(1 - \cos x)}{x}$$

12. Find the *x*-values (if any) at which the function $f(x) = -4x^2 - 13x - 11$ is not continuous. Which of the discontinuitites are removable?

13. Find the *x*-values (if any) at which the function $f(x) = \dfrac{x}{x^2 + 4}$ is not continuous. Which of the discontinuitites are removable?

14. Find the *x*-values (if any) at which the function $f(x) = \dfrac{x+8}{x^2 + 4x - 32}$ is not continuous. Which of the discontinuitites are removable?

15. Find the limit:

$$\lim_{x \to 5^+} \frac{x+13}{x-5}$$

16. Find the limit:

$$\lim_{x \to 0^-} \left(x^5 + \frac{1}{x} \right)$$

17. Find the derivative of the following function using the limiting process.

$$f(x) = 2x^2 - 8x - 1$$

18. Find the derivative of the following function using the limiting process.

$$f(x) = -5x^2 + 6x - 7$$

19. Find the derivative of the following function using the limiting process.

$$f(x) = -5x^3 + 8x^2 - 1$$

20. Find the derivative of the following function using the limiting process.

$$f(x) = \frac{4}{x+10}$$

21. Find the derivative of the following function using the limiting process.

$$f(x) = \frac{1}{x^3}$$

22. Find the derivative of the following function using the limiting process.

$$f(x) = \sqrt{7x-7}$$

23. Find the slope of the graph of the function at the given value.

$$f(x) = 3x^2 + \frac{4}{x^2} \text{ when } x = 3$$

24. Find the slope of the graph of the function at the given value.

$$f(x) = 4x^2 + 8x - \frac{5}{x^2} \text{ when } x = 4$$

25. Find the slope of the graph of the function at the given value.

$$f(x) = x(5x^5 + 4) \text{ when } x = 2$$

26. Find the slope of the graph of the function at the given value.

$$h(z) = z^{\frac{4}{7}} - z^{\frac{7}{8}} \text{ when } z = 5$$

27. Determine the point(s), (if any), at which the graph of the function has a horizontal tangent.

$$y(x) = x^3 + 6x^2 + 7$$

28. Determine the point(s), (if any), at which the graph of the function has a horizontal tangent.

$$y(x) = x^4 - 32x + 7$$

29. Determine the point(s), (if any), at which the graph of the function has a horizontal tangent.

$$y(x) = \frac{8}{x-9}$$

30. Determine the point(s), (if any), at which the graph of the function has a horizontal tangent.

$$y(x) = \frac{6x}{\left(x-4\right)^2}$$

31. Use the product rule to differentiate.

$$f(t) = \sqrt{t}\left(7 - t^6\right)$$

32. Use the product rule to differentiate.

$$R(t) = t^5 \cos t$$

33. Use the product rule to differentiate.

$$g(v) = v^{-3} \sin v$$

34. Use the product rule to differentiate.

$$g(s) = s^{-4} \cos s$$

35. Use the quotient rule to differentiate.

$$R(x) = \frac{2x}{x^3 + 10}$$

36. Use the quotient rule to differentiate.

$$P(t) = \frac{7 + t}{t^2 + 3}$$

37. Use the quotient rule to differentiate.

$$g(x) = \frac{\sin x}{x^2 + 2}$$

38. Find the second derivative of the function.

$$f(x) = 2x^{\frac{7}{5}}$$

39. Find the second derivative of the function.

$$H(s) = \frac{5s^2 + 4s - 5}{s}$$

40. Find the second derivative of the function.

$$Q(t) = t^5 \sec t$$

41. Find *dy/dx* by implicit differentiation.

$$x^2 + y^2 = 16$$

42. Find *dy/dx* by implicit differentiation.

$$x^2 + 5x + 15xy - y^2 = 16$$

43. Find *dy/dx* by implicit differentiation.

$$x^{6/7} + y^{8/5} = 9$$

44. Find *dy/dx* by implicit differentiation.

$$x^5 + 9x + 9xy - y^3 = 36$$

45. Find *dy/dx* by implicit differentiation.

$$x^5 + 3x + x^7 y - y^7 = 16$$

46. Find *dy/dx* by implicit differentiation.

$$\sin x + 9\cos 7y = 2$$

47. Find *dy/dx* by implicit differentiation and evaluate it at the given point.

$$x^3 - 4y^3 = -6, \qquad \left(3, \sqrt[3]{\frac{33}{4}}\right)$$

48. Find an equation of the tangent line to the graph of the function given below at the given point.

$$(y-10)^2 = 5(x-6), \qquad (34.80, -2.00).$$

(The coefficients below are given to two decimal places.)

49. Find an equation of the tangent line to the graph of the function given below at the given point.

$$9x^2 - 10xy + 2y^2 - 4 = 0, \qquad (-2, -2)$$

(The coefficients below are given to two decimal places.)

50. A point is moving along the graph of the function

$y = 6x^2 + 6$

such that *dx/dt* = 3 centimeters per second.

Find *dy/dt* for the given values of *x*.

(a) $x = 2$ (b) $x = 4$

51. A point is moving along the graph of the function

$y = \dfrac{1}{9x^2 + 7}$

such that *dx/dt* = 4 centimeters per second.

Find *dy/dt* when $x = 4$.

52. A point is moving along the graph of the function

$y = \sin 3x$

such that *dx/dt* = 8 centimeters per second.

Find *dy/dt* when $x = \dfrac{\pi}{11}$.

53. ***Area*** The radius, *r*, of a circle is decreasing at a rate of 4 centimeters per minute.

Find the rate of change of area, *A*, when the radius is 6

54. Determine whether Rolle's Theorem can be applied to the function $f(x) = (x+3)(x+2)(x+1)$ on the closed interval $[-3, -1]$. If Rolle's Theorem can be applied, find all numbers *c* in the open interval $(-3, -1)$ such that $f'(c) = 0$.

55. Determine whether Rolle's Theorem can be applied to the function $f(x) = (x-2)(x-3)^2$ on the closed interval $[2,3]$. If Rolle's Theorem can be applied, find all numbers *c* in the open interval $(2,3)$ such that $f'(c) = 0$.

56. Determine whether the Mean Value Theorem can be applied to the function $f(x) = x^2$ on the closed interval [4,10]. If the Mean Value Theorem can be applied, find all numbers *c* in the open interval (4,10) such that $f'(c) = \dfrac{f(10) - f(4)}{10 - 4}$.

57. Determine whether the Mean Value Theorem can be applied to the function $f(x) = x^3$ on the closed interval [0,8]. If the Mean Value Theorem can be applied, find all numbers *c* in the open interval (0,8) such that $f'(c) = \dfrac{f(8) - f(0)}{8 - 0}$.

58. The function $s(t) = t^3 - 12t^2 + 36t + 6$ describes the motion of a particle moving along a line.

(a) Find the velocity function of the particle at any time *t*;
(b) Identify the time intervals when the particle is moving in a positive direction;
(c) Identify the time intervals when the particle is moving in a negative direction; and
(d) Identify the times when the particle changes its direction.

59. The function $s(t) = 24t - t^2$ describes the motion of a particle moving along a line.

(a) Find the velocity function of the particle at any time *t*;
(b) Identify the time intervals when the particle is moving in a positive direction;
(c) Identify the time intervals when the particle is moving in a negative direction; and
(d) Identify the times when the particle changes its direction.

60. The function $s(t) = t^2 - 12t + 3$ describes the motion of a particle moving along a line.

(a) Find the velocity function of the particle at any time *t*;
(b) Identify the time intervals when the particle is moving in a positive direction;
(c) Identify the time intervals when the particle is moving in a negative direction; and
(d) Identify the times when the particle changes its direction.

61. Find the points of inflection and discuss the concavity of the function $f(x) = -\sin x + \cos x$ on the interval $(0, 2\pi)$.

62. Find the points of inflection and discuss the concavity of the function $f(x) = -7x - 2\cos x$ on the interval $[0, 2\pi]$.

63. Find the limit.
$$\lim_{x \to \infty} \frac{4x - 8}{8x - 7}$$

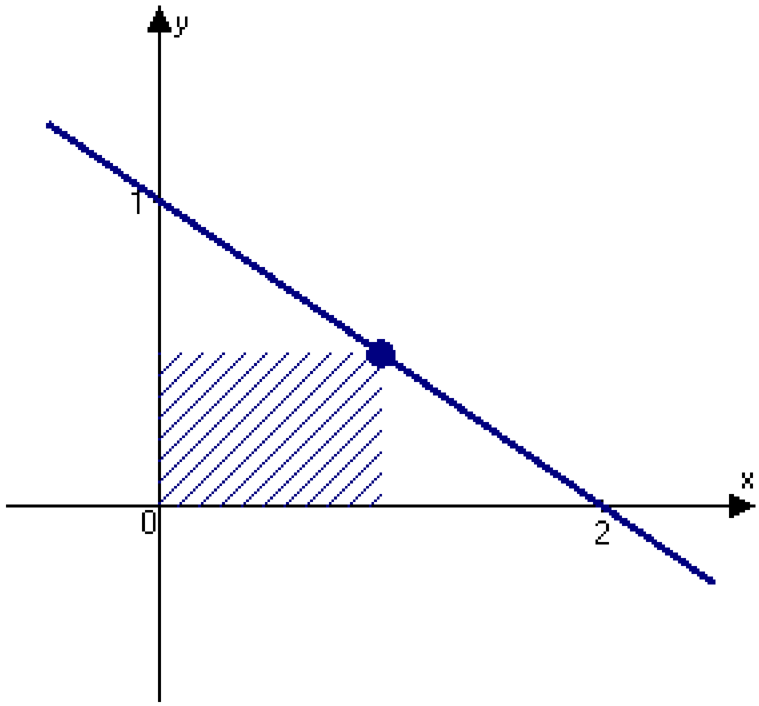
64. Find the limit.
$$\lim_{x \to \infty} \frac{-6x + 7}{-7x^2 - 5}$$

65. Find two positive numbers with product of 4 and sum that is minimum.

66. Find the length and width of a rectangle that has perimeter 40 meters and a maximum area.

67. Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 100 meters.

68. Find the dimensions of the rectangle of maximum area bounded by the x-axis and y-axis and the graph of $y = \dfrac{2-x}{2}$.



69. A rectangular page is to contain 169 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.

Answer Key

1. 7
2. 3
3. 9 , –1 , 6859
4. 3 , –3 , –15
5. $\frac{3^{1/2}}{2}$
6. $\frac{3^{1/2}}{2}$
7. -1
8. 75 , $x^2 + 5x + 25$
9. 5 , $-x + 14$
10. Does not exist.
11. 0
12. Continuous everywhere
13. Continuous everywhere
14. $x = -8$ (Removable), $x = 4$ (Not removable)
15. ∞
16. $-\infty$
17. $f'(x) = 4x - 8$
18. Both A and D
19. $f'(x) = -15x^2 + 16x$
20. $f'(x) = -\frac{4}{(x+10)^2}$
21. $f'(x) = -\frac{3}{x^4}$
22. $f'(x) = \frac{7}{2\sqrt{7x-7}}$
23. $f'(3) = \frac{478}{27}$
24. $f'(4) = \frac{1285}{32}$
25. $f'(2) = 964$
26. $h'(5) = \frac{4}{7(5)^{\frac{3}{7}}} - \frac{7}{8(5)^{\frac{1}{8}}}$
27. 0 and –4
28. 2
29. There are no points at which the graph has a horizontal tangent.
30. –4

31. $f'(t) = -6t^{5.5} + \dfrac{7-t^6}{2\sqrt{t}}$

32. $R'(t) = -t^5 \sin t + 5t^4 \cos t$

33. $g'(v) = v^{-3} \cos v - 3v^{-4} \sin v$

34. $g'(s) = -s^{-4} \sin s - 4s^{-5} \cos s$

35. $R'(x) = -\dfrac{2\left(-10+2x^3\right)}{\left(x^3+10\right)^2}$

36. $P'(t) = \dfrac{\left(3-14t-t^2\right)}{\left(t^2+3\right)^2}$

37. $g'(x) = \dfrac{\left((2+x^2)\cos x - 2x\sin x\right)}{\left(x^2+2\right)^2}$

38. $f''(x) = \dfrac{28}{25}x^{\frac{-3}{5}}$

39. $H''(s) = -\dfrac{10}{s^3}$

40. $Q''(t) = t^3 \sec t(20 + t^2 \sec^2 t + 10t \tan t + t^2 \tan^2 t)$

41. $\dfrac{dy}{dx} = -\dfrac{x}{y}$

42. $\dfrac{dy}{dx} = \dfrac{2x+5+15y}{2y-15x}$

43. $\dfrac{dy}{dx} = -\dfrac{15x^{-1/7}}{28y^{3/5}}$

44. $\dfrac{dy}{dx} = \dfrac{5x^4+9+9y}{3y^2-9x}$

45. $\dfrac{dy}{dx} = \dfrac{5x^4+3+7x^6y}{7y^6-x^7}$

46. $\dfrac{dy}{dx} = \dfrac{\cos x}{63\sin 7y}$

47. $\left.\dfrac{dy}{dx}\right|_{x=3} = \dfrac{9}{4\left(\sqrt[3]{\dfrac{33}{4}}\right)^2}$

48. $y = -0.21x + 5.25$

49. $y = 1.33x + 0.67$

50. $\dfrac{dy}{dt} = 72$ $\qquad$ $\dfrac{dy}{dt} = 144$

51. $\frac{dy}{dt} = -\frac{288}{22801}$

52. $\frac{dy}{dt} = 24\cos\left(\frac{3\pi}{11}\right)$

53. $\frac{dA}{dt} = -48\pi$

54. Both A and B

55. Rolle's Theorem applies; $\frac{7}{3}$

56. MVT applies; 7

57. MVT applies; $\frac{8\sqrt{3}}{3}$

58. (a) $v(t) = 3t^2 - 24t + 36$;
    (b) $(0,2)\cup(6,\infty)$;
    (c) $(2,6)$;
    (d) t = 2 and t = 6.

59. (a) $v(t) = 24 - 2t$;
    (b) $(0,12)$;
    (c) $(12,\infty)$;
    (d) t = 12.

60. (a) $v(t) = 2t - 12$;
    (b) $(6,\infty)$;
    (c) $(0,6)$;
    (d) t = 6.

61. None of the above

62. Concave downward on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; concave upward on $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$. Inflection points at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

63. $\frac{1}{2}$

64. 0

65. 2, 2

66. 10, 10

67. Square base side $\frac{5\sqrt{6}}{3}$; height $\frac{5\sqrt{6}}{3}$

68. Length 1; width 0.5

69. 15, 15