81 points

Trigonometry/Pre-Calculus Exam 7

Name: ______________________ Date: ____________ Period: ____________

Show all your work neatly and clearly. Calculators are allowed on the exam; however, each question is worth multiple points. Therefore, it is pertinent that you show all your work to receive the maximum amount of points on each question. Draw a box around your answer.

1. Factor; then use fundamental identities to simplify the expression below.

4

$\cot^2\alpha\tan^2\alpha+\cot^2\alpha$

$\cot^2\alpha(\tan^2\alpha+1)$

$\cot^2\alpha\cdot\sec^2\alpha$

$\frac{\cos^2\alpha}{\sin^2\alpha}\cdot\frac{1}{\cos^2\alpha}$

$\frac{1}{\sin^2\alpha}=\boxed{\csc^2\alpha}$

2. Multiply; then use fundamental identities to simplify the expression below.

3

$(2-2\cos x)(2+2\cos x)$

$4+4\cos x-4\cos x-4\cos^2 x$

$4-4\cos^2 x$

$4(1-\cos^2 x)=\boxed{4\sin^2 x}$

3. Add or subtract as indicated; then use fundamental identities to simplify the expression below.

4

$\frac{(1-\tan x)}{(1-\tan x)}\frac{1}{1+\tan x}+\frac{1}{1-\tan x}\frac{(1+\tan x)}{(1+\tan x)}$

$\frac{(1-\tan x)+(1+\tan x)}{1-\tan^2 x}=\frac{2}{1-(1-\sec^2 x)}=\frac{2}{\sec^2 x}=\boxed{2\cos^2 x}$

4. If $x=7\cot\theta$, use trigonometric substitution to write $\sqrt{49+x^2}$ as a trigonometric function of θ, where $0<\theta<\frac{\pi}{2}$.

5

$\sqrt{49+(7\cot\theta)^2}$

$\sqrt{49+49\cot^2\theta}$

$\sqrt{49(1+\cot^2\theta)}$

$\sqrt{49(\csc^2\theta)}=\boxed{7\csc\theta}$

16

5. Verify the identity shown below.

3

$$\frac{\sec\theta+\tan\theta}{\sec^2\theta}=\cos\theta(1+\sin\theta)$$

$\frac{\sec\theta}{\sec^2\theta}+\frac{\tan\theta}{\sec^2\theta}$

$\frac{1}{\sec\theta}+\frac{\sin\theta}{\cos\theta}\cdot\cos^2\theta$

$\cos\theta+\sin\theta\cdot\cos\theta$

$\cos\theta(1+\sin\theta)$ ✓

6. Verify the identity shown below.

4

$$(1+\csc\theta)(\sec\theta-\tan\theta)=\cot\theta$$

$\sec\theta-\tan\theta+\csc\theta\cdot\sec\theta-\csc\theta\tan\theta$

$\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}+\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}-\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}$

$\frac{\sin\theta\cdot-\sin\theta}{\sin\theta\cdot\cos\theta}+\frac{1}{\sin\theta\cos\theta}=\frac{-\sin^2\theta+1}{\sin\theta\cos\theta}=\frac{\cos^2\theta}{\sin\theta\cos\theta}=\boxed{\cot\theta}$ ✓

7. Verify the identity shown below.

5

$$\frac{(1-\sin\theta)}{(1-\sin\theta)}\frac{1-\sin\theta}{1+\sin\theta}=2\sec^2\theta-2\sec\theta\tan\theta-1$$

$\frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}=\frac{1-2\sin\theta+1-\cos^2\theta}{\cos^2\theta}=\frac{2-2\sin\theta-\cos^2\theta}{\cos^2\theta}$

$\boxed{=2\sec^2\theta-2\sec\theta\tan\theta-1}$ ✓

8. Verify the identity shown below.

2

$$\sec^2\mu-\cot^2\left(\frac{\pi}{2}-\mu\right)=1$$

$=\sec^2\mu-\tan^2x$

$=\boxed{1}$ ✓

14

9. Find all solutions of the following equation in the interval $[0, 2\pi)$.

$2\cos^2 x = 2 + \sin x$

4

$2\cos^2 x - \sin x - 2 = 0$

$2(1-\sin^2 x) - \sin x - 2 = 0$

$2 - 2\sin^2 x - \sin x - 2 = 0$

$2\sin^2 x + \sin x = 0$

$\sin x(2\sin x + 1) = 0$

$\sin x = 0 \qquad \sin x = -1/2$

$x = 0, \pi \qquad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

10. Solve the multiple-angle equation in the interval $[0, 2\pi)$.

PERIOD $\frac{\pi}{2}$

$\tan 2x = -1$

4

$2x = \frac{3\pi}{4} \qquad 2x = \frac{7\pi}{4}$

$\boxed{x = \frac{3\pi}{8},} \quad \boxed{x = \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}}$

11. Find the exact value of the given expression using a sum or difference formula.

$\cos\frac{5\pi}{12} = \cos 75^\circ = \cos(45^\circ + 30^\circ)$

4

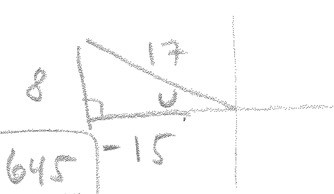
$= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

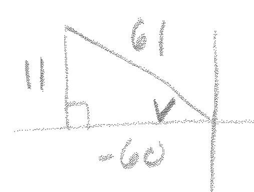
$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

12. Find the exact value of $\sin(u+v)$ given that $\sin u = \frac{8}{17}$ and $\cos v = -\frac{60}{61}$. (Both u and v are in Quadrant II.)

5

$\sin(u+v) = \sin u \cdot \cos v + \cos u \sin v$





$\left(\frac{8}{17}\right)\left(\frac{-60}{61}\right) + \left(\frac{-15}{17}\right)\left(\frac{11}{61}\right) = \boxed{\frac{-645}{1037}}$

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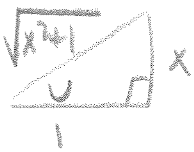
7 13. Write the given expression as an algebraic expression.

$\sin(\arctan x - \arcsin x)$

$u = \arctan x \quad v = \arcsin x$

$\tan u = x \quad \sin v = x$



$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{\sqrt{1-x^2}}{x}\right) - \left(\frac{1}{\sqrt{x^2+1}}\right)(x)$

$\boxed{\frac{\sqrt{1-x^2} - x}{\sqrt{x^2+1}}}$

14. Simplify the given expression algebraically.

$\sin(\pi + x) = \sin\pi \cdot \cos x + \cos\pi \cdot \sin x$

3 $\quad 0 \cdot \cos x + (-1) \cdot \sin x$

$\boxed{-\sin x}$

15. Find the exact solutions of the given equation in the interval $[0, 2\pi)$.

$\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

5 $\quad 2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = 1/2$

$\boxed{x = 0, \pi \quad x = \pi/3, \frac{5\pi}{3}}$

16. Use a double angle formula to rewrite the given expression.

$10\cos^2 x - 5$

4 $\quad 5(2\cos^2 x - 1)$

$\boxed{5\cos 2x}$

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17. Use the power-reducing formulas to rewrite the given expression in terms of the first power of the cosine.

$\sin^4 x$ $= \sin^2 x \cdot \sin^2 x$

5

$\left[\frac{1-\cos 2x}{2}\right]\left[\frac{1-\cos 2x}{2}\right]$

18. Use the half-angle formula to simplify the given expression.

$\sqrt{\frac{1+\cos 24x}{2}}$ $= \cos \frac{24x}{2} = \boxed{\cos 12x}$

2

19. Use the product-to-sum formula to write the given product as a sum or difference.

$8\sin\frac{\pi}{6}\sin\frac{\pi}{6}$

4 $8 \cdot \frac{1}{2}\left[\cos\left(\frac{\pi}{6}-\frac{\pi}{6}\right)-\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)\right]$

$4\left(\cos(0)-\cos \pi/3\right)$

20. Use the sum-to-product formulas to find the exact value of the given expression.

$\sin 150° - \sin 30°$

4 $2\cos\left(\frac{150°+30°}{2}\right)\sin\left(\frac{150°-30°}{2}\right)$

$2\cos 90° \sin 60°$

$2 \cdot 0 \cdot \frac{\sqrt{3}}{2} = \boxed{0}$

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