81 POINTS

Trigonometry/Pre-Calculus Exam 7

Name: ______________________ Date: ____________ Period: ____________

Show all your work neatly and clearly. Calculators are allowed on the exam; however, each question is worth multiple points. Therefore, it is pertinent that you show all your work to receive the maximum amount of points on each question. Draw a box around your answer.

4

1. Factor; then use fundamental identities to simplify the expression below.

$$\left(\sin^3 x - \sin^2 x\right)\left(-\sin x + 1\right)$$

$\sin^2 x(\sin x - 1) - 1(\sin x - 1)$

$(\sin^2 x - 1)(\sin x - 1)$

$-(\cos^2 x)(\sin x - 1)$

3

2. Multiply; then use fundamental identities to simplify the expression below.

$$(\tan x + 1)^2$$

$(\tan x + 1)(\tan x + 1)$

$\tan^2 x + 2\tan x + 1$

$\tan^2 x + 1 + 2\tan x$

$\sec^2 x + 2\tan x$

4

3. Add or subtract as indicated; then use fundamental identities to simplify the expression below.

$$(1-\csc x)\frac{1}{1+\csc x} - \frac{1}{1-\csc x}(1+\csc x)$$

$$\frac{(1-\csc x)-(1+\csc x)}{(1+\csc x)(1+\csc x)} = \frac{-2\csc x}{1-\csc^2 x} = \frac{-2\csc x}{-\cot^2 x} = \boxed{\frac{2\csc x}{\cot^2 x}}$$

$$= 2\cdot\frac{1}{\sin x}\cdot\frac{\sin^2 x}{\cos^2 x} = 2\sin x\cdot\frac{1}{\cos x}\cdot\frac{1}{\cos x} = \boxed{2\tan x\sec x}$$

5

4. If $x = 9\tan\theta$, use trigonometric substitution to write $\sqrt{81+x^2}$ as a trigonometric function of θ, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

$$\sqrt{81+(9\tan\theta)^2} = \sqrt{81+81\tan^2\theta} = \sqrt{81(1+\tan^2\theta)} = \sqrt{81\cdot\sec^2\theta}$$

$$= \boxed{9\sec\theta}$$

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5. Verify the identity shown below.

3

$$\frac{1+\sin\theta}{\tan\theta} = \cos\theta + \cot\theta$$ $= \cot\theta + \cos\theta$

$\frac{1}{\tan\theta} + \frac{\sin\theta}{\tan\theta}$

$\cot\theta + \frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}}$

6. Verify the identity shown below.

4

$$\left(\tan^2\theta+1\right)\left(\cos^2\theta-1\right) = 1-\sec^2\theta$$

$\sec^2\theta \cdot (\cancel{\sin}\cos^2\theta - 1)$

$\tan^2\theta\cos^2\theta - \tan^2\theta + \cos^2\theta - 1$

$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} - \tan^2\theta - 1 + \cos^2\theta$

$\sin^2\theta - \sec^2\theta + \cos^2\theta$

$1 - \sec^2\theta$ ✓

7. Verify the identity shown below.

5

$$\frac{1+\tan\theta}{1+\cot\theta} = \tan\theta$$

$\frac{1+\frac{\sin\theta}{\cos\theta}}{1+\frac{\cos\theta}{\sin\theta}} = \frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\sin\theta}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓

8. Verify the identity shown below.

2

$$\sec^2\left(\frac{\pi}{2}-y\right)-1 = \cot^2 y$$

$\csc^2 y - 1 = \cot^2 y$ ✓

14

9. Solve the following equation.

(4)

$$\cos^2 x + \cos x = 0$$

$\cos x(\cos x + 1) = 0$

$\cos x = 0 \quad \cos x + 1 = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \quad \cos x = -1$

$\boxed{x = \pi}$

$$\boxed{\begin{aligned} x &= \frac{\pi}{2} + 2\pi n,\ n \in \mathbb{Z} \\ x &= \frac{3\pi}{2} + 2\pi n,\ n \in \mathbb{Z} \\ x &= \pi + 2\pi n,\ n \in \mathbb{Z} \end{aligned}}$$

10. Solve the multiple-angle equation in the interval $[0, 2\pi)$.

(5)

$$\sin 2x = \frac{1}{2}$$

PERIOD $\frac{2\pi}{2} = \pi$

$2\sin x \cos x = \frac{1}{2}$

$\sin x = \frac{1}{4} \quad \cos x = \frac{1}{4}$

$2x = \frac{\pi}{6} \quad 2x = \frac{5\pi}{6}$

$\boxed{x = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}}$

11. Find the exact value of the given expression using a sum or difference formula.

(4)

$\cos\frac{7\pi}{12} = \cos 105^\circ = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

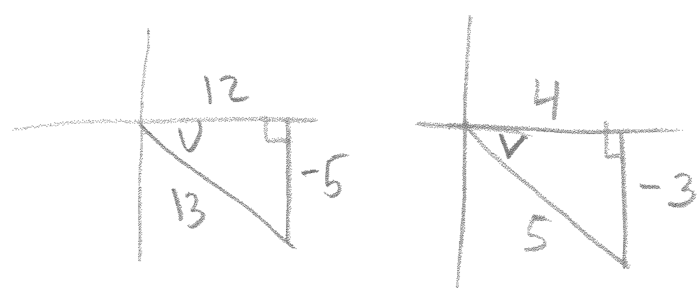
$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

12. Find the exact value of $\tan(u+v)$ given that $\sin u = -\frac{5}{13}$ and $\cos v = \frac{4}{5}$. (Both u and v are in Quadrant IV.)

(4)

$$\frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\frac{(-5/12) + (-3/4)}{1 - (-5/12)(-3/4)} = \boxed{\frac{-56}{33}}$$



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13. Write the given expression as an algebraic expression.

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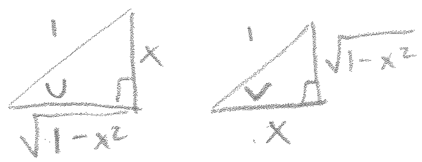
$\sin(\arcsin x + \arccos x)$

$u = \arcsin x$ $\quad$ $v = \arccos x$

$\sin u = x$ $\quad$ $\cos v = x$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$



$\sin(\arcsin x)\cos(\arccos x) + \cos(\arcsin x)\sin(\arccos x)$

$x^2 + \sqrt{1-x^2} \cdot \sqrt{1-x^2}$

$x^2 + 1 - x^2 = \boxed{1}$

14. Simplify the given expression algebraically.

3

$\sin\left(\frac{3\pi}{2} - x\right)$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\sin\frac{3\pi}{2} \cdot \cos x - \cos\frac{3\pi}{2}\sin x$

$-1 \cdot \cos x - 0 \cdot \sin x$

$\boxed{-\cos x}$

15. Find the exact solutions of the given equation in the interval $[0, 2\pi)$.

5

$\sin 4x = -2\sin 2x$

$2\sin[2(2x)]$

$2\sin 2u \cdot \cos 2u + 2\sin 2u = 0$

$\sin 2u(\cos 2u + 1) = 0$

$p: \pi$ $\quad$ $\sin 2u = 0$ $\quad$ $\cos 2u = -1$ $\quad$ $p = \pi$

$2u = 0$ $\quad$ $2u = \pi$ $\quad$ $u = \pi$

$\boxed{u = 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}}$

16. Use a double angle formula to rewrite the given expression.

4

$14\cos^2 x - 7$

$\cos 2x = 2\cos_x^2 - 1$

$\frac{\cos 2x + 1}{2} = \cos_x^2$

$14\left[\frac{\cos 2x + 1}{2}\right] - 7 =$

$7\cos 2x + 7 - 7$

$\boxed{7\cos 2x}$

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17. Use the power-reducing formulas to rewrite the given expression in terms of the first power of the cosine.

5

$\sin^2 x \cos^2 x$

$\left[\frac{1-\cos 2x}{2}\right]\left[\frac{1+\cos 2x}{2}\right]$

$\frac{1}{4}\left[1-\cos^2 2x\right] = \frac{1}{4}\left[1-\left(\frac{1+\cos 4x}{2}\right)\right]$

$\frac{1}{4}\left[\frac{2-1+\cos 4x}{2}\right]$

$=\frac{1}{4}\left[\frac{1+\cos 4x}{2}\right]$

$=\frac{1}{8}\left[1+\cos 4x\right]$

18. Use the half-angle formula to simplify the given expression.

2

$\sqrt{\frac{1+\cos 4x}{2}} = \cos\frac{4x}{2} = \cos 2x$

19. Use the product-to-sum formula to write the given product as a sum or difference.

4

$8\sin\frac{\pi}{8}\cos\frac{\pi}{8}$ $\quad \sin u\cos v = \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$

$8\cdot\frac{1}{2}\left[\sin\left(\frac{\pi}{8}+\frac{\pi}{8}\right)+\sin\left(\frac{\pi}{8}-\frac{\pi}{8}\right)\right]$

$\boxed{4\left(\sin\frac{\pi}{4}+\sin 0\right)}$

$4\left(\sqrt{2}/2+0\right) = \boxed{2\sqrt{2}}$

20. Use the sum-to-product formulas to find the exact value of the given expression.

4

$\cos 150° - \cos 30°$ $\quad \cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$

$-2\sin\left(\frac{150+30}{2}\right)\sin\left(\frac{150-30}{2}\right)$

$-2\sin 90°\cdot\sin 60°$

$-2\cdot 1\cdot\frac{\sqrt{3}}{2} = \boxed{-\sqrt{3}}$

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