Teaching Theorems Multiple-choice Examlpes

Multiple Choice
Identify the choice that best completes the statement or answers the question.

x	0	1	2
$f(x)$	1	k	2

____ 1.

The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if $k=$

a. 0
b. $\frac{1}{2}$
c. 1
d. 2
e. 3

____ 2. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c)=3$.

a. None
b. I only
c. I and II only
d. I and III only
e. I, II and III

____ 3. The function f is continuous for $-2 \le x \le 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

a. There exists c, where $-2 < c < 1$, such that $f(c) = 0$.
b. There exists c, where $-2 < c < 1$, such that $f'(c) = 0$.
c. There exists c, where $-2 < c < 1$, such that $f(c) = 3$.
d. There exists c, where $-2 < c < 1$, such that $f'(c) = 3$.
e. There exists c, where $-2 \le c \le 1$ such that $f(c) \ge f(x)$ for all x on the closed interval $-2 \le x \le 1$.

____ 4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

a. $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.
b. $f'(c) = 0$ for come c such that $a < c < b$.
c. f has a minimum value on $a \le x \le b$.
d. f has a maximum value on $a \le x \le b$.
e. $\int_a^b f(x)dx$ exists.

(2)

x	0	1	2	3	4
$f(x)$	2	3	4	3	2

___ 5.

The function f is continuous and differentiable on the closed interval [0,4]. The table above gives selected values of f on this interval. Which of the following statements must be true?

a. The minimum value of f on [0, 4] is 2.
b. The maximum value of f on [0, 4] is 4.
c. $f(x) > 0$ for $0 < x < 4$
d. $f'(x) < 0$ for $2 < x < 4$
e. There exists c, with $0 < c < 4$, for which $f'(c) = 0$.

___ 6. Let $f(x) = \int_0^{x^2} \sin t\,dt$. At how many points in the closed interval $[0, \sqrt{\pi}\,]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

a. Zero
b. One
c. Two
d. Three
e. Four

___ 7. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1,4]?

a. 0.456
b. 1.244
c. 2.164
d. 2.342
e. 2.452

___ 8. If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

a. -0.012
b. 0
c. 0.016
d. 0.376
e. 0.629

___ 9. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$

a. -3
b. -2
c. 2
d. 3
e. 18

___ 10. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right)dt\right) =$

a. $-\cos\left(x^6\right)$
b. $\sin\left(x^3\right)$
c. $\sin\left(x^6\right)$
d. $2x\sin\left(x^3\right)$
e. $2x\sin\left(x^6\right)$

2006 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

6. A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

   (a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\,dt$ using a trapezoidal approximation with the three subintervals determined by the table.

   (b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\,dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\,dt$.

   * (c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

   (d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

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t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   * (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

   * (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

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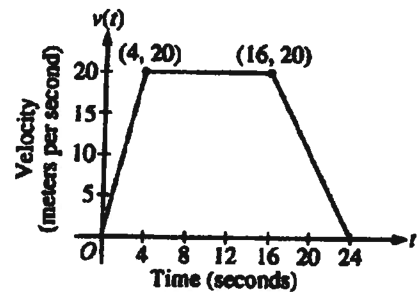

5. A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

   (a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

   (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

   (c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

   * (d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?