Related Rates

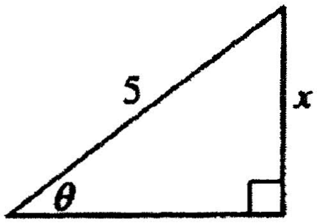
Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

a. 57.60
b. 57.88
c. 59.20
d. 60.00
e. 67.40

____ 2.



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

a. 3
b. $\frac{15}{4}$
c. 4
d. 9
e. 12

____ 3. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

a. $-(0.2)\pi C$
b. $-(0.1)C$
c. $-\frac{(0.1)C}{2\pi}$
d. $(0.1)^2 C$
e. $(0.1)^2 \pi C$

Name: ______________________

____ 4. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

a. A is always increasing.
b. A is always decreasing.
c. A is decreasing only when $b < h$.
d. A is decreasing only when $b > h$.
e. A remains constant.

____ 5. The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

a. 0.04π m²/sec

b. 0.4π m²/sec

c. 4π m²/sec

d. 20π m²/sec

e. 100π m²/sec

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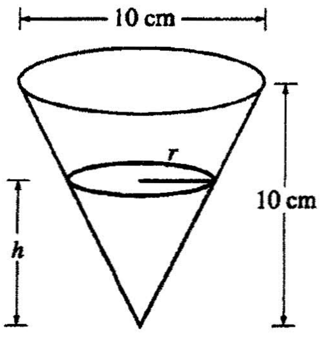

5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

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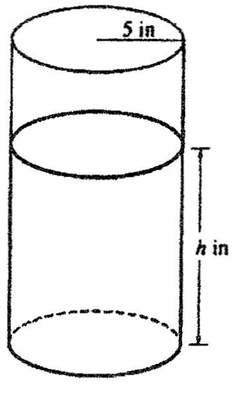
2003 SCORING GUIDELINES

Question 5

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

AP Calculus AB-3 **2008**

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

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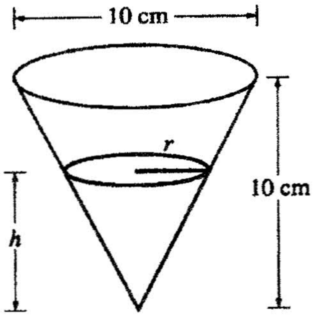
Question 5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

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(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

(a) When $h = 5$, $r = \frac{5}{2}$; $V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi$ cm^3	1 : V when $h = 5$
(b) $\frac{r}{h} = \frac{5}{10}$, so $r = \frac{1}{2}h$ $V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3$; $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$ $\left.\frac{dV}{dt}\right\rvert_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi$ cm^3/hr OR $\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$; $\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$ $\left.\frac{dV}{dt}\right\rvert_{h=5, r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$ $= -\frac{15}{8}\pi$ cm^3/hr	5 { 1 : $r = \frac{1}{2}h$ in (a) or (b) 1 : { V as a function of one variable in (a) or (b) OR $\frac{dr}{dt}$ 2 : $\frac{dV}{dt}$ < −2 > chain rule or product rule error 1 : evaluation at $h = 5$
(c) $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} = -\frac{3}{40}\pi h^2$ $= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10}\cdot$ area The constant of proportionality is $-\frac{3}{10}$.	2 { 1 : shows $\frac{dV}{dt} = k \cdot$ area 1 : identifies constant of proportionality
units of cm^3 in (a) and cm^3/hr in (b)	**1 : correct units in (a) and (b)**

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Question 5

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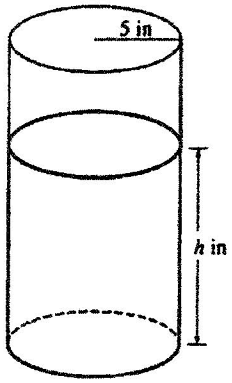

(a) Show that $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

(a) $V = 25\pi h$

$$\frac{dV}{dt} = 25\pi\frac{dh}{dt} = -5\pi\sqrt{h}$$

$$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$$

$$3: \begin{cases} 1: \dfrac{dV}{dt} = -5\pi\sqrt{h} \\ 1: \text{computes } \dfrac{dV}{dt} \\ 1: \text{shows result} \end{cases}$$

(b) $\dfrac{dh}{dt} = -\dfrac{\sqrt{h}}{5}$

$$\frac{1}{\sqrt{h}}\,dh = -\frac{1}{5}\,dt$$

$$2\sqrt{h} = -\frac{1}{5}t + C$$

$$2\sqrt{17} = 0 + C$$

$$h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$$

$$5: \begin{cases} 1: \text{separates variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition } h = 17 \\ \quad\ \text{when } t = 0 \\ 1: \text{solves for } h \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(c) $\left(-\dfrac{1}{10}t + \sqrt{17}\right)^2 = 0$

$$t = 10\sqrt{17}$$

1 : answer

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

(c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

Solution	Scoring
(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000\ \text{cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5$ cm/min. $\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$ $2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$ $\frac{dh}{dt} = 0.038$ or 0.039 cm/min	$4: \begin{cases} 1: \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2: \text{expression for } \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(b) $\frac{dV}{dt} = 2000 - R(t)$, so $\frac{dV}{dt} = 0$ when $R(t) = 2000$. This occurs when $t = 25$ minutes. Since $\frac{dV}{dt} > 0$ for $0 < t < 25$ and $\frac{dV}{dt} < 0$ for $t > 25$, the oil slick reaches its maximum volume 25 minutes after the device begins working.	$3: \begin{cases} 1: R(t) = 2000 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(c) The volume of oil, in cm^3, in the slick at time $t = 25$ minutes is given by $60{,}000 + \int_0^{25} (2000 - R(t))\, dt$.	$2: \begin{cases} 1: \text{limits and initial condition} \\ 1: \text{integrand} \end{cases}$