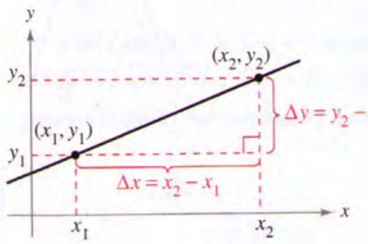
## Section P.2 Linear Models and Rates of Change

- **Find the slope of a line passing through two points.**
- **Write the equation of a line with a given point and slope.**
- **Interpret slope as a ratio or as a rate in a real-life application.**
- **Sketch the graph of a linear equation in slope-intercept form.**
- **Write equations of lines that are parallel or perpendicular to a given line.**




$\Delta y = y_2 - y_1$ = change in $y$
$\Delta x = x_2 - x_1$ = change in $x$
**Figure P.12**

### The Slope of a Line

The **slope** of a nonvertical line is a measure of the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right. Consider the two points $(x_1, y_1)$ and $(x_2, y_2)$ on the line in Figure P.12. As you move from left to right along this line, a vertical change of

$$\Delta y = y_2 - y_1 \qquad \text{Change in } y$$

units corresponds to a horizontal change of

$$\Delta x = x_2 - x_1 \qquad \text{Change in } x$$

units. ($\Delta$ is the Greek uppercase letter *delta*, and the symbols $\Delta y$ and $\Delta x$ are read "delta $y$" and "delta $x$.")

**Definition of the Slope of a Line**

The **slope** $m$ of the nonvertical line passing through $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2.$$

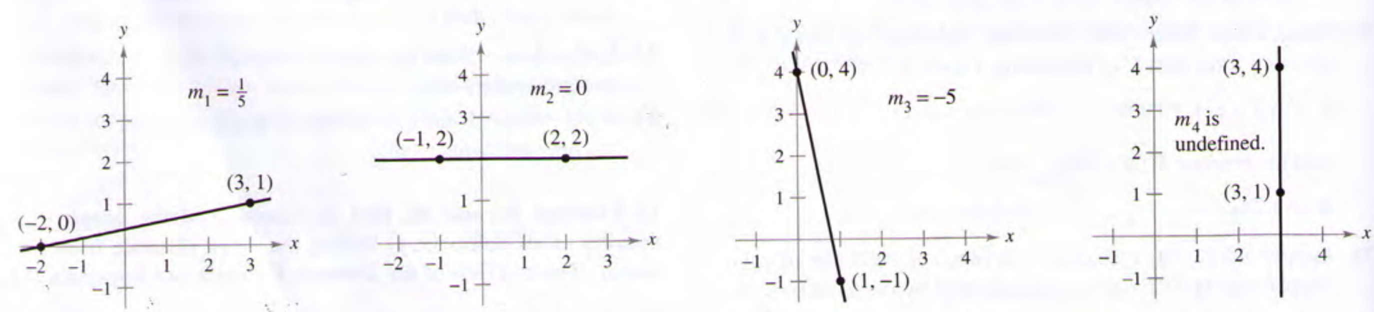
Slope is not defined for vertical lines.

NOTE When using the formula for slope, note that

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-(y_1 - y_2)}{-(x_1 - x_2)} = \frac{y_1 - y_2}{x_1 - x_2}.$$

So, it does not matter in which order you subtract *as long as* you are consistent and both "subtracted coordinates" come from the same point.

Figure P.13 shows four lines: one has a positive slope, one has a slope of zero, one has a negative slope, and one has an "undefined" slope. In general, the greater the absolute value of the slope of a line, the steeper the line is. For instance, in Figure P.13, the line with a slope of $-5$ is steeper than the line with a slope of $\frac{1}{5}$.




If $m$ is positive, then the line rises from left to right.
**Figure P.13**

If $m$ is zero, then the line is horizontal.

If $m$ is negative, then the line falls from left to right.

If $m$ is undefined, then the line is vertical.

**EXPLORATION**

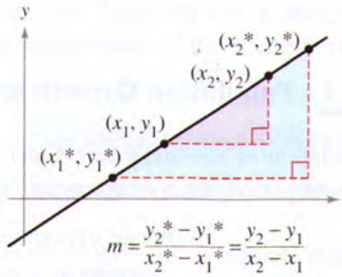
***Investigating Equations of Lines***
Use a graphing utility to graph each of the linear equations. Which point is common to all seven lines? Which value in the equation determines the slope of each line?

**a.** $y - 4 = -2(x + 1)$
**b.** $y - 4 = -1(x + 1)$
**c.** $y - 4 = -\frac{1}{2}(x + 1)$
**d.** $y - 4 = 0(x + 1)$
**e.** $y - 4 = \frac{1}{2}(x + 1)$
**f.** $y - 4 = 1(x + 1)$
**g.** $y - 4 = 2(x + 1)$

Use your results to write an equation of a line passing through $(-1, 4)$ with a slope of $m$.

## Equations of Lines

*Any* two points on a nonvertical line can be used to calculate its slope. This can be verified from the similar triangles shown in Figure P.14. (Recall that the ratios of corresponding sides of similar triangles are equal.)




Any two points on a nonvertical line can be used to determine its slope.
**Figure P.14**

You can write an equation of a nonvertical line if you know the slope of the line and the coordinates of one point on the line. Suppose the slope is $m$ and the point is $(x_1, y_1)$. If $(x, y)$ is any other point on the line, then

$$\frac{y - y_1}{x - x_1} = m.$$

This equation, involving the two variables $x$ and $y$, can be rewritten in the form $y - y_1 = m(x - x_1)$, which is called the **point-slope equation of a line.**

> **Point-Slope Equation of a Line**
>
> An equation of the line with slope $m$ passing through the point $(x_1, y_1)$ is given by
>
> $$y - y_1 = m(x - x_1).$$

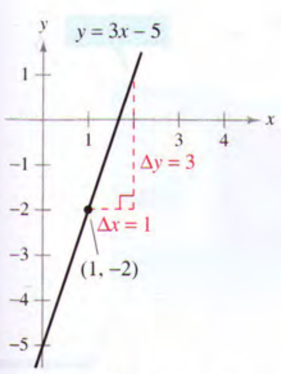
### *EXAMPLE 1* Finding an Equation of a Line

Find an equation of the line that has a slope of 3 and passes through the point $(1, -2)$.

**Solution**

| | |
|---|---|
| $y - y_1 = m(x - x_1)$ | Point-slope form |
| $y - (-2) = 3(x - 1)$ | Substitute $-2$ for $y_1$, 1 for $x_1$, and 3 for $m$. |
| $y + 2 = 3x - 3$ | Simplify. |
| $y = 3x - 5$ | Solve for $y$. |

(See Figure P.15.)



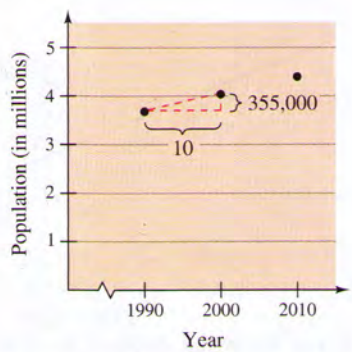

The line with a slope of 3 passing through the point $(1, -2)$
**Figure P.15**

NOTE Remember that only nonvertical lines have a slope. Consequently, vertical lines cannot be written in point-slope form. For instance, the equation of the vertical line passing through the point $(1, -2)$ is $x = 1$.

## Ratios and Rates of Change

The slope of a line can be interpreted as either a *ratio* or a *rate*. If the $x$- and $y$-axes have the same unit of measure, the slope has no units and is a **ratio.** If the $x$- and $y$-axes have different units of measure, the slope is a rate or **rate of change.** In your study of calculus, you will encounter applications involving both interpretations of slope.

### EXAMPLE 2 Population Growth and Engineering Design




Population of Kentucky in census years
**Figure P.16**

**a.** The population of Kentucky was 3,687,000 in 1990 and 4,042,000 in 2000. Over this 10-year period, the average rate of change of the population was

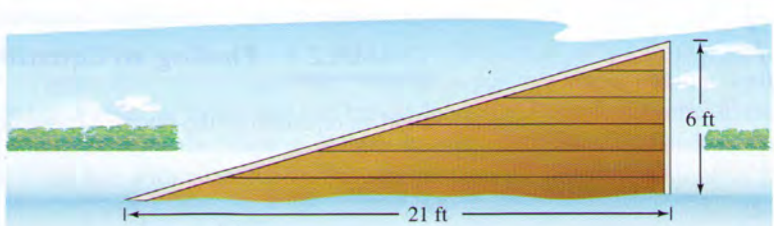
$$\begin{aligned}\text{Rate of change} &= \frac{\text{change in population}}{\text{change in years}}\\ &= \frac{4{,}042{,}000 - 3{,}687{,}000}{2000 - 1990}\\ &= 35{,}500 \text{ people per year.}\end{aligned}$$

If Kentucky's population continues to increase at this same rate for the next 10 years, it will have a 2010 population of 4,397,000 (see Figure P.16). *(Source: U.S. Census Bureau)*

**b.** In tournament water-ski jumping, the ramp rises to a height of 6 feet on a raft that is 21 feet long, as shown in Figure P.17. The slope of the ski ramp is the ratio of its height (the rise) to the length of its base (the run).

$$\begin{aligned}\text{Slope of ramp} &= \frac{\text{rise}}{\text{run}} \qquad \text{Rise is vertical change, run is horizontal change.}\\ &= \frac{6 \text{ feet}}{21 \text{ feet}}\\ &= \frac{2}{7}\end{aligned}$$

In this case, note that the slope is a ratio and has no units.




Dimensions of a water-ski ramp
**Figure P.17**

The rate of change found in Example 2(a) is an **average rate of change.** An average rate of change is always calculated over an interval. In this case, the interval is [1990, 2000]. In Chapter 2 you will study another type of rate of change called an *instantaneous rate of change*.

## Graphing Linear Models

Many problems in analytic geometry can be classified in two basic categories: (1) Given a graph, what is its equation? and (2) Given an equation, what is its graph? The point-slope equation of a line can be used to solve problems in the first category. However, this form is not especially useful for solving problems in the second category. The form that is better suited to sketching the graph of a line is the **slope-intercept** form of the equation of a line.

**The Slope-Intercept Equation of a Line**

The graph of the linear equation

$$y = mx + b$$

is a line having a *slope* of $m$ and a *y-intercept* at $(0, b)$.

### *EXAMPLE 3* Sketching Lines in the Plane

Sketch the graph of each equation.

**a.** $y = 2x + 1$ **b.** $y = 2$ **c.** $3y + x - 6 = 0$

**Solution**

**a.** Because $b = 1$, the $y$-intercept is $(0, 1)$. Because the slope is $m = 2$, you know that the line rises two units for each unit it moves to the right, as shown in Figure P.18(a).

**b.** Because $b = 2$, the $y$-intercept is $(0, 2)$. Because the slope is $m = 0$, you know that the line is horizontal, as shown in Figure P.18(b).

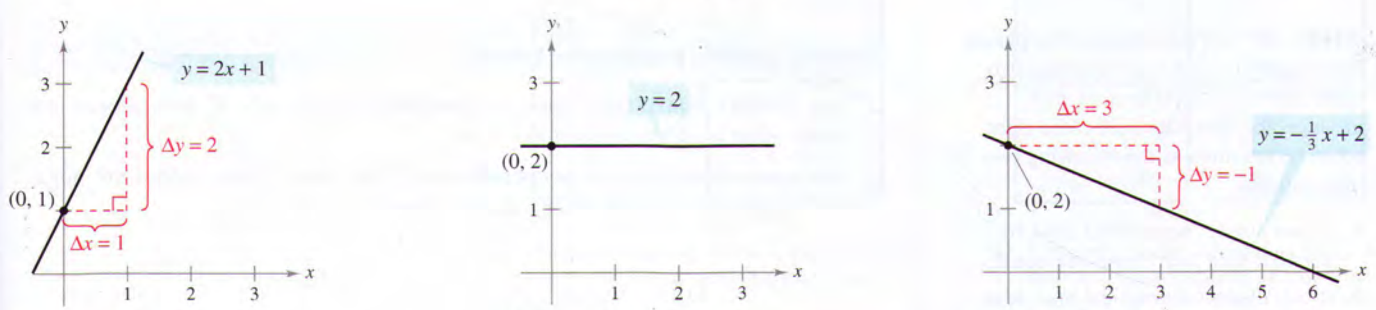
**c.** Begin by writing the equation in slope-intercept form.

$$3y + x - 6 = 0 \qquad \text{Write original equation.}$$

$$3y = -x + 6 \qquad \text{Isolate } y\text{-term on the left.}$$

$$y = -\frac{1}{3}x + 2 \qquad \text{Slope-intercept form}$$

In this form, you can see that the $y$-intercept is $(0, 2)$ and the slope is $m = -\frac{1}{3}$. This means that the line falls one unit for every three units it moves to the right, as shown in Figure P.18(c).




(a) $m = 2$; line rises (b) $m = 0$; line is horizontal (c) $m = -\frac{1}{3}$; line falls

**Figure P.18**

Because the slope of a vertical line is not defined, its equation cannot be written in the slope-intercept form. However, the equation of any line can be written in the **general form**

$$Ax + By + C = 0$$ General form of the equation of a line

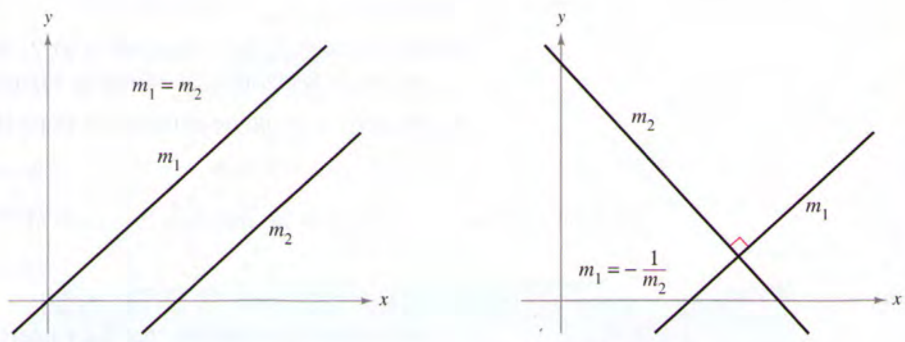
where $A$ and $B$ are not *both* zero. For instance, the vertical line given by $x = a$ can be represented by the general form $x - a = 0$.

**Summary of Equations of Lines**

1. General form: $Ax + By + C = 0, \quad (A, B \neq 0)$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Point-slope form: $y - y_1 = m(x - x_1)$
5. Slope-intercept form: $y = mx + b$

## Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular, as shown in Figure P.19. Specifically, nonvertical lines with the same slope are parallel and nonvertical lines whose slopes are negative reciprocals are perpendicular.




Parallel lines Perpendicular lines
**Figure P.19**

**STUDY TIP** In mathematics, the phrase "if and only if" is a way of stating two implications in one statement. For instance, the first statement at the right could be rewritten as the following two implications.

a. If two distinct nonvertical lines are parallel, then their slopes are equal.
b. If two distinct nonvertical lines have equal slopes, then they are parallel.

**Parallel and Perpendicular Lines**

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal—that is, if and only if $m_1 = m_2$.
2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other—that is, if and only if

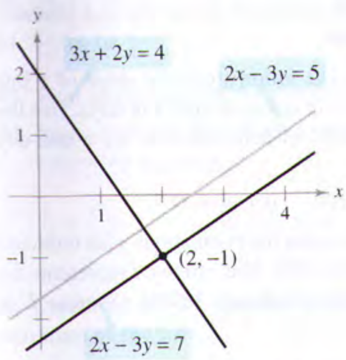
$$m_1 = -\frac{1}{m_2}.$$

## EXAMPLE 4 Finding Parallel and Perpendicular Lines

Find the general forms of the equations of the lines that pass through the point $(2, -1)$ and are

**a.** parallel to the line $2x - 3y = 5$ **b.** perpendicular to the line $2x - 3y = 5$.

(See Figure P.20.)




Lines parallel and perpendicular to $2x - 3y = 5$
**Figure P.20**

**Solution** By writing the linear equation $2x - 3y = 5$ in slope-intercept form, $y = \frac{2}{3}x - \frac{5}{3}$, you can see that the given line has a slope of $m = \frac{2}{3}$.

**a.** The line through $(2, -1)$ that is parallel to the given line also has a slope of $\frac{2}{3}$.

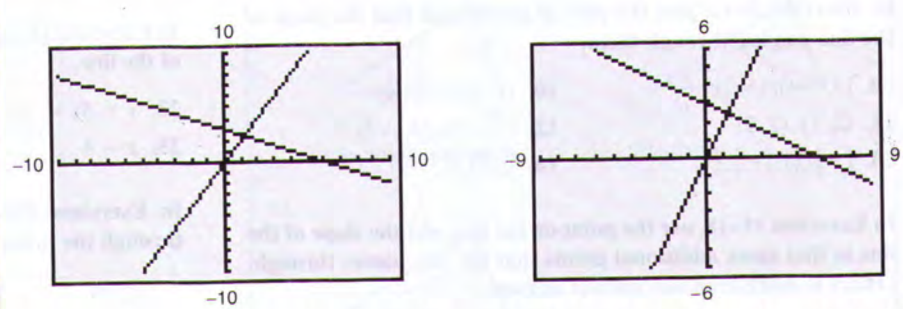
$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-1) = \tfrac{2}{3}(x - 2) \qquad \text{Substitute.}$$
$$3(y + 1) = 2(x - 2) \qquad \text{Simplify.}$$
$$2x - 3y - 7 = 0 \qquad \text{General form}$$

Note the similarity to the original equation.

**b.** Using the negative reciprocal of the slope of the given line, you can determine that the slope of a line perpendicular to the given line is $-\frac{3}{2}$. So, the line through the point $(2, -1)$ that is perpendicular to the given line has the following equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-1) = -\tfrac{3}{2}(x - 2) \qquad \text{Substitute.}$$
$$2(y + 1) = -3(x - 2) \qquad \text{Simplify.}$$
$$3x + 2y - 4 = 0 \qquad \text{General form}$$

**TECHNOLOGY PITFALL** The slope of a line will appear distorted if you use different tick-mark spacing on the $x$- and $y$-axes. For instance, the graphing calculator screens in Figures P.21(a) and P.21(b) both show the lines given by $y = 2x$ and $y = -\frac{1}{2}x + 3$. Because these lines have slopes that are negative reciprocals, they must be perpendicular. In Figure P.21(a), however, the lines don't appear to be perpendicular because the tick-mark spacing on the $x$-axis is not the same as that on the $y$-axis. In Figure P.21(b), the lines appear perpendicular because the tick-mark spacing on the $x$-axis is the same as on the $y$-axis. This type of viewing window is said to have a *square setting*.




**(a)** Tick-mark spacing on the $x$-axis is not the same as tick-mark spacing on the $y$-axis.

**(b)** Tick-mark spacing on the $x$-axis is the same as tick-mark spacing on the $y$-axis.

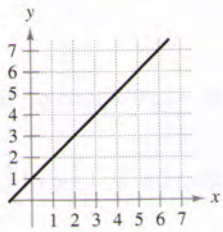
**Figure P.21**

## Exercises for Section P.2

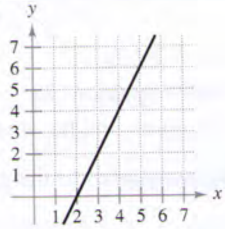
See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

**In Exercises 1–6, estimate the slope of the line from its graph. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

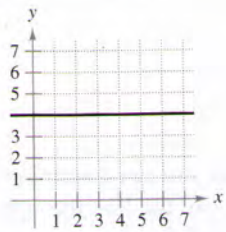
**1.**



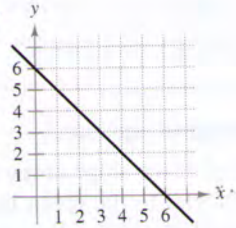

**2.**



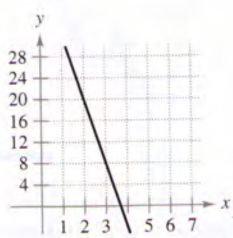

**3.**



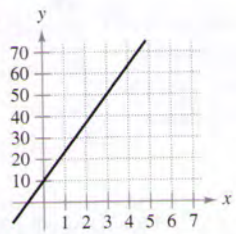

**4.**




**5.**




**6.**




**In Exercises 7 and 8, sketch the lines through the point with the indicated slopes. Make the sketches on the same set of coordinate axes.**

| | *Point* | *Slopes* | | | |
|---|---|---|---|---|---|
| **7.** | (2, 3) | (a) 1 | (b) $-2$ | (c) $-\frac{3}{2}$ | (d) Undefined |
| **8.** | $(-4, 1)$ | (a) 3 | (b) $-3$ | (c) $\frac{1}{3}$ | (d) 0 |

**In Exercises 9–14, plot the pair of points and find the slope of the line passing through them.**

**9.** $(3, -4), (5, 2)$

**10.** $(1, 2), (-2, 4)$

**11.** $(2, 1), (2, 5)$

**12.** $(3, -2), (4, -2)$

**13.** $\left(-\frac{1}{2}, \frac{2}{3}\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$

**14.** $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

**In Exercises 15–18, use the point on the line and the slope of the line to find three additional points that the line passes through. (There is more than one correct answer.)**

| | *Point* | *Slope* | | *Point* | *Slope* |
|---|---|---|---|---|---|
| **15.** | (2, 1) | $m = 0$ | **16.** | $(-3, 4)$ | $m$ undefined |
| **17.** | (1, 7) | $m = -3$ | **18.** | $(-2, -2)$ | $m = 2$ |

**19.** ***Conveyor Design*** A moving conveyor is built to rise 1 meter for each 3 meters of horizontal change.

(a) Find the slope of the conveyor.

(b) Suppose the conveyor runs between two floors in a factory. Find the length of the conveyor if the vertical distance between floors is 10 feet.

**20.** ***Rate of Change*** Each of the following is the slope of a line representing daily revenue $y$ in terms of time $x$ in days. Use the slope to interpret any change in daily revenue for a one-day increase in time.

(a) $m = 400$ (b) $m = 100$ (c) $m = 0$

**21.** ***Modeling Data*** The table shows the populations $y$ (in millions) of the United States for 1996–2001. The variable $t$ represents the time in years, with $t = 6$ corresponding to 1996. *(Source: U.S. Bureau of the Census)*

| $t$ | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|
| $y$ | 269.7 | 272.9 | 276.1 | 279.3 | 282.3 | 285.0 |

(a) Plot the data by hand and connect adjacent points with a line segment.

(b) Use the slope of each line segment to determine the year when the population increased least rapidly.

**22.** ***Modeling Data*** The table shows the rate $r$ (in miles per hour) that a vehicle is traveling after $t$ seconds.

| $t$ | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| $r$ | 57 | 74 | 85 | 84 | 61 | 43 |

(a) Plot the data by hand and connect adjacent points with a line segment.

(b) Use the slope of each line segment to determine the interval when the vehicle's rate changed most rapidly. How did the rate change?

**In Exercises 23–26, find the slope and the $y$-intercept (if possible) of the line.**

**23.** $x + 5y = 20$

**24.** $6x - 5y = 15$

**25.** $x = 4$

**26.** $y = -1$

**In Exercises 27–32, find an equation of the line that passes through the point and has the indicated slope. Sketch the line.**

| | *Point* | *Slope* | | *Point* | *Slope* |
|---|---|---|---|---|---|
| **27.** | (0, 3) | $m = \frac{3}{4}$ | **28.** | $(-1, 2)$ | $m$ undefined |
| **29.** | (0, 0) | $m = \frac{2}{3}$ | **30.** | (0, 4) | $m = 0$ |
| **31.** | $(3, -2)$ | $m = 3$ | **32.** | $(-2, 4)$ | $m = -\frac{3}{5}$ |

**In Exercises 33–42, find an equation of the line that passes through the points, and sketch the line.**

33. $(0, 0), (2, 6)$
34. $(0, 0), (-1, 3)$
35. $(2, 1), (0, -3)$
36. $(-3, -4), (1, 4)$
37. $(2, 8), (5, 0)$
38. $(-3, 6), (1, 2)$
39. $(5, 1), (5, 8)$
40. $(1, -2), (3, -2)$
41. $\left(\frac{1}{2}, \frac{7}{2}\right), \left(0, \frac{3}{4}\right)$
42. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

43. Find an equation of the vertical line with $x$-intercept at 3.

44. Show that the line with intercepts $(a, 0)$ and $(0, b)$ has the following equation.

$$\frac{x}{a} + \frac{y}{b} = 1, \qquad a \neq 0, b \neq 0$$

**In Exercises 45–48, use the result of Exercise 44 to write an equation of the line.**

45. $x$-intercept: $(2, 0)$
$y$-intercept: $(0, 3)$

46. $x$-intercept: $\left(-\frac{2}{3}, 0\right)$
$y$-intercept: $(0, -2)$

47. Point on line: $(1, 2)$
$x$-intercept: $(a, 0)$
$y$-intercept: $(0, a)$
$(a \neq 0)$

48. Point on line: $(-3, 4)$
$x$-intercept: $(a, 0)$
$y$-intercept: $(0, a)$
$(a \neq 0)$

**In Exercises 49–56, sketch a graph of the equation.**

49. $y = -3$
50. $x = 4$
51. $y = -2x + 1$
52. $y = \frac{1}{3}x - 1$
53. $y - 2 = \frac{3}{2}(x - 1)$
54. $y - 1 = 3(x + 4)$
55. $2x - y - 3 = 0$
56. $x + 2y + 6 = 0$

***Square Setting*** **In Exercises 57 and 58, use a graphing utility to graph both lines in each viewing window. Compare the graphs. Do the lines appear perpendicular? Are the lines perpendicular? Explain.**

57. $y = x + 6, \; y = -x + 2$

(a)
Xmin = -10
Xmax = 10
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1

(b)
Xmin = -15
Xmax = 15
Xscl = 1
Ymin = -10
Ymax = 10
Yscl = 1

58. $y = 2x - 3, \; y = -\frac{1}{2}x + 1$

(a)
Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -5
Ymax = 5
Yscl = 1

(b)
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -4
Ymax = 4
Yscl = 1

**In Exercises 59–64, write an equation of the line through the point (a) parallel to the given line and (b) perpendicular to the given line.**

| | *Point* | *Line* | | *Point* | *Line* |
|---|---|---|---|---|---|
| 59. | $(2, 1)$ | $4x - 2y = 3$ | 60. | $(-3, 2)$ | $x + y = 7$ |
| 61. | $\left(\frac{3}{4}, \frac{7}{8}\right)$ | $5x - 3y = 0$ | 62. | $(-6, 4)$ | $3x + 4y = 7$ |
| 63. | $(2, 5)$ | $x = 4$ | 64. | $(-1, 0)$ | $y = -3$ |

***Rate of Change*** **In Exercises 65–68, you are given the dollar value of a product in 2004 *and* the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value $V$ of the product in terms of the year $t$. (Let $t = 0$ represent 2000.)**

| | *2004 Value* | *Rate* |
|---|---|---|
| 65. | \$2540 | \$125 increase per year |
| 66. | \$156 | \$4.50 increase per year |
| 67. | \$20,400 | \$2000 decrease per year |
| 68. | \$245,000 | \$5600 decrease per year |

**In Exercises 69 and 70, use a graphing utility to graph the parabolas and find their points of intersection. Find an equation of the line through the points of intersection and graph the line in the same viewing window.**

69. $y = x^2$
$y = 4x - x^2$

70. $y = x^2 - 4x + 3$
$y = -x^2 + 2x + 3$

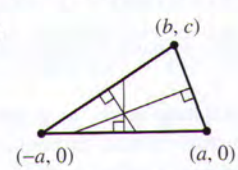
**In Exercises 71 and 72, determine whether the points are collinear. (Three points are *collinear* if they lie on the same line.)**

71. $(-2, 1), (-1, 0), (2, -2)$
72. $(0, 4), (7, -6), (-5, 11)$

## Writing About Concepts

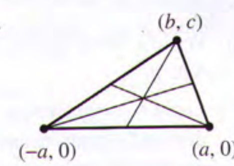
**In Exercises 73–75, find the coordinates of the point of intersection of the given segments. Explain your reasoning.**

73.



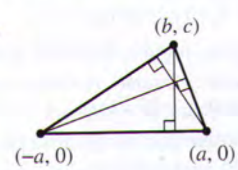

Perpendicular bisectors

74.




Medians

75.




Altitudes

76. Show that the points of intersection in Exercises 73, 74, and 75 are collinear.

**77.** ***Temperature Conversion*** Find a linear equation that expresses the relationship between the temperature in degrees Celsius $C$ and degrees Fahrenheit $F$. Use the fact that water freezes at 0°C (32°F) and boils at 100°C (212°F). Use the equation to convert 72°F to degrees Celsius.

**78.** ***Reimbursed Expenses*** A company reimburses its sales representatives \$150 per day for lodging and meals plus 34¢ per mile driven. Write a linear equation giving the daily cost $C$ to the company in terms of $x$, the number of miles driven. How much does it cost the company if a sales representative drives 137 miles on a given day?

**79.** ***Career Choice*** An employee has two options for positions in a large corporation. One position pays \$12.50 per hour *plus* an additional unit rate of \$0.75 per unit produced. The other pays \$9.20 per hour *plus* a unit rate of \$1.30.

(a) Find linear equations for the hourly wages $W$ in terms of $x$, the number of units produced per hour, for each option.

(b) Use a graphing utility to graph the linear equations and find the point of intersection.

(c) Interpret the meaning of the point of intersection of the graphs in part (b). How would you use this information to select the correct option if the goal were to obtain the highest hourly wage?

**80.** ***Straight-Line Depreciation*** A small business purchases a piece of equipment for \$875. After 5 years the equipment will be outdated, having no value.

(a) Write a linear equation giving the value $y$ of the equipment in terms of the time $x$, $0 \leq x \leq 5$.

(b) Find the value of the equipment when $x = 2$.

(c) Estimate (to two-decimal-place accuracy) the time when the value of the equipment is \$200.

**81.** ***Apartment Rental*** A real estate office handles an apartment complex with 50 units. When the rent is \$580 per month, all 50 units are occupied. However, when the rent is \$625, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent $p$ and the demand $x$ is linear. (*Note:* The term *demand* refers to the number of occupied units.)

(a) Write a linear equation giving the demand $x$ in terms of the rent $p$.

(b) *Linear extrapolation* Use a graphing utility to graph the demand equation and use the *trace* feature to predict the number of units occupied if the rent is raised to \$655.

(c) *Linear interpolation* Predict the number of units occupied if the rent is lowered to \$595. Verify graphically.

**82.** ***Modeling Data*** An instructor gives regular 20-point quizzes and 100-point exams in a mathematics course. Average scores for six students, given as ordered pairs $(x, y)$ where $x$ is the average quiz score and $y$ is the average test score, are (18, 87), (10, 55), (19, 96), (16, 79), (13, 76), and (15, 82).

(a) Use the regression capabilities of a graphing utility to find the least squares regression line for the data.

(b) Use a graphing utility to plot the points and graph the regression line in the same viewing window.

(c) Use the regression line to predict the average exam score for a student with an average quiz score of 17.

(d) Interpret the meaning of the slope of the regression line.

(e) The instructor adds 4 points to the average test score of everyone in the class. Describe the changes in the positions of the plotted points and the change in the equation of the line.

**83.** ***Tangent Line*** Find an equation of the line tangent to the circle $x^2 + y^2 = 169$ at the point (5, 12).

**84.** ***Tangent Line*** Find an equation of the line tangent to the circle $(x - 1)^2 + (y - 1)^2 = 25$ at the point $(4, -3)$.

***Distance*** **In Exercises 85–90, find the distance between the point and line, or between the lines, using the formula for the distance between the point $(x_1, y_1)$ and the line $Ax + By + C = 0$.**

$$\textbf{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

**85.** Point: (0, 0)
Line: $4x + 3y = 10$

**86.** Point: (2, 3)
Line: $4x + 3y = 10$

**87.** Point: $(-2, 1)$
Line: $x - y - 2 = 0$

**88.** Point: (6, 2)
Line: $x = -1$

**89.** Line: $x + y = 1$
Line: $x + y = 5$

**90.** Line: $3x - 4y = 1$
Line: $3x - 4y = 10$

**91.** Show that the distance between the point $(x_1, y_1)$ and the line $Ax + By + C = 0$ is

$$\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

**92.** Write the distance $d$ between the point (3, 1) and the line $y = mx + 4$ in terms of $m$. Use a graphing utility to graph the equation. When is the distance 0? Explain the result geometrically.

**93.** Prove that the diagonals of a rhombus intersect at right angles. (A rhombus is a quadrilateral with sides of equal lengths.)

**94.** Prove that the figure formed by connecting consecutive midpoints of the sides of any quadrilateral is a parallelogram.

**95.** Prove that if the points $(x_1, y_1)$ and $(x_2, y_2)$ lie on the same line as $(x_1^*, y_1^*)$ and $(x_2^*, y_2^*)$, then

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Assume $x_1 \neq x_2$ and $x_1^* \neq x_2^*$.

**96.** Prove that if the slopes of two nonvertical lines are negative reciprocals of each other, then the lines are perpendicular.

***True or False?*** **In Exercises 97 and 98, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

**97.** The lines represented by $ax + by = c_1$ and $bx - ay = c_2$ are perpendicular. Assume $a \neq 0$ and $b \neq 0$.

**98.** It is possible for two lines with positive slopes to be perpendicular to each other.