* use calc

Limits & Continuity - ANSWERS

(1) $\lim_{x\to a} f(x) = 2$. $\lim_{x\to b} f(x) = DNE$

(B) so (B) is true.

*(2) $\lim_{x\to 1} f(x) = 2$ so $\sin 2 = .909$

(A)

(3) I. $\lim_{x\to 4} f(x) = 3$ lim exists on I and II

(D) II. $\lim_{x\to 4} f(x) = 4$

III. $\lim_{x\to 4} f(x) = DNE$ because different from left & right

(4) Graph of 'f' is continuous @ a, c & e.

(A) Graph of 'f' is not differentiable @ a. c is diff and so is e.

(5) limits @ ∞ (or $-\infty$) compare exponents of variables.

(C) so $\frac{1}{4}$ $\lim_{x\to\infty} \frac{x^3}{4x^3}$ all other terms are irrelevant because they approach 0.

(6) $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$ put in 'a' for x. Get $\frac{0}{0}$ so

(B) factor (or use L'Hôpital if you know that)

$\lim_{x\to a} \frac{\cancel{(x-a)(x+a)}}{\cancel{(x^2-a^2)}(x^2+a^2)}$. sub 'a' back in for x.

$\cancel{(x-a)(x+a)}$ $\lim_{x\to a} \frac{1}{a^2+a^2} = \frac{1}{2a^2}$

(7) For 'f' to be continuous, the denominator

(A) can't ever equal zero. If $x^2 - a = 0 \Rightarrow$ $x^2 = a$ so a can't be any positive value.

(8) for $0 < x \le 2$ you are coming toward 2 from the left so $\ln x$ is $\ln 2$.

(E) $x^2 \ln 2$ is $4 \ln 2$ when $2 < x \le 4$ for coming into 2 from the right.

Since $\ln 2 \ne 4\ln 2$, the answer is E.

(9) Looking @ I, II and III, we have to check for continuity & differentiability. So

(D) $\lim_{x\to 3} f(x)$ has to be the same which will make I and II true. $\lim_{x\to 3^-} (x+2) = 5$

$\lim_{x\to 3^+} (4x-7) = 5$ so

I and II are true.

Now take der. of $f(x)$

$$f'(x) = \begin{cases} 1, & \text{if } x \le 3 \\ 4, & \text{if } x > 3 \end{cases}$$

so III is Not true

(10) You need to recognize that $\lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = 5$

(C) is this.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

which is the definition of the derivative

so, the der. as h approaches 0 is 5 which means $f'(2) = 5$

so f is increasing which implies f is cont & differentiable @ $x=2$, I and II true

BUT, we don't know anything for sure about the der. of f @ $x=2$ being cont.