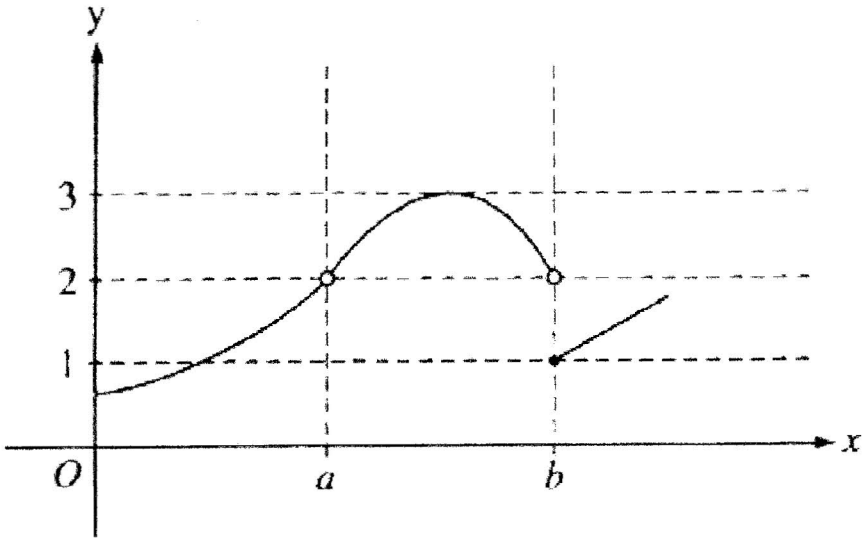
Name: _____ Class: _____ Date: _____ ID: A

Limits and Continuity

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1.



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

b. $\lim_{x \to a} f(x) = 2$

c. $\lim_{x \to b} f(x) = 2$

d. $\lim_{x \to b} f(x) = 1$

e. $\lim_{x \to a} f(x)$ does not exist

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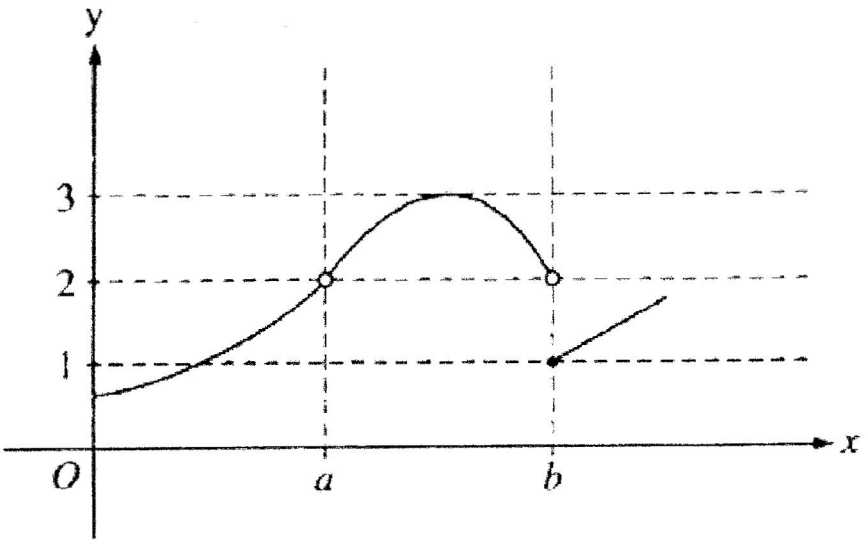

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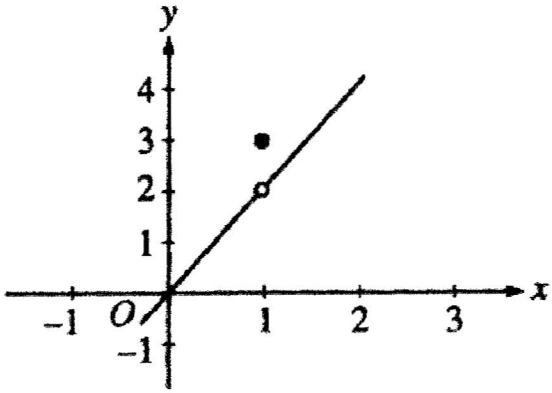


Graph of f

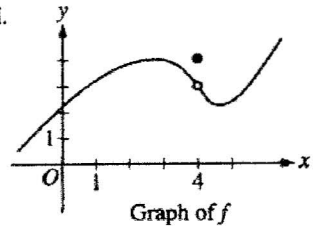
2.

The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1} \sin\left(f(x)\right)$ is

a. 0.909
b. 0.841
c. 0.141
d. -0.416
e. nonexistent

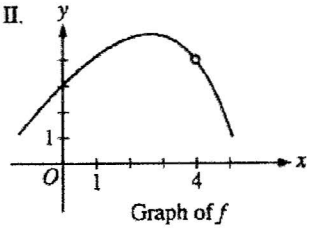
3. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I.



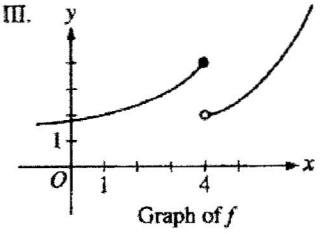
Graph of f

II.



Graph of f

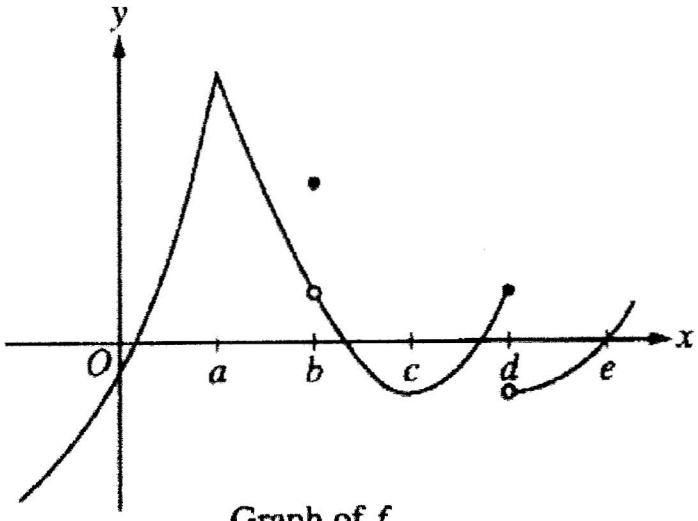
III.



Graph of f

a. I only
b. II only
c. III only
d. I and II only
e. I and III only

Name: _____ **ID: A**



Graph of f

4.

The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

a. a
b. b
c. c
d. d
e. e

5. $\lim_{x \to \infty} \dfrac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

a. 4
b. 1
c. $\dfrac{1}{4}$
d. 0
e. -1

6. If $a \neq 0$, then $\lim_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

a. $\dfrac{1}{a^2}$
b. $\dfrac{1}{2a^2}$
c. $\dfrac{1}{6a^2}$
d. 0
e. nonexistent

4

Name: _____ **ID: A**

7. Let f be the function given by $f(x) = \dfrac{(x-1)\left(x^2-4\right)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

   a. None
   b. 1 only
   c. 2 only
   d. 4 only
   e. 1 and 4 only

8. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ the $\lim\limits_{x \to 2} f(x)$ is

   a. $\ln 2$
   b. $\ln 8$
   c. $\ln 16$
   d. 4
   e. nonexistent

9.
$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

   Let f be the function given above. Which of the following statements are true about f?

   I. $\lim\limits_{x \to 3} f(x)$ exists
   II. f is continuous at $x = 3$.
   III. f is differentiable at $x = 3$.

   a. None
   b. I only
   c. II only
   d. I and II only
   e. I, II, and III

10. Let f be a function such that $\lim\limits_{h \to 0} \dfrac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

   I. f is continuous at x = 2.
   II. f is differentiable at x = 2.
   III. The derivative of f is continuous at x = 2.

   a. I only
   b. II only
   c. I and II only
   d. I and III only
   e. II and III only

5

1998 AP Calculus AB Free-Response Questions Q 2

2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

2003 AB Q6

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

   (a) Is f continuous at $x = 3$? Explain why or why not.
   (b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.
   (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

   where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

GO ON TO THE NEXT PAGE

1998 Calculus AB Scoring Guidelines

2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim_{x \to -\infty} 2xe^{2x} = 0$

$\lim_{x \to \infty} 2xe^{2x} = \infty$ or DNE

2 $\begin{cases} 1: & 0 \text{ as } x \to -\infty \\ 1: & \infty \text{ or DNE as } x \to \infty \end{cases}$

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and $f'(x) > 0$ for all $x > -1/2$

–or–

(ii) $f'(x)$ $-$ $+$ ($-1/2$)

and $x = -1/2$ is the only critical number

3 $\begin{cases} 1: & \text{solves } f'(x) = 0 \\ 1: & \text{evaluates } f \text{ at student's critical point} \\ & 0/1 \text{ if not local minimum from student's derivative} \\ 1: & \text{justifies absolute minimum value} \\ & 0/1 \text{ for a local argument} \\ & 0/1 \text{ without explicit symbolic derivative} \end{cases}$

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

3 $\begin{cases} 1: & \text{sets } y' = be^{bx}(1 + bx) = 0 \\ 1: & \text{solves student's } y' = 0 \\ 1: & \text{evaluates } y \text{ at a critical number and gets a value independent of } b \end{cases}$

Note: 0/3 if only considering specific values of b

AP® CALCULUS AB
2003 SCORING GUIDELINES

Question 6

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

(a) f is continuous at $x = 3$ because
$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 2.$
Therefore, $\lim_{x \to 3} f(x) = 2 = f(3).$

$2: \begin{cases} 1: \text{answers "yes" and equates the values of the left- and right-hand limits} \\ 1: \text{explanation involving limits} \end{cases}$

(b)
$$\begin{aligned} \int_0^5 f(x)\,dx &= \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx \\ &= \frac{2}{3}(x+1)^{3/2}\Big|_0^3 + \left(5x - \frac{1}{2}x^2\right)\Big|_3^5 \\ &= \left(\frac{16}{3} - \frac{2}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right) = \frac{20}{3} \end{aligned}$$

Average value: $\frac{1}{5}\int_0^5 f(x)\,dx = \frac{4}{3}$

$4: \begin{cases} 1: k\int_0^3 f(x)\,dx + k\int_3^5 f(x)\,dx \\ \quad (\text{where } k \ne 0) \\ 1: \text{antiderivative of } \sqrt{x+1} \\ 1: \text{antiderivative of } 5 - x \\ 1: \text{evaluation and answer} \end{cases}$

(c) Since g is continuous at $x = 3$, $2k = 3m + 2$.

$$g'(x) = \begin{cases} \dfrac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$\lim_{x \to 3^-} g'(x) = \frac{k}{4}$ and $\lim_{x \to 3^+} g'(x) = m$

Since these two limits exist and g is differentiable at $x = 3$, the two limits are equal. Thus $\frac{k}{4} = m$.

$8m = 3m + 2$; $m = \frac{2}{5}$ and $k = \frac{8}{5}$

$3: \begin{cases} 1: 2k = 3m + 2 \\ 1: \frac{k}{4} = m \\ 1: \text{values for } k \text{ and } m \end{cases}$