The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is true?

a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$
b. $\lim_{x \to a} f(x) = 2$
c. $\lim_{x \to b} f(x) = 2$
d. $\lim_{x \to b} f(x) = 1$
e. $\lim_{x \to a} f(x)$ does not exist
The graph of the function $f$ is shown in the figure above. Which of the following statements about $f$ is true?

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c. $\lim_{x \to b} f(x) = 2$

d. $\lim_{x \to b} f(x) = 1$

e. $\lim_{x \to a} f(x)$ does not exist
2. The graph of the function $f$ is shown in the figure above. The value of $\lim_{x \to 1} \sin(f(x))$ is

a. 0.909  
b. 0.841  
c. 0.141  
d. -0.416  
e. nonexistent

3. For which of the following does $\lim_{x \to 4} f(x)$ exist?

I. 

![Graph of f](image1)

II. 

![Graph of f](image2)

III. 

![Graph of f](image3)

a. I only  
b. II only  
c. III only  
d. I and II only  
e. I and III only
4. The graph of a function \( f \) is shown above. At which value of \( x \) is \( f \) continuous, but not differentiable?

a. \( a \)
b. \( b \)
c. \( c \)
d. \( d \)
e. \( e \)

5. \( \lim_{x \to 0} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} = \)

a. 4
b. 1
c. \( \frac{1}{4} \)
d. 0
e. -1

6. If \( a \neq 0 \), then \( \lim_{x \to a} \frac{x^2 - a^2}{x^2 - a^4} \) is

a. \( \frac{1}{a^2} \)
b. \( \frac{1}{2a^2} \)
c. \( \frac{1}{6a^2} \)
d. 0
e. nonexistent
7. Let \( f \) be the function given by \( f(x) = \frac{(x-1)(x^2-4)}{x^2-a} \). For what positive values of \( a \) is \( f \) continuous for all real numbers \( x \)?
   a. None  
   b. 1 only  
   c. 2 only  
   d. 4 only  
   e. 1 and 4 only

8. If \( f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x < 4, \end{cases} \) the \( \lim_{x \to 2} f(x) \) is
   a. \( \ln 2 \)  
   b. \( \ln 8 \)  
   c. \( \ln 16 \)  
   d. 4  
   e. nonexistent

9. \( f(x) = \begin{cases} x+2 & \text{if } x \leq 3 \\ 4x-7 & \text{if } x > 3 \end{cases} \)
   Let \( f \) be the function given above. Which of the following statements are true about \( f \)?
   I. \( \lim_{x \to 3} f(x) \) exists  
   II. \( f \) is continuous at \( x = 3 \)  
   III. \( f \) is differentiable at \( x = 3 \)
   a. None  
   b. I only  
   c. II only  
   d. I and II only  
   e. I, II, and III

10. Let \( f \) be a function such that \( \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5 \). Which of the following must be true?
   I. \( f \) is continuous at \( x = 2 \)  
   II. \( f \) is differentiable at \( x = 2 \)  
   III. The derivative of \( f \) is continuous at \( x = 2 \)
   a. I only  
   b. II only  
   c. I and II only  
   d. I and III only  
   e. II and III only
2. Let $f$ be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of $f$. Justify that your answer is an absolute minimum.

(c) What is the range of $f$?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where $b$ is a nonzero constant. Show that the absolute minimum value of $bxe^{bx}$ is the same for all nonzero values of $b$.

6. Let $f$ be the function defined by

$$f(x) = \begin{cases} \sqrt{x} + 1 & \text{for } 0 \leq x \leq 3 \\ 5 - x & \text{for } 3 < x \leq 5. \end{cases}$$

(a) Is $f$ continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.

(c) Suppose the function $g$ is defined by

$$g(x) = \begin{cases} k\sqrt{x} + 1 & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5, \end{cases}$$

where $k$ and $m$ are constants. If $g$ is differentiable at $x = 3$, what are the values of $k$ and $m$?
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2. Let $f$ be the function given by $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

(b) Find the absolute minimum value of $f$. Justify that your answer is an absolute minimum.

(c) What is the range of $f$?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where $b$ is a nonzero constant. Show that the absolute minimum value of $bxe^{bx}$ is the same for all nonzero values of $b$.

| (a) $\lim_{x \to -\infty} 2xe^{2x} = 0$ | 2 | 1: $0$ as $x \to -\infty$  
| $\lim_{x \to \infty} 2xe^{2x} = \infty$ or DNE | \{ | 1: $\infty$ or DNE as $x \to \infty$ |
| (b) $f'(x) = 2e^{2x} + 2x \cdot 2e^{2x} = 2e^{2x}(1 + 2x) = 0$ | \{ | 1: solves $f'(x) = 0$ |
| if $x = -1/2$ | \{ | 1: evaluates $f$ at student’s critical point |
| $f(-1/2) = -1/e$ or $-0.368$ or $-0.367$ | \{ | 0/1 if not local minimum from student’s derivative |
| $-1/e$ is an absolute minimum value because: | \{ | 1: justifies absolute minimum value |
| (i) $f'(x) < 0$ for all $x < -1/2$ and | \{ | 0/1 for a local argument |
| $f'(x) > 0$ for all $x > -1/2$ | \{ | 0/1 without explicit symbolic derivative |
| -or- | | Note: 0/3 if no absolute minimum based on student’s derivative |
| (ii) $\frac{f'(x)}{-1/2} = +$ | | 1: answer |
| and $x = -1/2$ is the only critical number | | Note: must include the left-hand endpoint; exclude the right-hand “endpoint” |
| (c) Range of $f = [-1/e, \infty)$ | \{ | 1: sets $y' = be^{bx}(1 + bx) = 0$ |
| or $[-0.367, \infty)$ | \{ | 1: solves student’s $y' = 0$ |
| or $[-0.368, \infty)$ | \{ | 1: evaluates $y$ at a critical number |
| (d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$ | \{ | and gets a value independent of $b$ |
| if $x = -1/b$ | \{ | Note: 0/3 if only considering specific values of $b$ |
| At $x = -1/b$, $y = -1/e$ | | 3 |
| $y$ has an absolute minimum value of $-1/e$ for all nonzero $b$ | | 1: |

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Let $f$ be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5 - x & \text{for } 3 < x \leq 5. \end{cases}$$

(a) Is $f$ continuous at $x = 3$? Explain why or why not.
(b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.
(c) Suppose the function $g$ is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5, \end{cases}$$

where $k$ and $m$ are constants. If $g$ is differentiable at $x = 3$, what are the values of $k$ and $m$?

(a) $f$ is continuous at $x = 3$ because

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 2.$$ 

Therefore, $\lim_{x \to 3} f(x) = 2 = f(3)$.

(b) $\int_0^5 f(x) \, dx = \int_0^3 f(x) \, dx + \int_3^5 f(x) \, dx$

$$= \left[ \frac{2}{3} (x + 1)^{3/2} + \left( 5x - \frac{1}{2} x^2 \right) \right]_3^5
= \left( \frac{16}{3} - \frac{2}{3} \right) + \left( \frac{25}{2} - \frac{21}{2} \right) = \frac{20}{3}$$

Average value: $\frac{1}{5} \int_0^5 f(x) \, dx = \frac{4}{5}$

(c) Since $g$ is continuous at $x = 3$, $2k = 3m + 2$.

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ \frac{m}{x} & \text{for } 3 < x < 5 \end{cases}$$

$$\lim_{x \to 3^-} g'(x) = \frac{k}{4} \text{ and } \lim_{x \to 3^+} g'(x) = m$$

Since these two limits exist and $g$ is differentiable at $x = 3$, the two limits are equal. Thus $\frac{k}{4} = m$.

$$8m = 3m + 2; \quad m = \frac{2}{5} \text{ and } k = \frac{8}{5}$$