QUIZ 30 POINTS EXAM 61 POINTS

Section 4.4-4.6 & 7.1-7-2 Exam

Name: ______________________________ Date: ____________ Period: ___________

Show all work neatly and clearly. You will be graded on your completeness and ability to derive the correct answer.

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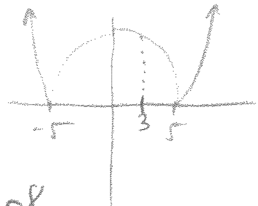
1. $\int_3^8 9|x^2-25|\,dx$



$9\int_3^5 (25-x^2)\,dx + 9\int_5^8 (x^2-25)\,dx$

$9\left[25x-\frac{x^3}{3}\right]_3^5 + 9\left[\frac{x^3}{3}-25x\right]_5^8$

$156 \quad + \quad 486$

$\boxed{642}$

5

2. $\int u^4(2+u^5)^5\,du$

$=\frac{1}{5}\int 5u^4(2+u^5)^5\,du$

$=\frac{1}{5}\cdot\frac{(2+u^5)^6}{6}$

$\boxed{\frac{(2+u^5)^6}{30}+c}$

3

3. $\int 5^x \ln e\,dx$

$\ln e\int 5^x\,dx$

$\boxed{1\cdot\frac{5^x}{\ln 5}+c}$

2

4. $\int 1-\csc x\cot x\,dx$

$\boxed{x+\csc x+c}$

7

5. $\int x^2\sqrt{1-x}\,dx$

$u=1-x$

$du=-dx$

$-du=dx$

$x=1-u$

$-\int(1-u)^2\sqrt{u}\,du$

$=-\int(1-2u+u^2)u^{1/2}\,du$

$=-\int(u^{1/2}-2u^{3/2}+u^{5/2})\,du$

$-\left[\frac{2}{3}u^{3/2}-2\cdot\frac{2}{5}u^{5/2}+\frac{2}{7}u^{7/2}+c\right]$

$\boxed{=-\frac{2}{3}(1-x)^{3/2}+\frac{4}{5}(1-x)^{5/2}-\frac{2}{7}(1-x)^{7/2}+c}$

2

6. $\int -\csc^2 x\sqrt{\cot x}\,dx$

$\boxed{\frac{2(\cot x)^{3/2}}{3}+c}$

24

3

7. $\int \frac{x}{x^2+4}\,dx$

$= \frac{1}{2}\int \frac{2x}{x^2+4}\,dx = \boxed{\frac{\ln(x^2+4)}{2}+c}$

3

8. $\int \frac{\cos x}{\sin^5 x}\,dx$

$= -\int \frac{-\cos x}{\sin^5 x} = \boxed{\frac{(\sin x)^{-4}}{4}+c}$

2

Find y'

9. $y = \int_0^{\sin x} \sqrt{t}\ \mathrm{dt}$

$y' = \cos x \sqrt{\sin x}$

2

10. $y = \int_0^{3x} \sqrt{1+t^3}\ \mathrm{dt}$

$y' = 3\sqrt{1+27x^3}$

11. Find the average value of the function over the given interval and all values x in the interval for which the function equals its average value

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$f(x) = \frac{x^2+5}{x^2},\quad 1 \le x \le 3$

$f(c) = \frac{1}{3-1}\int_1^3 (1+5x^{-2})\,dx$

$= \frac{1}{2}\left[x + \frac{5x^{-1}}{-1}\right]_1^3 =$

$\frac{1}{2}\left[\frac{4}{3}+4\right]$

$\frac{1}{2}\cdot\frac{16}{3} = \boxed{\frac{8}{3}}$

$f(c) = \frac{8}{3}$

$\frac{c^2+5}{c^2} = \frac{8}{3}$

$1 + \frac{5}{c^2} = \frac{8}{3}$

$\frac{5}{c^2} = \frac{5}{3}$

$5c^2 = 15$

$c^2 = 3$

$c = \pm\sqrt{3}$

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12. Sketch and shade the enclosed figure. Find the area of the region bounded by the graphs of the equations.

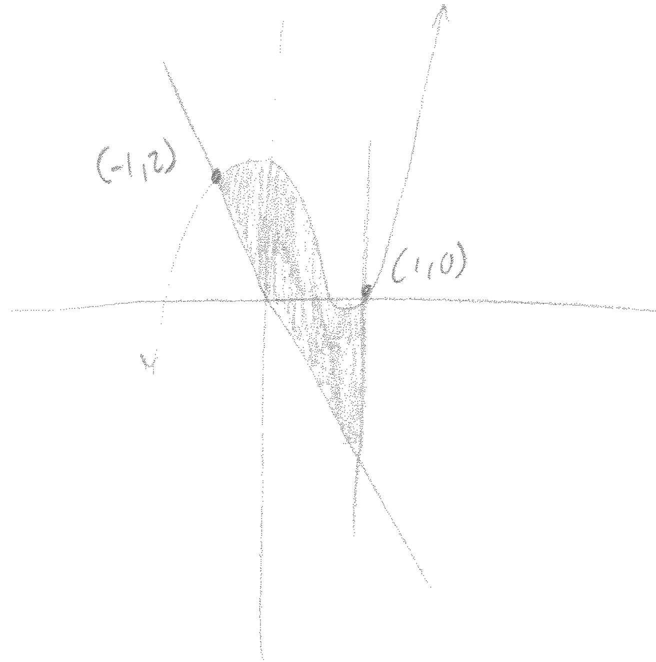
$f(x) = x^3 - 2x + 1, \; g(x) = -2x, \quad x = 1$

6

$$A = \int_{-1}^{1} \left(x^3 - 2x + 1 - (-2x)\right)\, dx$$

$$= \int_{-1}^{1} (x^3 + 1)\, dx$$

$$= x^4 + x \Big]_{-1}^{1} = \boxed{2}$$



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13. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations $y = \sin x \;\; y = 0, 0 \le x \le \frac{\pi}{2}$ about the line y = 10

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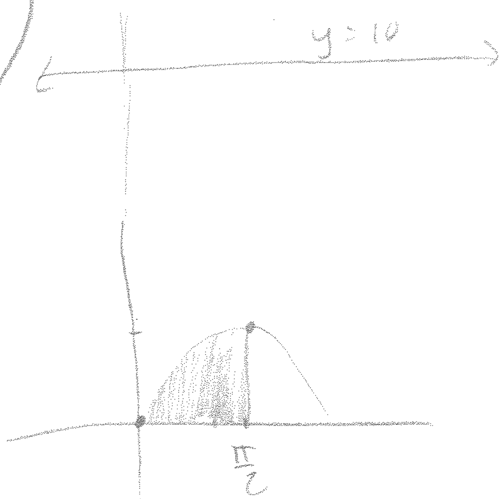
$R(x) = 10$

$r(x) = (10 - \sin x)^2$



$$V = \pi \int_0^{\frac{\pi}{2}} [10^2 - (10 - \sin x)^2] \, dx$$

$$V = \pi \int_0^{\pi/2} 100 - (100 - 20\sin x + \sin^2 x) \, dx$$

$$V = \pi \int_0^{\pi/2} (+20\sin x - \sin^2 x) \, dx \approx 60.3645$$

NO RULE

***BONUS Problem:** Write the integral that would give the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 9$ and the cross section perpendicular to the x-axis are squares.

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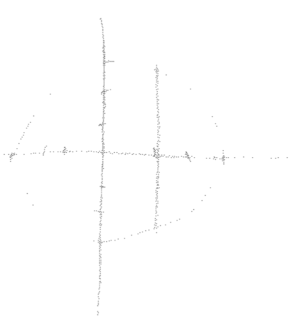
$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$

SQUARE

$2\sqrt{9-x^2}$



$$V = \int_{-3}^{3} (2\sqrt{9 - x^2})^2 \, dx$$

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