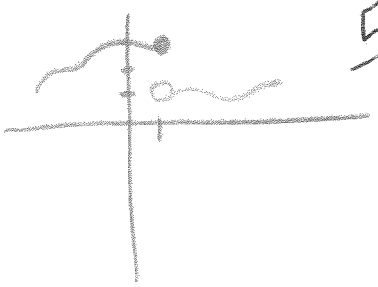
84 points

Trigonometry/Pre-Calculus Exam 9

Name: _____ Date: _____ Period: _____

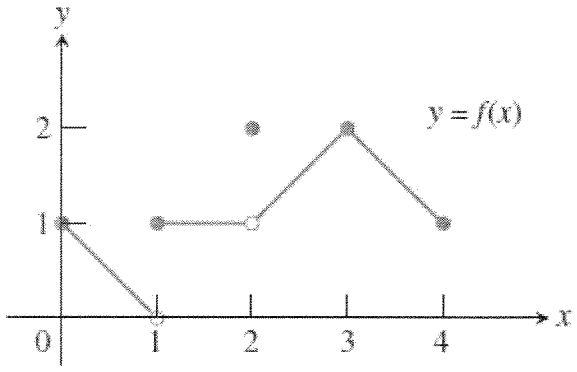
Show all your work neatly and clearly. Calculators are allowed on the exam; however, unless otherwise stated show all work when deriving answers.

1. If the limit of $\lim_{x\to 1^-} f(x) = 3$ exists and $f(1) = 3$, can you conclude anything about $\lim_{x\to 1} f(x)$? Explain your reasoning.



5 WE MUST HAVE THE RIGHT & LEFT LIMIT TO BE EQUIVALENT TO FIND A LIMIT. NOT ENOUGH INFORMATION TO DRAW ANY CONCLUSIONS.

2. Given the following graph, compute each of the following



(a) $f(0) =$ 1

(b) $\lim_{x\to -4^-} f(x) =$ 1

(c) $\lim_{x\to -4^+} f(x) =$ DNE

(d) $f(4) =$ 1

(e) $f(1) =$ 1

(f) $\lim_{x\to 1^-} f(x) =$ 0

(g) $\lim_{x\to 1^+} f(x) =$ 1

(h) $\lim_{x\to 1} f(x) =$ DNE

(i) $f(2) =$ 2

(j) $f(3) =$ 2

(j) $\lim_{x\to 2^-} f(x) =$ 1

(k) $\lim_{x\to 2^+} f(x) =$ 1

(l) $\lim_{x\to 2} f(x) =$ 1

$$f(x) = \begin{cases} -x+1, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ x-1, & 2 < x \le 3 \\ -x+5, & 3 < x \le 4. \end{cases}$$

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Questions 3-4, complete the table and use the result to estimate the limit.

3. $\lim_{x\to 4} \dfrac{[x/(x+1)] - (4/5)}{x-4} \approx$ .04 OR 1/25 (2)

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	.04082	.04008	.04001	.03996	.03992	.03913

3 — ½ PT EA

4. $\lim_{x\to 0} \dfrac{1}{x^2} \approx$ DNE, UNBOUNDED (2)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	100	10,000	1,000,000	1,000,000	10,000	100

3 — ½ PT EA

Questions 5-8, find the limit using mathematical techniques. Do not use a graphing calculator to derive your answer.

5. $\lim_{x\to 2} \dfrac{2-x}{x^2-4}$ (3)

$\lim_{x\to 2} \dfrac{-1(x-2)}{(x+2)(x-2)}$

$\lim_{x\to 2} \dfrac{-1}{x+2} = \dfrac{-1}{4}$

6. $\lim_{x\to 4} \dfrac{x^2+x-6}{x^2-9} = \dfrac{4^2+4-6}{4^2-9} = 2$ OR (3)

$\lim_{x\to 4} \dfrac{(x+3)(x-2)}{(x+3)(x-3)}$

$\lim_{x\to 4} \dfrac{x-2}{x-3} = \dfrac{4-2}{4-3} = 2$

7. $\lim_{x\to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x}$

$4(1) - \frac{1}{4}(x+4)$

$4(x+4) \quad 4(x+4)$

$\lim_{x\to 0} \dfrac{\frac{4-x-4}{4(x+4)}}{x}$ 5

$\lim_{x\to 0} \dfrac{-x}{4x(x+4)}$

$\lim_{x\to 0} \dfrac{-1}{4(x+4)} = \boxed{\frac{-1}{16}}$

8. $\lim_{x\to -1} \dfrac{x^3+1}{x+1}$

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$\lim_{x\to -1} \dfrac{(x+1)(x^2-x+1)}{(x+1)}$

$\lim_{x\to -1} x^2-x+1 = \boxed{3}$

9. Given $f(x) = 3x^2 + x$

(a) Find $f'(x)$ using the limit process

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$f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$\lim_{h\to 0} \dfrac{3(x+h)^2+(x+h)-(3x^2+x)}{h}$

$\lim_{h\to 0} \dfrac{3(x^2+2xh+h^2)+x+h-3x^2-x}{h}$

$\lim_{h\to 0} \dfrac{3x^2+6xh+3h^2+x+h-3x^2-x}{h}$

$\lim_{h\to 0} \dfrac{h(6x+3h+1)}{h} = \boxed{6x+1}$

(b) Find $f'(-5)$ and $f'(2)$

$f'(x) = 6x + 1$

$f'(-5) = 6(-5) + 1 = -29$ 2

$f'(2) = 6(2) + 1 = 13$ 2

(c) Determine any points on the graph of f at which the tangent line is horizontal

$f'(x) = 0$

$6x + 1 = 0$ 3

$x = -1/6$

10. Describe the process of finding the area of a region bounded by the graph of a nonnegative, continuous function f, the x-axis, and the vertical lines $x = a$ and $x = b$.

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11. Use the limit process to find the area of the region between the graph of the function and the x-axis over the specified interval.

$f(x) = 3x^3$; [0,1]

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \left(\frac{b-a}{n}\right)i\right)\left(\frac{b-a}{n}\right)$

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$w = \frac{1-0}{n} = \frac{1}{n}$ $\quad h = f\left(\frac{i}{n}\right) = 3\left(\frac{i}{n}\right)^3 = 3\frac{i^3}{n^3}$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3i^3}{n^3}\right)\left(\frac{1}{n}\right)$

$A = \int_0^1 3x^3\,dx = \frac{3x^4}{4}\Big]_0^1 = \frac{3}{4}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3i^3}{n^4}$

$\lim_{n\to\infty} \frac{3}{n^4} \sum_{i=1}^{n} i^3$

$\lim_{n\to\infty} \left(\frac{3}{n^4} \cdot \frac{n^2(n+1)^2}{4}\right)$

$\lim_{n\to\infty} \frac{3(n^2+2n+1)}{4n^2} = \boxed{\frac{3}{4}}$

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12. Sketch the graph of a function that has at least one positive and at least one negative derivative.

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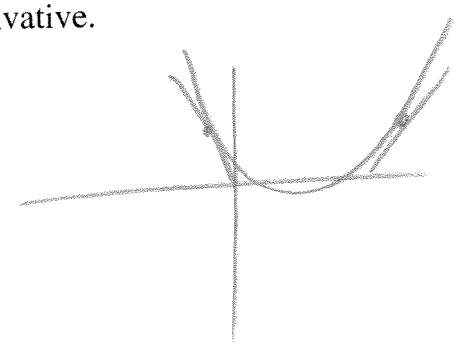

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