84 pts

Trigonometry/Pre-Calculus Exam 9

Name: _____ Date: _____ Period: _____

Show all your work neatly and clearly. Calculators are allowed on the exam: however, unless otherwise stated show all work when deriving answers.

1. If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.

NO. WE CAN TAKE LIMITS @ UNDEFINED VALUES.

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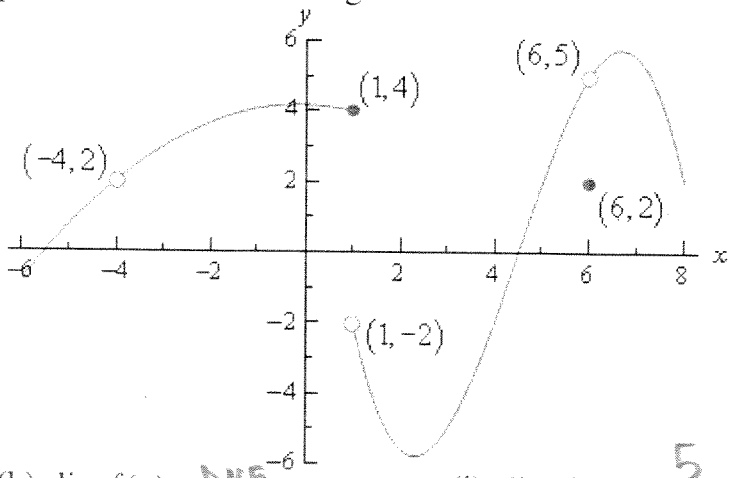
2. Given the following graph, compute each of the following



(a) $f(-4) =$ UNDEF.

(b) $\lim_{x\to -4^-} f(x) =$ 2

(c) $\lim_{x\to -4^+} f(x) =$ 2

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(d) $\lim_{x\to -4} f(x) =$ 2

(e) $f(1) =$ 4

(f) $\lim_{x\to 1^-} f(x) =$ 4

(g) $\lim_{x\to 1^-} f(x) =$ -2

(h) $\lim_{x\to 1} f(x) =$ DNE

(i) $f(0) =$ 4.1

(j) $f(6) =$ 2

(j) $\lim_{x\to 6^-} f(x) =$ 5

(k) $\lim_{x\to 6^+} f(x) =$ 5

(l) $\lim_{x\to 6} f(x) =$ 5

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Questions 3-4, complete the table and use the result to estimate the limit.

3. $\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3} \approx$ 2 $-\frac{1}{4}$ OR $-.25$

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x	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$f(x)$	$-.2485$	$-.2498$	$-.25$	$-.25$	$-.2502$	$-.2516$

$\frac{1}{2}$ PT EA

4. $\lim_{x \to 0} \frac{\cos x - 1}{x} \approx$ 2 0

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x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	$.04996$	$.005$	$.0005$	$-.0005$	$-.005$	$-.05$

$\frac{1}{2}$ PT EA

Questions 5-8, find the limit using mathematical techniques. Do not use a graphing calculator to derive your answer.

5. $\lim_{x \to 3} \frac{x^2+x-6}{x^2-9}$ 3

$\lim_{x \to 3} \frac{(x+3)(x-2)}{(x+3)(x-3)}$

$\lim_{x \to 3} \frac{x-2}{x-3} =$ DNE

6. $\lim_{x \to 4} \frac{x^2-5x+4}{x^2-2x-8}$ 3

$\lim_{x \to 4} \frac{(x-4)(x-1)}{(x-4)(x+2)}$

$\lim_{x \to 4} \frac{x-1}{x+2} = \frac{3}{6} = \frac{1}{2}$

7. $\lim_{x\to 0}\dfrac{\frac{3}{3+x}-\frac{1}{3}}{x}$

$\frac{3(1)}{3(3+x)} - \frac{1}{3}\cdot\frac{(3+x)}{(3+x)}$

$\lim_{x\to 0}\dfrac{\frac{3-3-x}{3(3+x)}}{x}$

$\lim_{x\to 0}\dfrac{\frac{-x}{3(3+x)}}{x}$

$\lim_{x\to 0}\dfrac{-1}{9+3x} = \boxed{\frac{-1}{9}}$

8. $\lim_{x\to 2}\dfrac{x^3-8}{x-2}$

$x^3 - 2^3$

$\lim_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{x-2}$

$\lim_{x\to 2} x^2+2x+4 = \boxed{12}$

9. Given $f(x) = x - 3x^2$

(a) Find $f'(x)$ using the limit process

$f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h) - 3(x+h)^2 - (x-3x^2)}{h}$

$\lim_{h\to 0}\dfrac{x+h-3(x^2+2xh+h^2)-x+3x^2}{h}$

$\lim_{h\to 0}\dfrac{x+h-3x^2-6xh-3h^2-x+3x^2}{h}$

$\lim_{h\to 0}\dfrac{h(1-6x-3h)}{h}$

$\lim_{h\to 0} 1-6x-3h = \boxed{1-6x}$

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(b) Find $f'(-1)$ and $f'(4)$

$f'(x) = 1 - 6x$

$f'(-1) = 1 - 6(-1) = 7$ 2

$f'(4) = 1 - 6(4) = -23$ 2

(c) Determine any points on the graph of f at which the tangent line is horizontal

$f'(x) = 0$

$1 - 6x = 0$ 3

$-6x = -1$

$x = 1/6$

10. Describe the process of finding the area of a region bounded by the graph of a nonnegative, continuous function f, the x-axis, and the vertical lines $x = a$ and $x = b$.

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10. Use the limit process to find the area of the region between the graph of the function and the x-axis over the specified interval.

$f(x) = x^3 + 1; [0,1]$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{(b-a)}{n} i\right)\left(\frac{b-a}{n}\right)$$

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$$w = \frac{1-0}{n} = \frac{1}{n}$$

$$H = f\left(\frac{i}{n}\right) = \left(\frac{i}{n}\right)^3 + 1 = \frac{i^3}{n^3} + 1$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^3}{n^3} + 1\right)\left(\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^3}{n^4} + \frac{1}{n}\right)$$

$$\lim_{n\to\infty} \left[\frac{1}{n^4}\sum_{i=1}^{n} i^3 + \frac{1}{n}\sum_{i=1}^{n} 1\right]$$

$$\lim_{n\to\infty} \left(\frac{1}{n^4} \cdot \frac{n^2(n+1)^2}{4} + 1\right)$$

$$\lim_{n\to\infty} \left(\frac{n^2+2n+1}{4n^2} + 1\right)$$

$$\frac{1}{4} + 1 = \boxed{\frac{5}{4}}$$

$$A = \int_0^1 (x^3+1)\,dx$$

$$\frac{x^4}{4} + x\Big]_0^1 = \frac{1}{4} + 1 = \frac{5}{4}$$

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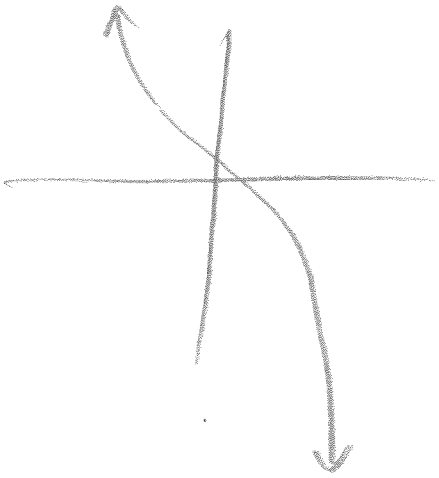
12. Sketch the graph of a function whose derivative is always negative



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