84 POINTS

Trigonometry/Pre-Calculus Exam 9

Name: _____ Date: _____ Period: _____

Show all your work neatly and clearly. Calculators are allowed on the exam; however, unless otherwise stated show all work when deriving answers.

1. If the limit of $\lim_{x\to 1^-} f(x)$ exists and $\lim_{x\to 1^+} f(x)$ exists as well, can you conclude anything about $\lim_{x\to 1} f(x)$? Explain your reasoning.

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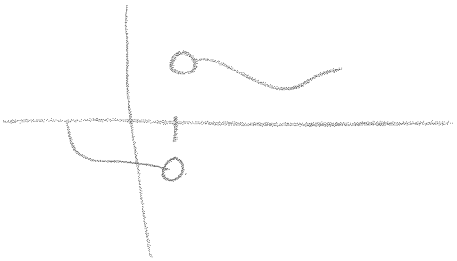
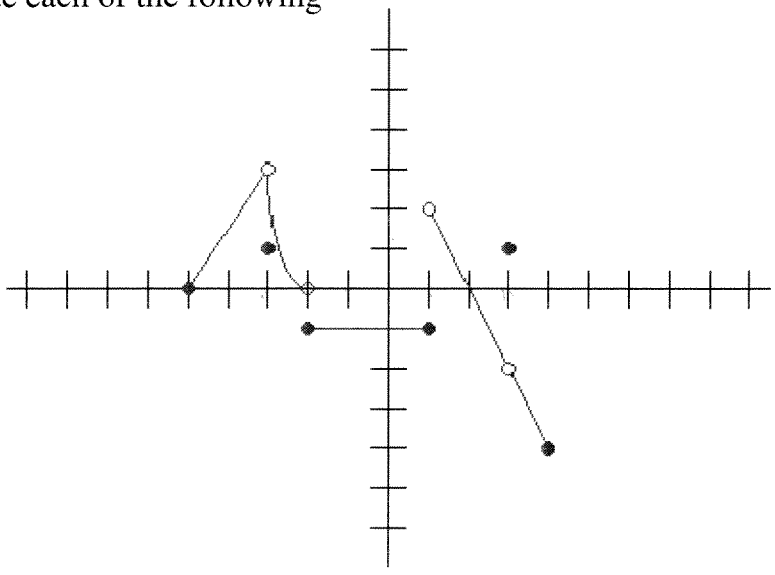


IF A LEFT & RIGHT LIMIT EXIST, THAT DOES NOT IMPLY THEY ARE THE VALUE. SEE FIGURE. NO CONCLUSION

2. Given the following graph, compute each of the following



1 (a) $f(-3) =$ 1

1 (b) $\lim_{x\to -3^-} f(x) =$ 3

1 (c) $\lim_{x\to -3^+} f(x) =$ 3

1 (d) $\lim_{x\to -3} f(x) =$ 3

1 (e) $f(1) =$ -1

1 (f) $\lim_{x\to 1^-} f(x) =$ -1

1 (g) $\lim_{x\to 1^+} f(x) =$ 2

1 (h) $\lim_{x\to 1} f(x) =$ DNE

1 (i) $f(0) =$ -1

1 (j) $f(3) =$ 1

1 (j) $\lim_{x\to 3^-} f(x) =$ -2

1 (k) $\lim_{x\to 3^+} f(x) =$ -2

1 (l) $\lim_{x\to 3} f(x) =$ -2

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Questions 3-4, complete the table and use the result to estimate the limit.

3. $\lim_{x\to 3} \dfrac{[1/(x+1)] - (1/4)}{x-3} \approx$ $-1/16$ OR $-.0625$ 2

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	-0.64	-.0627	-.0625	-.0625	-.0623	-.061

3 ½ PT EA

4. $\lim_{x\to 0} \dfrac{\sin x}{x} \approx$ 1 2

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	.999	.999	1	1	.999	.998

3 ½ PT EA

Questions 5-8, find the limit using mathematical techniques. Do not use a graphing calculator to derive your answer.

5. $\lim_{x\to 2} \dfrac{2-x}{x^2-4}$

$\lim_{x\to 2} \dfrac{-1(x-2)}{(x+2)(x-2)}$ 3

$\lim_{x\to 2} \dfrac{-1}{x+2} = \boxed{-\frac{1}{4}}$

6. $\lim_{x\to 4} \dfrac{x^2+x-6}{x^2-9}$

$\lim_{x\to 4} \dfrac{(x+3)(x-2)}{(x+3)(x-3)}$ 3

$\lim_{x\to 4} \dfrac{x-2}{x-3} = \dfrac{2}{1} = \boxed{2}$

7. $\lim_{x\to 0}\dfrac{\frac{1}{x+4}-\frac{1}{4}}{x}$

$\frac{4(1)}{4(x+4)} - \frac{1}{4}\cdot\frac{(x+4)}{(x+4)}$

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$\lim_{x\to 0}\dfrac{\frac{4-x-4}{4(x+4)}}{x} = \lim_{x\to 0}\dfrac{\frac{-x}{4(x+4)}}{x}$

$= \lim_{x\to 0}\dfrac{-1}{4(x+4)}$

$= \boxed{\frac{-1}{16}}$

8. $\lim_{x\to -1}\dfrac{x^3+1}{x+1} = \lim_{x\to -1}\dfrac{(x+1)(x^2-x+1)}{(x+1)}$

3

$= \lim_{x\to -1} x^2-x+1$

$= (-1)^2-(-1)+1$

$= \boxed{3}$

9. Given $f(x) = -2x^2 + x - 1$

(a) Find $f'(x)$ using the limit process

$f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\dfrac{-2(x+h)^2+(x+h)-1-(-2x^2+x-1)}{h}$

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$= \lim_{h\to 0}\dfrac{-2(x^2+2xh+h^2)+x+h-1+2x^2-x+1}{h}$

$= \lim_{h\to 0}\dfrac{-2x^2-4xh-2h^2+x+h-1+2x^2-x+1}{h}$

$= \lim_{h\to 0}\dfrac{h(-4x-2h+1)}{h} = \lim_{h\to 0} -4x-2h+1 = \boxed{-4x+1}$

(b) Find $f'(-5)$ and $f'(2)$

$f'(x) = -4x+1$

2 $f'(-5) = -4(-5)+1 = 21$

2 $f'(2) = -4(2)+1 = -7$

(c) Determine any points on the graph of f at which the tangent line is horizontal

3 $f'(x) = 0$

$-4x+1 = 0$

$-4x = -1$

$x = 1/4$

10. Describe the process of finding the area of a region bounded by the graph of a nonnegative, continuous function f, the x-axis, and the vertical lines $x = a$ and $x = b$.

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11. Use the limit process to find the area of the region between the graph of the function and the x-axis over the specified interval.

$f(x) = 2x^2 + 1;\ [0,1]$ (a, b)

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$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \left(\frac{b-a}{n}\right)i\right)\left(\frac{b-a}{n}\right)$

$w = \frac{1-0}{n} = \frac{1}{n}$ 3

$H: f\left(\frac{i}{n}\right) = 2\left(\frac{i}{n}\right)^2 + 1 = \frac{2i^2}{n^2} + 1$ 3

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i^2}{n^2} + 1\right)\left(\frac{1}{n}\right)$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i^2}{n^3} + \frac{1}{n}\right)$

$\lim_{n\to\infty} \left(\frac{2}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n}\sum_{i=1}^{n} 1\right)$

$\lim_{n\to\infty} \left(\frac{2}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{n}\cdot n\right)$ 3

$\lim_{n\to\infty}\left(\frac{2n^2+3n+1}{3n^2} + 1\right)$

$\frac{2}{3} + 1 = \boxed{\frac{5}{3}}$

$\int_0^1 (2x^2+1)\,dx$

$\left[\frac{2x^3}{3} + x\right]_0^1 = \frac{5}{3}$

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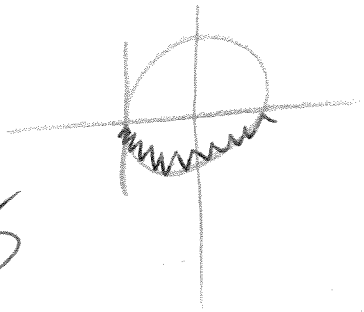
12. Sketch the graph of a function where a particular point does not have a derivative. Explain why the derivative does not exist.

VERTICAL LINES HAVE UNDEFINED SLOPES



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