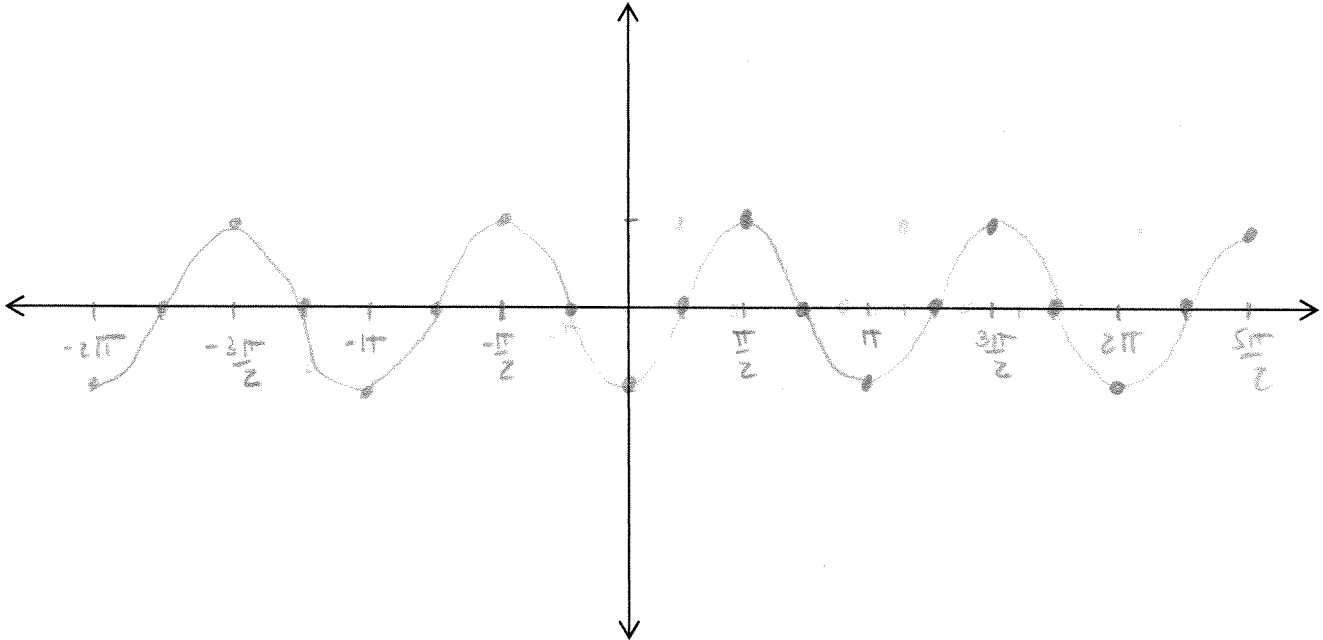Trigonometry/Pre-Calculus Exam 6 (Sections 4.5-4.7)

Name: _____ Date: _____ Period: _____

**Show all work** neatly and clearly in order to receive full credit.   SL packet is allowed, however, calculators are not allowed.

1.   Sketch the graph (include two full periods) of $f(x) = \cos(2x - \pi)$ and find the following (5 points):



(i) Maximum (1 pt)          (ii) Minimum (1 Pt)          (iii) Amplitude (1 pt)          (iv) Period (1 pt)

$1$                                    $-1$                                    $1$                                    $\pi$

(v) Zeros (2 pts)

$x = \dfrac{\pi}{4} + \dfrac{\pi}{2}n, \ n \in \mathbb{Z}$

y-INTERCEPT (1 pt)

$(0, -1)$

(v)   Complete the table by giving 5 points on the graph of $f(x)$ (5 points)   $2x - \pi = \theta$

$2x = \theta + \pi$

$x = \dfrac{\theta + \pi}{2}$

| $x$ | $\frac{-3\pi}{4}$ | $\frac{-\pi}{2}$ | $\frac{\pi}{4}$ | $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ | $\frac{9\pi}{4}$ | $\frac{5\pi}{2}$ | $\frac{11\pi}{4}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $0$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ | $-1$ | $0$ | $1$ | $0$ |

| $\theta$ | $0$ | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ |
|---|---|---|---|---|---|
| $\cos\theta$ | $1$ | $0$ | $-1$ | $0$ | $1$ |

| $x$ | $\pi/2$ | $3\pi/4$ | $\pi$ | $5\pi/4$ | $3\pi/2$ |
|---|---|---|---|---|---|
| $\cos(2x-\pi)$ | $1$ | $0$ | $-1$ | $0$ | $1$ |

$\dfrac{3\pi}{2} - \dfrac{\pi}{2}$

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**IB SL Y1 Unit 3 Exam 5 Page 1**

$\dfrac{3\pi}{4} - \dfrac{\pi}{2} \cdot 2 = \dfrac{\pi}{4}$

2. Sketch the graph (including two full periods) of $f(x) = .5\sin(x+\pi)+3$ and find the following (5 points):
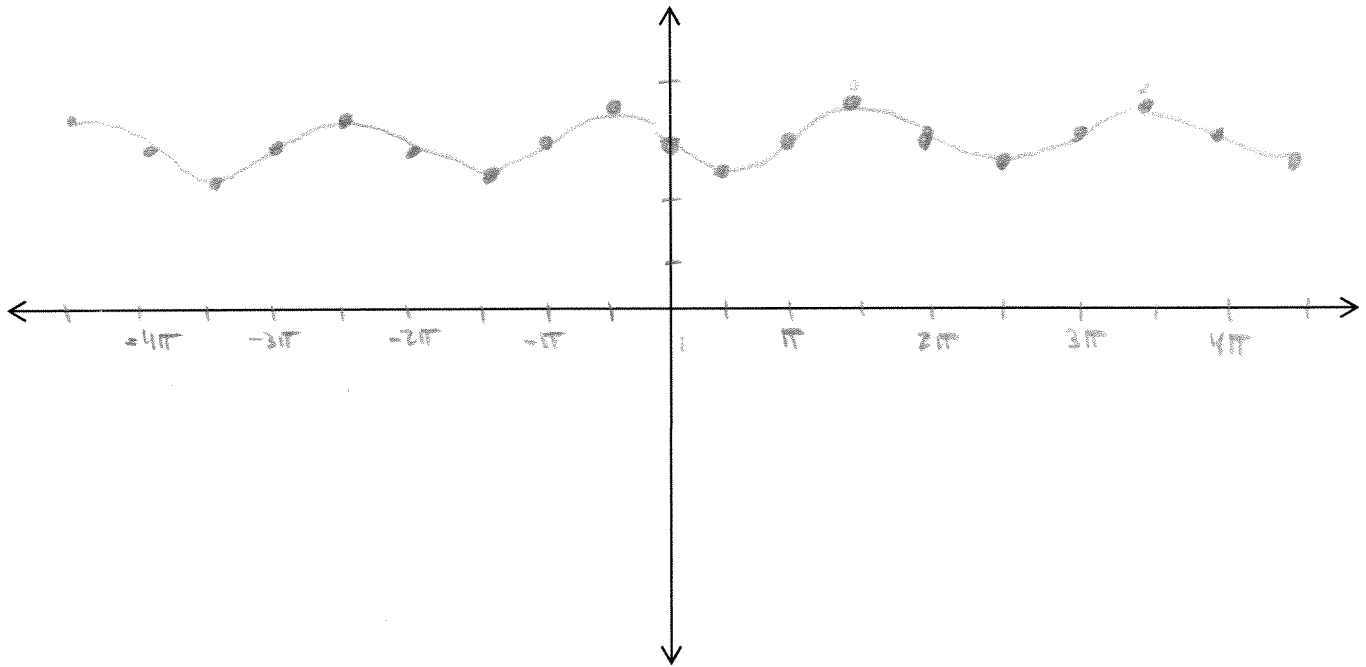


(i) Maximum (1 pt)

$3.5$

(ii) Minimum (1 Pt)

$2.5$

(iii) Amplitude (1 pt)

$\frac{1}{2}$

(iv) Period (1 pt)

$2\pi$

(v) ~~Zeros (2 pts)~~ (1)
Y-INTERCEPT

$(0,3)$

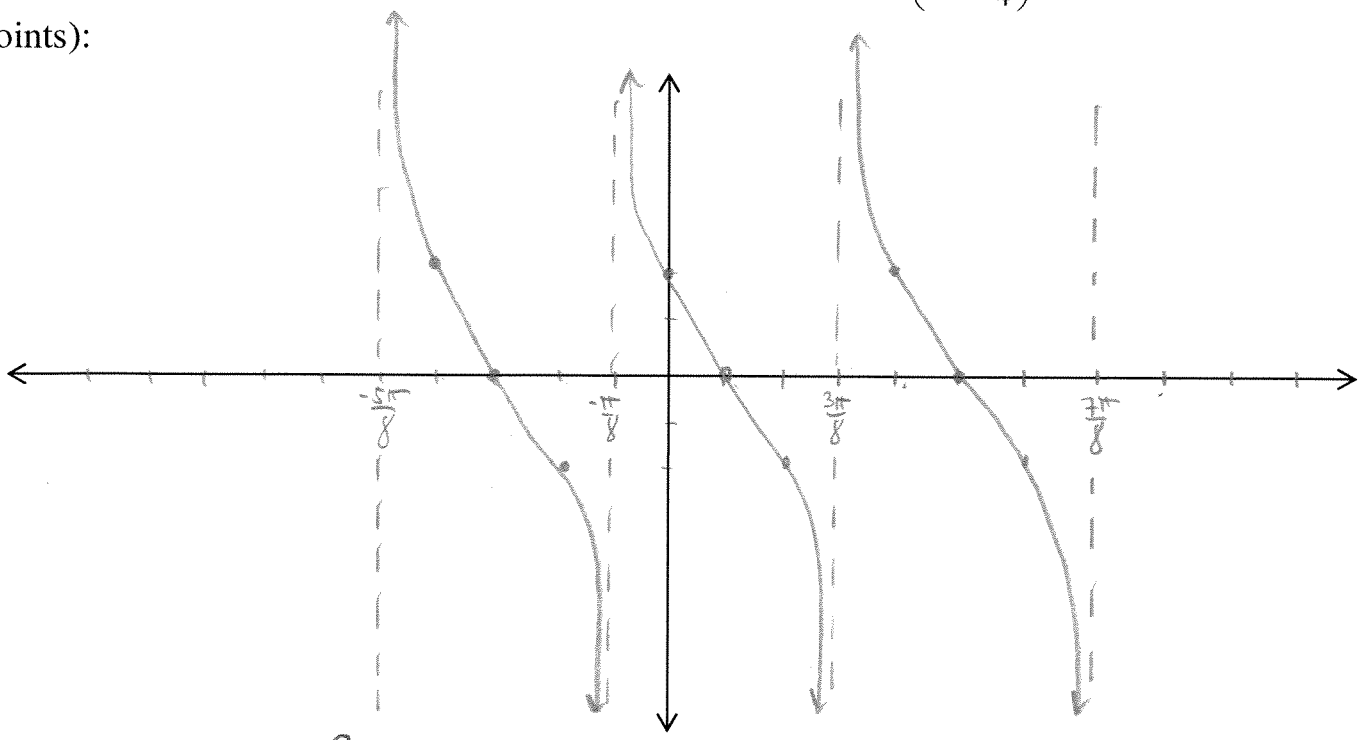(v) Complete the table by giving 5 points on the graph of f(x) (5 points).

$x + \pi = 0$
$x = 0 - \pi$

| $x$ | $-\frac{7\pi}{2}$ | $-3\pi$ | $-\frac{5\pi}{2}$ | $-2\pi$ | $-\frac{3\pi}{2}$ | $-\pi$ | $-\frac{\pi}{2}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ | $\frac{5\pi}{2}$ | $3\pi$ | $\frac{7\pi}{2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 3.5 | 3 | 2.5 | 3 | 3.5 | 3 | 3.5 | 3 | 2.5 | 3 | 3.5 | 3 | 2.5 | 3 | 3.5 |

| Q | 0 | $\pi/2$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
|---|---|---|---|---|---|
| sinθ | 0 | 1 | 0 | -1 | 0 |

| x | $.5\sin(x+\pi)+3$ |
|---|---|
| $-\pi$ | +3 |
| $-\frac{\pi}{2}$ | 3.5 |
| 0 | 3 |
| $\frac{\pi}{2}$ | 2.5 |
| $\pi$ | 3 |

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3. Sketch the graph (including two full periods) of $f(x) = 2\cot\left(2x + \dfrac{\pi}{4}\right)$ and find the following (5 points):



(i) What are the zeros? (3 pt)

$x = \dfrac{\pi}{8} + \dfrac{\pi}{2}n, \ n \in \mathbb{Z}$

(ii) What are the equations of the vertical asymptotes? (2)

$x = \dfrac{3}{8}\pi + \dfrac{\pi}{2}n, \ \in \mathbb{Z}$

(iii) What is the period? (1)

$\dfrac{\pi}{2}$

DOMAIN (2)

$\mathbb{R}, x \neq \dfrac{3\pi}{8} + \dfrac{\pi}{2}n \ (2), \ n \in \mathbb{Z}$

(v) Complete the table by giving 5 points on the graph of $f(x)$ (5 points).
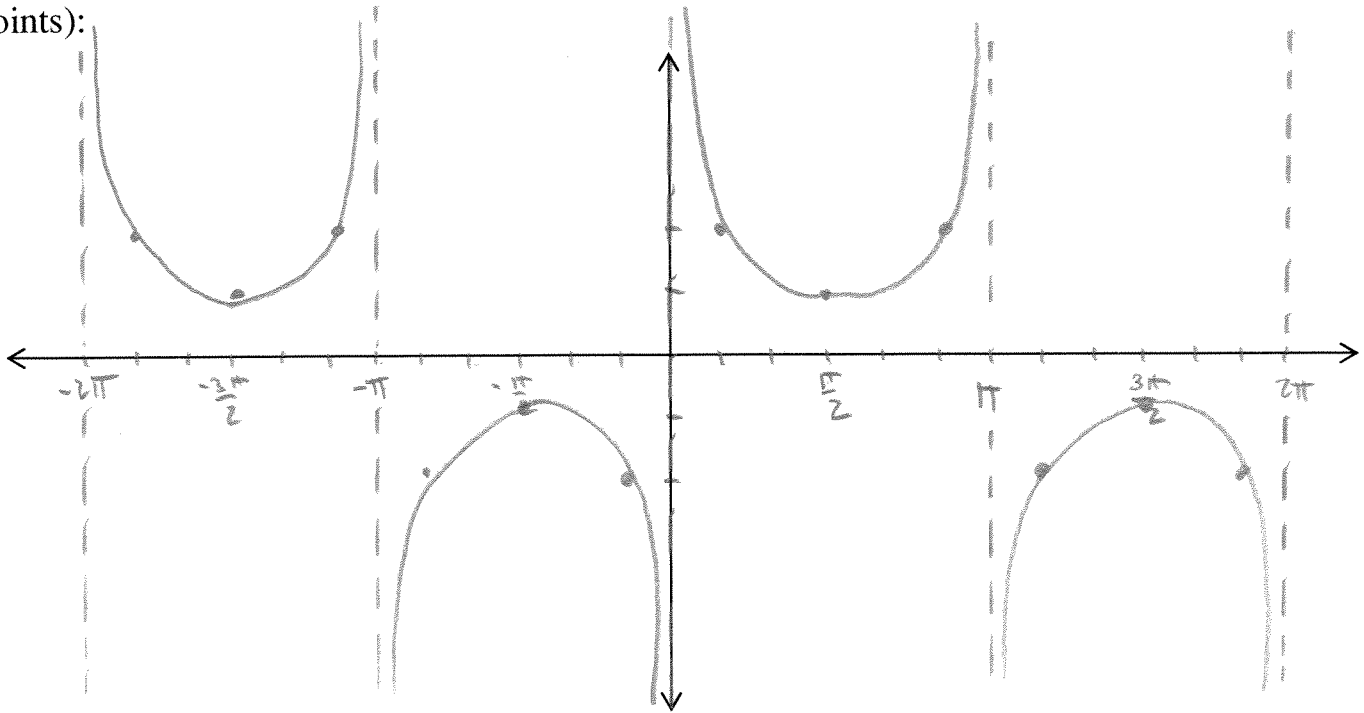
$2x + \dfrac{\pi}{4} = 0$

$2x = 0 - \dfrac{\pi}{4}$

$x = \dfrac{1}{2} \cdot 0 - \dfrac{\pi}{8}$

| $x$ | | $-\frac{5\pi}{8}$ | $-\frac{\pi}{2}$ | $-\frac{3\pi}{8}$ | $-\frac{\pi}{4}$ | $-\frac{\pi}{8}$ | $0$ | $\frac{\pi}{8}$ | $\frac{\pi}{4}$ | $\frac{3\pi}{8}$ | $\frac{\pi}{2}$ | $\frac{5\pi}{8}$ | $\frac{3\pi}{4}$ | $\frac{7\pi}{8}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | $U$ | $2$ | $0$ | $-2$ | $U$ | $2$ | $0$ | $-2$ | $U$ | $2$ | $0$ | $-2$ | $U$ |

| $\theta$ | $0$ | $\pi/4$ | $\pi/2$ | $3\pi/4$ | $\pi$ |
|---|---|---|---|---|---|
| $\cot\theta$ | $U$ | $1$ | $0$ | $-1$ | $U$ |

| $x$ | $-\frac{\pi}{8}$ | $0$ | $\frac{\pi}{8}$ | $\frac{\pi}{4}$ | $\frac{3\pi}{8}$ |
|---|---|---|---|---|---|
| $2\cot\left(2x + \frac{\pi}{4}\right)$ | $U$ | $2$ | $0$ | $-2$ | $U$ |

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4. Sketch the graph (including two full periods) of $f(x) = \csc(\pi - x)$ and find the following (5 points):



(i) ~~What are the zeros? (3 pt)~~

Domain (2)

$\mathbb{R}, x \neq \pi n, n \in \mathbb{Z}$

(ii) What are the equations of the vertical asymptotes? (2)

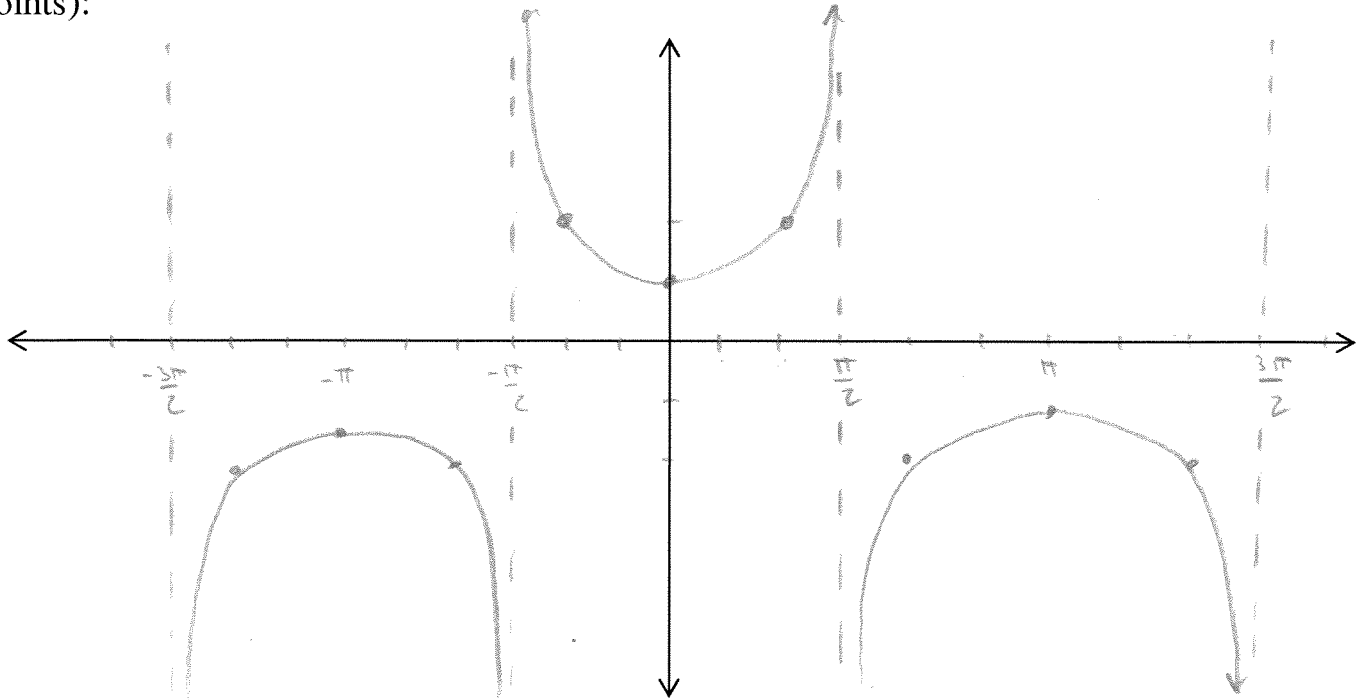$x = \pi n, n \in \mathbb{Z}$

(iii) What is the period? (1)

$2\pi$

(v) Complete the table by giving 5 points on the graph of $f(x)$ (5 points).

$\pi - x = \theta$

$-x = \theta - \pi$

$x = \pi - \theta$

| $x$ | | | $-\pi$ | $-\frac{5\pi}{6}$ | $-\frac{\pi}{2}$ | $-\frac{\pi}{6}$ | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{2}$ | $\frac{5\pi}{6}$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{3\pi}{2}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | $\cup$ | $-2$ | $-1$ | $-2$ | $\cup$ | $2$ | $1$ | $2$ | $\cup$ | $-2$ | $-1$ | $-2$ | $\cup$ |

| $\theta$ | $0$ | $\pi/6$ | $\pi/2$ | $5\pi/6$ | $\pi$ | $\frac{7\pi}{6}$ | $\frac{3\pi}{2}$ | $\frac{11\pi}{6}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $\csc\theta$ | $\cup$ | $2$ | $1$ | $2$ | $\cup$ | $-2$ | $-1$ | $-2$ | $\cup$ |

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5. Sketch the graph (including two full periods) of $f(x) = -\sec(x + \pi)$ and find the following (5 points):



(i) What are the zeros? (3 pt)

DOMAIN (2)

$\mathbb{R}, x \neq \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

(ii) What are the equations of the vertical asymptotes? ( 2 )

$x = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

(iii) What is the period? ( 1 )

$2\pi$

(v) Complete the table by giving 5 points on the graph of $f(x)$ (5 points).

$x + \pi = \theta$

$x = \theta - \pi$

| $x$ | | | | | $\frac{3\pi}{2}$ | $-\frac{4\pi}{3}$ | $-\pi$ | $-\frac{2\pi}{3}$ | $-\frac{\pi}{2}$ | $-\frac{\pi}{3}$ | $0$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | U | -2 | -1 | -2 | U | 2 | 1 | 2 | U |

| $\theta$ | $-\pi/2$ | $-\pi/3$ | $0$ | $\pi/3$ | $\pi/2$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|---|---|---|---|---|
| $\sec\theta$ | U | 2 | 1 | 2 | U | -2 | -1 | -2 | U |

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6. Sketch the graph (including two full periods) of $f(x) = -\dfrac{1}{2}\tan(\pi x)$ and find the following (5 points):
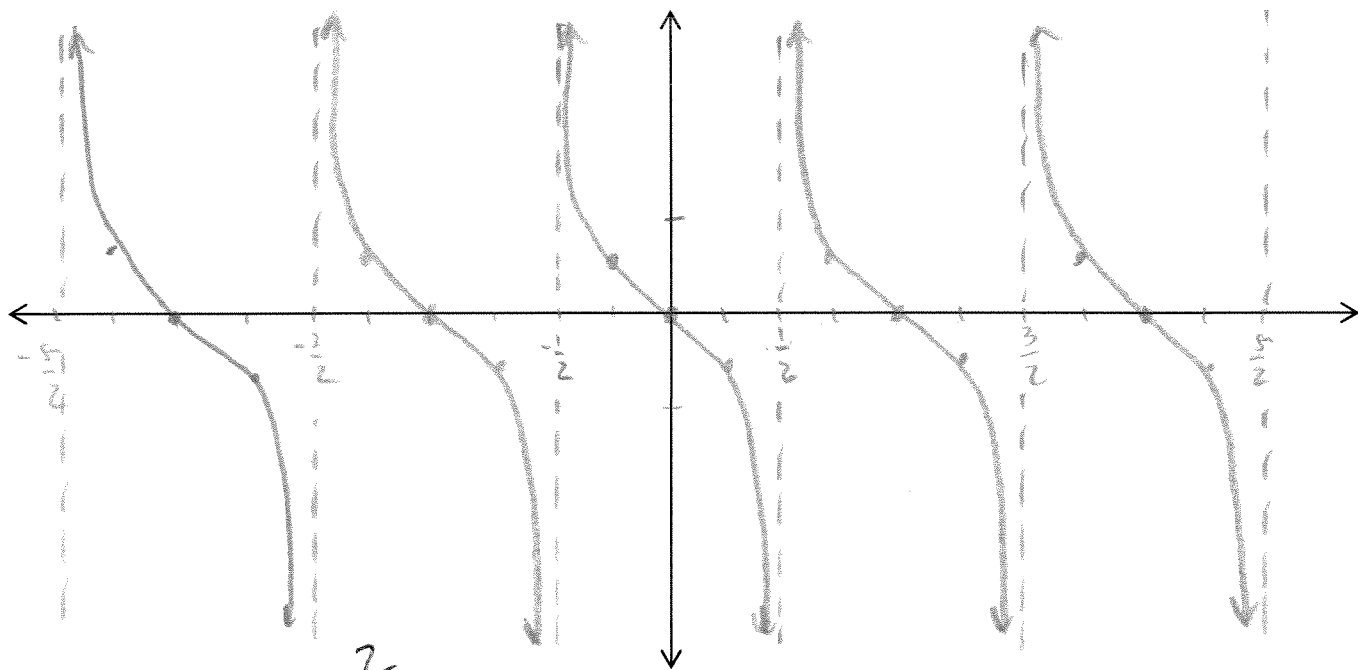


(i) What are the zeros? (3 pt)

$x = n,\ n \in \mathbb{Z}$

(ii) What are the equations of the vertical asymptotes? $(2)$

$x = \dfrac{1}{2} + n,\ n \in \mathbb{Z}$

(iii) What is the period? $(1)$

$1$

(v) Complete the table by giving 5 points on the graph of $f(x)$ (5 points).

$\pi x = \theta$

$x = \dfrac{1}{\pi} \cdot \theta$

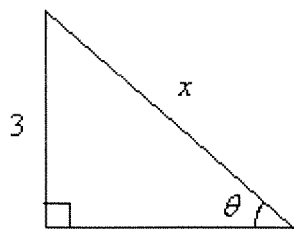| $x$ | $-\frac{7}{4}$ | $-\frac{3}{2}$ | $-\frac{5}{4}$ | $-1$ | $-\frac{3}{4}$ | $-\frac{1}{2}$ | $-\frac{1}{4}$ | $0$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | $1$ | $\frac{5}{4}$ | $\frac{3}{2}$ | $\frac{7}{4}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | U | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | U | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | U | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | U | $\frac{1}{2}$ |

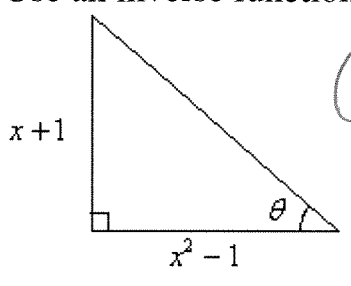| $\theta$ | $-\pi/2$ | $-\pi/4$ | $0$ | $\pi/4$ | $\pi/2$ |
|---|---|---|---|---|---|
| Tan $\theta$ | U | $-1$ | $0$ | $1$ | U |

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7. Use an inverse function to write $\theta$ as a function of $x$. ①

$$\theta = \arcsin\left(\frac{3}{x}\right)$$

8. Use an inverse function to write $\theta$ as a function of $x$.

② $\theta = \arctan\left(\frac{x+1}{x^2-1}\right) = \arctan\left(\frac{1}{x-1}\right)$

9. Use the properties of inverse trigonometric functions to evaluate $\cos\left[\arccos\left(-0.79\right)\right]$. ② SINCE $-1 \le -.79 \le 1$, WE HAVE $-.79$
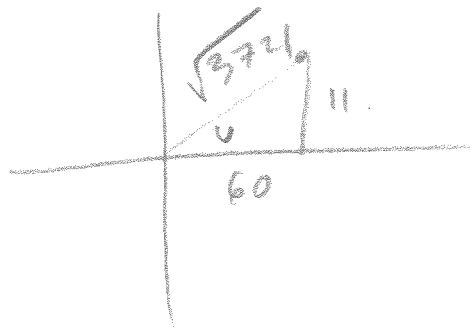
10. Use the properties of inverse trigonometric functions to evaluate $\arctan\left[\tan\left(\frac{3\pi}{8}\right)\right]$ ② SINCE $-\frac{\pi}{2} < \frac{3\pi}{8} < \frac{\pi}{2}$, WE HAVE $\frac{3\pi}{8}$

**11.** Find the exact value of $\csc\left(\arctan\dfrac{11}{60}\right)$. ⑤ $\Rightarrow \tan u = \dfrac{11}{60}$
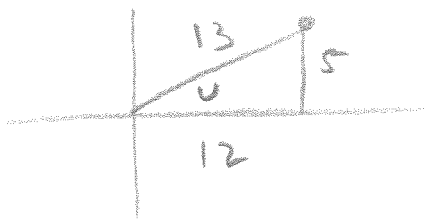
let $u = \arctan\dfrac{11}{60}$ , $\csc u$

$\csc u = \dfrac{1}{\sin u} = \boxed{\dfrac{\sqrt{3721}}{11}} = \boxed{\dfrac{61}{11}}$

$\sqrt{3721}$ , 11 , 60

**12.** Find the exact value of $\tan\left(\sin^{-1}\dfrac{5}{13}\right)$. ⑤

let $u = \sin^{-1} 5/13$ , $\tan u$

$\tan u = \boxed{\dfrac{5}{12}}$

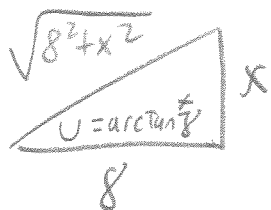13 , u , 5 , 12

**13.** Write an algebraic expression that is equivalent to $\sin\left(\arctan\dfrac{x}{8}\right)$. ⑤

LET $u = \arctan\dfrac{x}{8}$

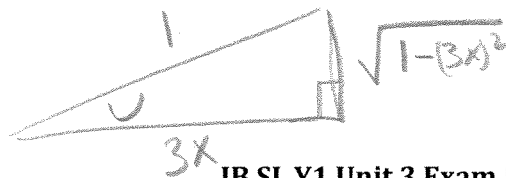$\tan u = \tan\left(\arctan\dfrac{x}{8}\right) = \dfrac{x}{8}$

$\sqrt{8^2+x^2}$ , $x$ , $u = \arctan\dfrac{x}{8}$ , 8

$\sin u = \dfrac{x}{\sqrt{8^2+x^2}} = \dfrac{x\sqrt{64+x^2}}{64+x^2}$

**14.** Write an algebraic expression that is equivalent to $\tan(\arccos 3x)$. ⑤

let $u = \arccos 3x$

$\cos u = \cos(\arccos 3x) = 3x, \quad -1 \le 3x \le 1$

$1$ , $\sqrt{1-(3x)^2}$ , $3x$

$\sqrt{1^2 + (3x)^2}$

$\tan u = \boxed{\dfrac{\sqrt{1-9x^2}}{3x}}$

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