1) $f(x) = x^3 - 9x$

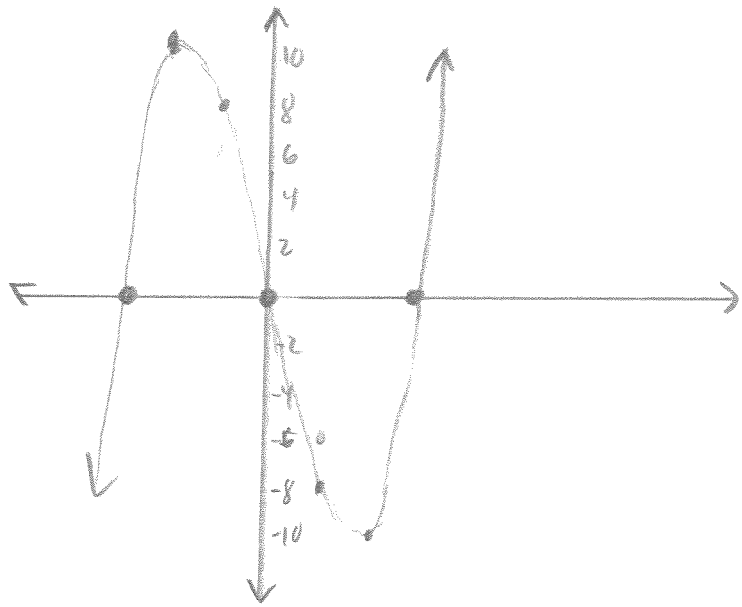
(a) $x(x^2-9)$
$x(x+3)(x-3)$

b) $x=0 \quad x+3=0 \quad x-3=0$
$x=0 \quad x=-3 \quad x=3$

$f(-1) = 8$
$f(-2) = 10$
$f(1) = -8$
$f(2) = -10$

ODD FUNCTION

c)



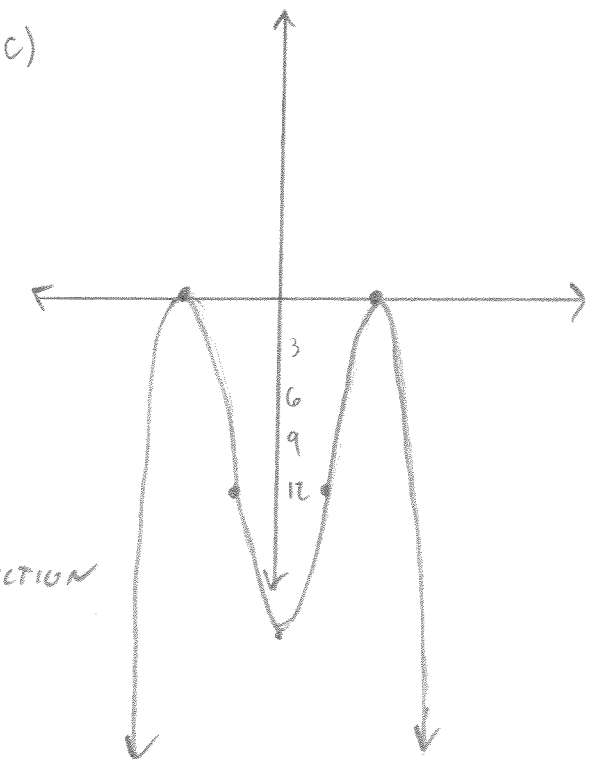
2) $h(x) = -x^4 + 9x^2 - 20$

a) $0 = -x^4 + 9x^2 - 20$
$0 = x^4 - 9x^2 + 20$
$0 = (x^2-5)(x^2-4)$
$0 = x^2 - 5 \quad 0 = x^2 - 4$
$x = \pm\sqrt{5}i \quad x = \pm 2$
$(x+\sqrt{5}i)(x-\sqrt{5}i)(x+2)(x-2)$

b) $x = \pm\sqrt{5}i \quad x = \pm 2$

$f(-1) = -12 = f(1) \Rightarrow$ EVEN FUNCTION
$f(0) = -20$

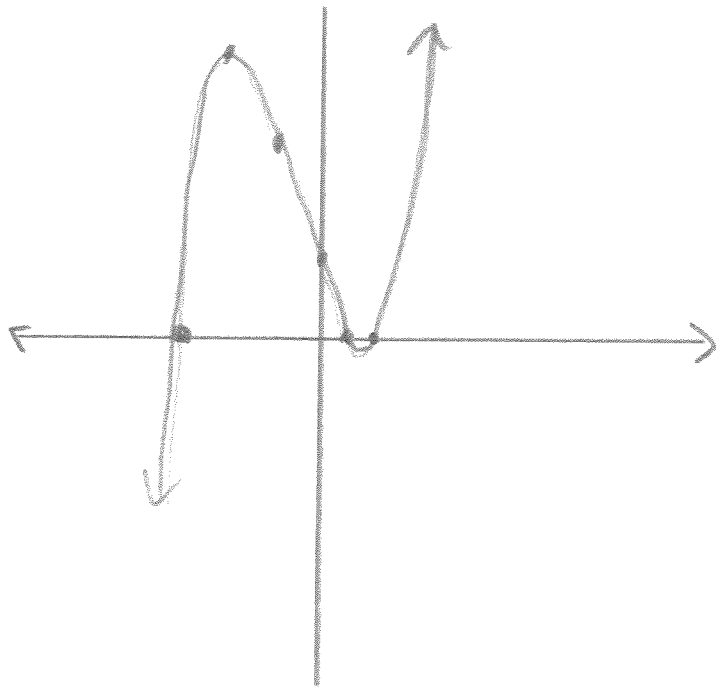
c)



3) $K(x) = 2x^3 + 3x^2 - 8x + 3$

a) $\frac{p}{q} = \frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

-3	2	3	-8	3
		-6	9	-3
	2	-3	1	0



$(x+3)(2x^2 - 3x + 1)$

a) $(x+3)(2x-1)(x-1)$

b) $x = -3 \quad x = \frac{1}{2} \quad x = 1$

c) $f(-2) = 15$
$f(-1) = 12$
$f(0) = 3$

4) $l(x) = 3x^3 - 5x^2 + 6x - 4$

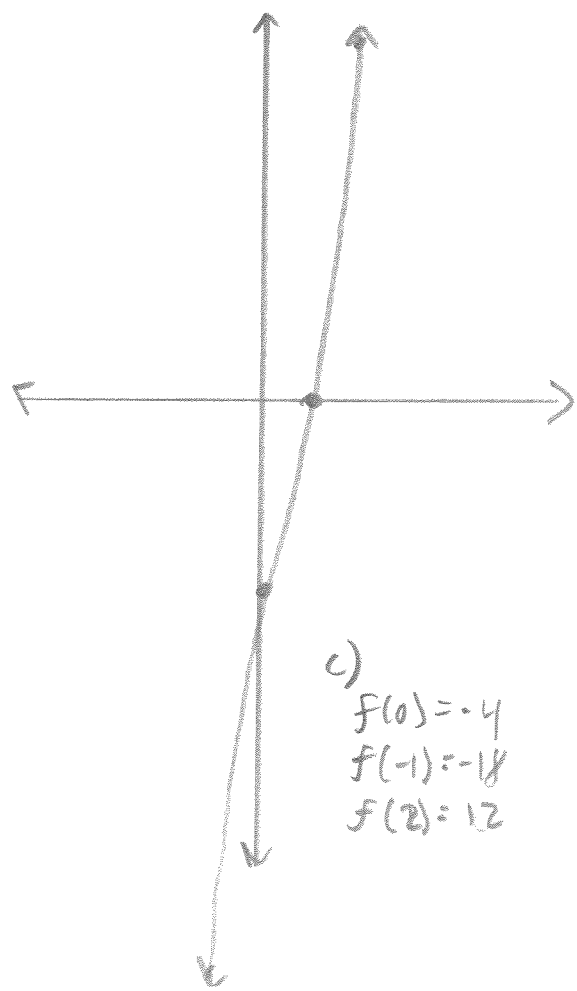
a) $\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3}$

1	3	-5	6	-4
		3	-2	$+4$
	3	-2	4	0

$(x-1)(3x^2 - 2x + 4)$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{-44}}{6} = \frac{2 \pm 2i\sqrt{10}}{6}$$
$$= 1 \pm i\sqrt{10}$$



c)
$f(0) = -4$
$f(-1) = -18$
$f(2) = 12$

a) $(x-1)(x-(1+i\sqrt{10}))(x-(1-i\sqrt{10}))$
b) $x = 1, 1 \pm \sqrt{10}\, i$

5) $m(x) = 4x^4 - 17x^2 + 4$

a) $\frac{p}{q}: \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 4}$

$$\begin{array}{r|rrrrr} -2 & 4 & 0 & -17 & 0 & 4 \\ & & -8 & 16 & 2 & -4 \\ \hline & 4 & -8 & -1 & 2 & 0 \end{array}$$

$(x+2)(4x^3 - 8x^2 - x + 2)$

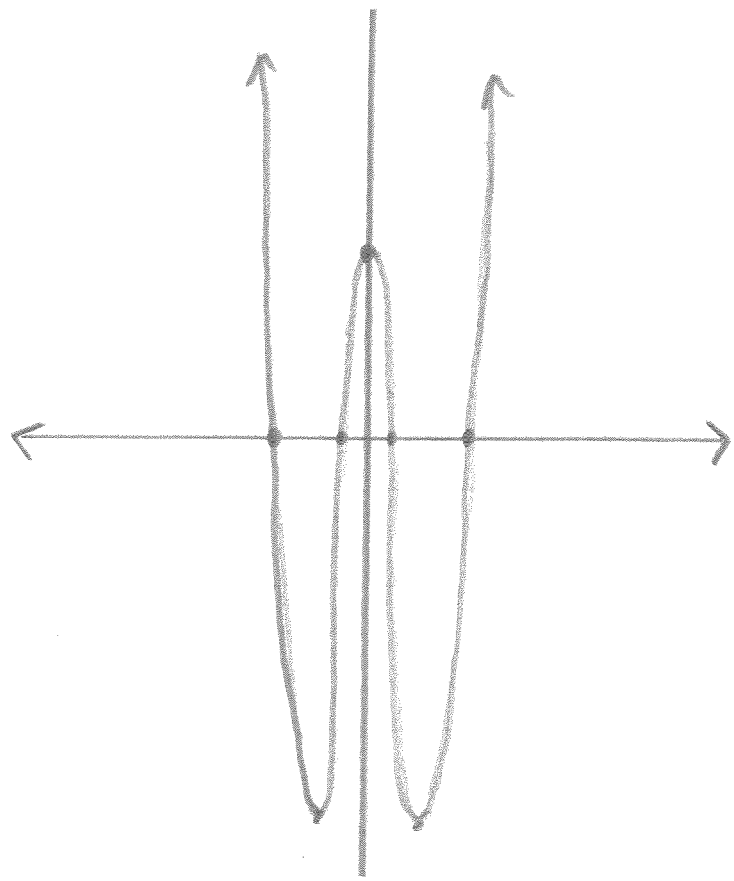
$$\begin{array}{r|rrrr} 2 & 4 & -8 & -1 & 2 \\ & & 8 & 0 & -2 \\ \hline & 4 & 0 & -1 & 0 \end{array}$$

$(x+2)(x-2)(4x^2 - 1) = 0$

a) $(x+2)(x-2)(2x+1)(2x-1) = 0$

b) $x = 2, -2, 1/2, -1/2$

c) $f(0) = 4$
$f(1) = f(-1) = -9 \Rightarrow$ EVEN FUNCTION



6) $x^4 - x^3 - 2x - 4$

a) $\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1}$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$(x+1)(x^3 - 2x^2 + 2x - 4)$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$(x+1)(x-2)(x^2+2) = 0$

$x^2 = -2$

$x = \pm\sqrt{2}\,i$

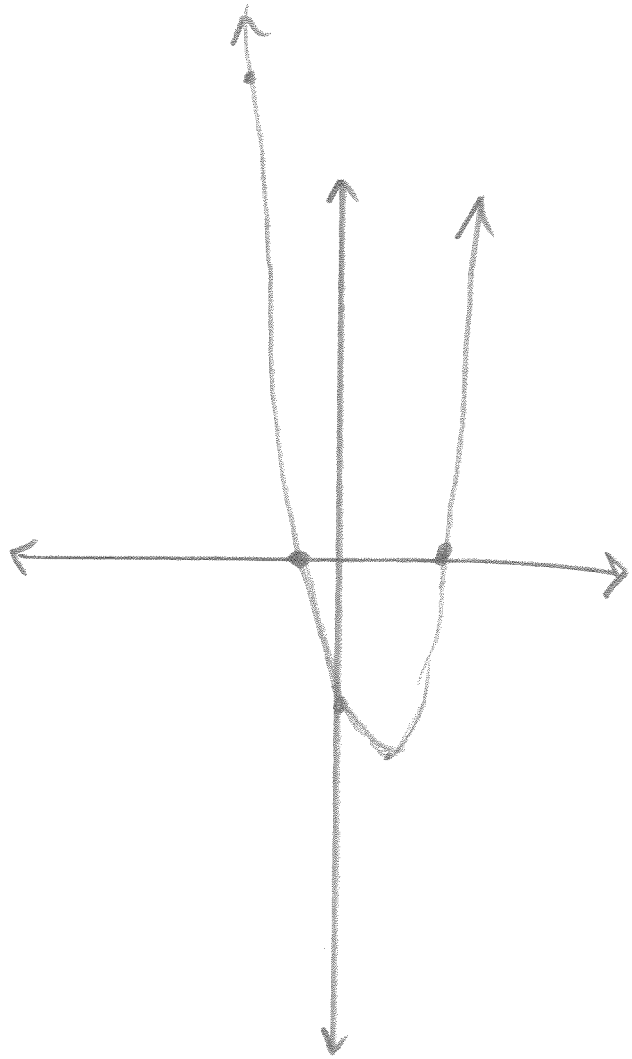
a) $(x+1)(x-2)(x+\sqrt{2}\,i)(x-\sqrt{2}\,i)$

b) $x = -1, 2, \pm\sqrt{2}\,i$

c) $f(0) = -4$

$f(1) = -6$

$f(-2) = 24$



① $f(x) = \dfrac{x^2-16}{x-4}$

a) $f(0) = \dfrac{(0)^2-16}{0-4} = 4$

$(0,4)$

b) 1ST SIMPLIFY

$\dfrac{x^2-16}{x-4} = \dfrac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}}$

$f(x) = x+4$; $x = 4$

$0 = x+4$

$x = -4$

$(-4,0)$

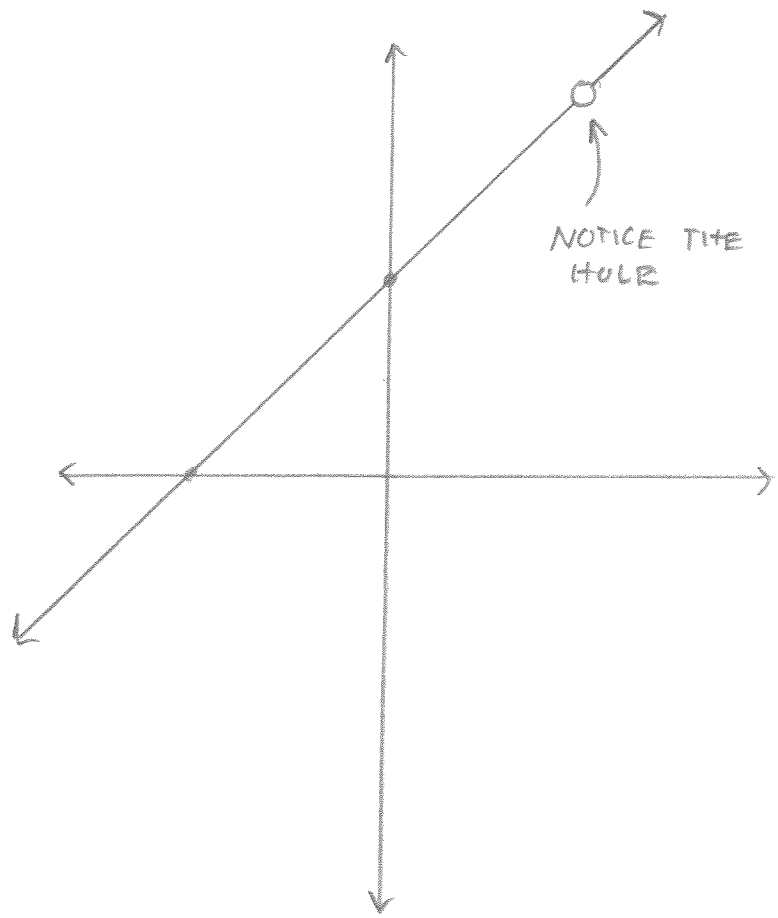
c) NO VA

d) NO HA

e) NO SA

f) $x = 4$

$(4,8)$ IS A HOLE



2) $h(x) = \dfrac{x^2-9}{x-3}$

(a) $h(0) = \dfrac{-9}{-3} = 3$

$(0,3)$

(b) $\dfrac{x^2-9}{x-3} = \dfrac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$

$h(x) = x+3, \; x \neq 3$

$0 = x+3$

$-3 = x$

$(-3,0)$

(c) VA: NONE

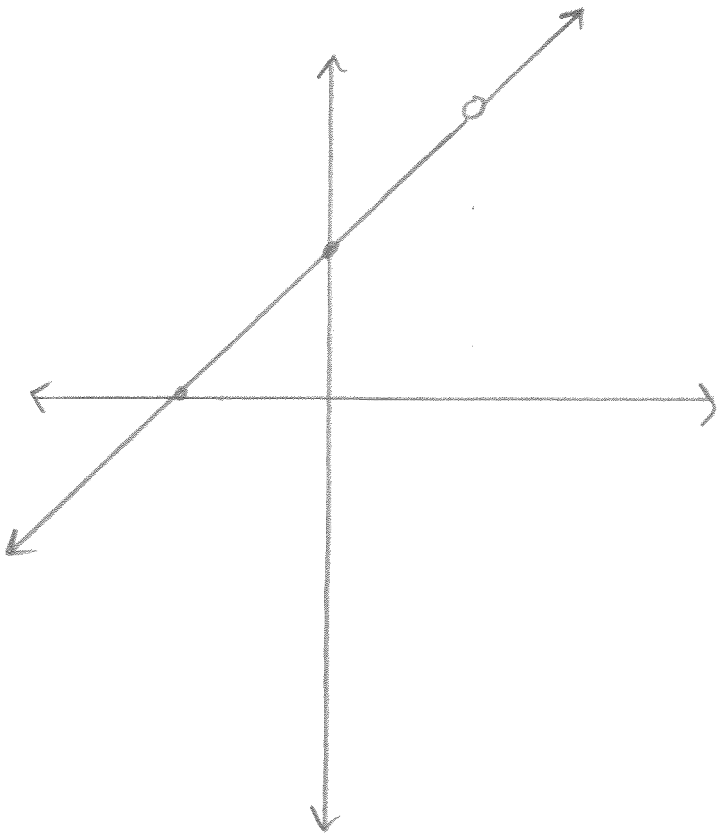
(d) HA: NONE

(e) NONE

(f) $x-3=0$

$x=3$

$(3,6)$



3) $K(x) = \dfrac{2x-1}{x^2+1}$

(a) $K(0) = \dfrac{2(0)-1}{(0)^2+1} = -1$

$(0,-1)$

(b) $2x-1=0$

$2x=1$

$x=1/2$

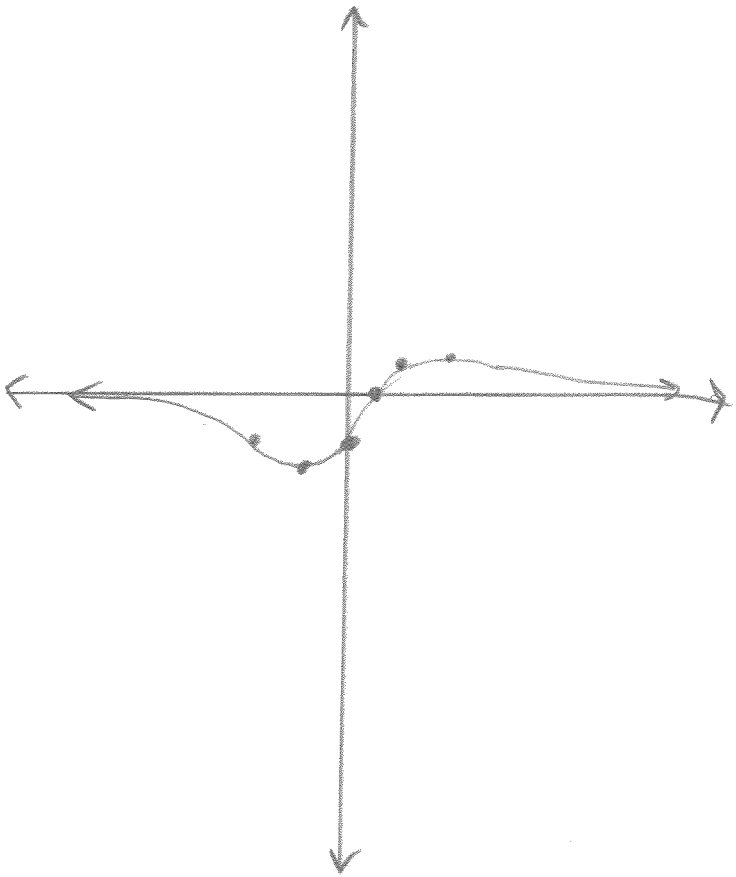
$(1/2, 0)$

(c) VA: NONE

(d) HA: $y=0$

(e) SA: NONE

(f) HOLES: NONE



ADDITIONAL POINTS

$f(1) = 1/2$

$f(2) = 3/5$

$f(3) = 1/2$

$f(-1) = -1.5$

$f(-2) = -1$

$f(-3) = -.7$

4) $f(x) = \dfrac{x^2 - x - 2}{x - 1}$

SINCE $n > m$, WE WILL HAVE A SLANTED ASYMPTOTE

$$\begin{array}{r|l} & x \\ \hline x-1 & x^2 - x - 2 \\ & -x^2 + x \\ \hline & -2 \end{array}$$

SLANTED ASYMPTOTE

$$x - \frac{2}{x-1}$$

a) $f(0) = \dfrac{-2}{-1} = 2$

$(0, 2)$

b) $0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2, -1$

$(2, 0), (-1, 0)$

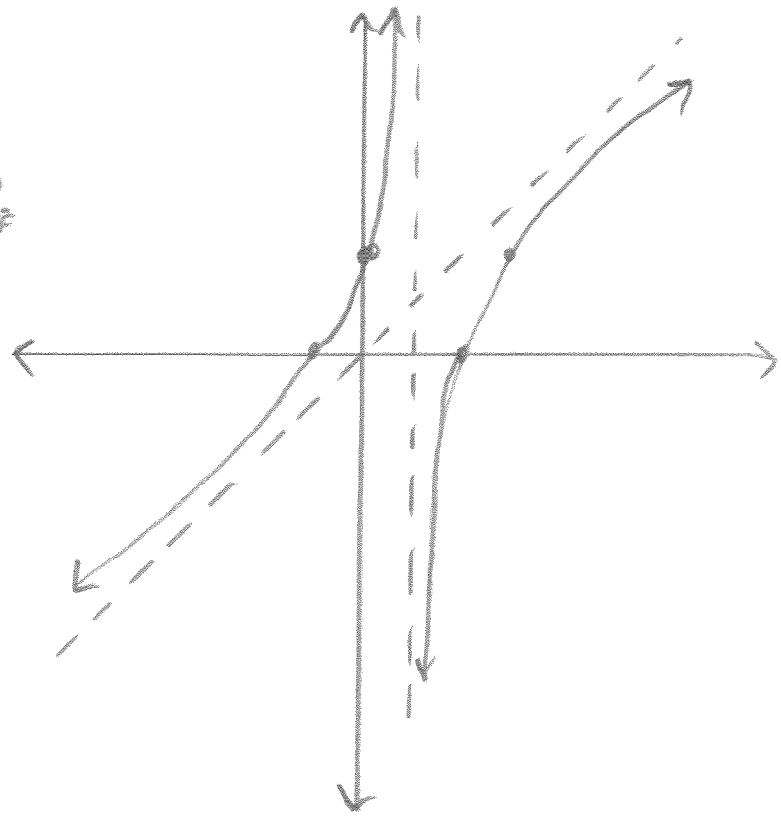
c) VA $x = 1$

d) NO HA

e) SEE ABOVE

$y = x$

f) NO HOLES



ADDITIONAL POINTS

$f(2) = 0$

$f(3) = 2$

$f(4) = 3.3$

5) $m(x) = \frac{x^2 - x}{x+1} = \frac{x(x-1)}{x+1}$

(a) $m(0) = \frac{0}{1} = 0 \quad (0,0)$

(b) $0 = x(x-1)$

$x = 0 \quad x = 1$

(c) $x = -1$

(d) NONE

(e)

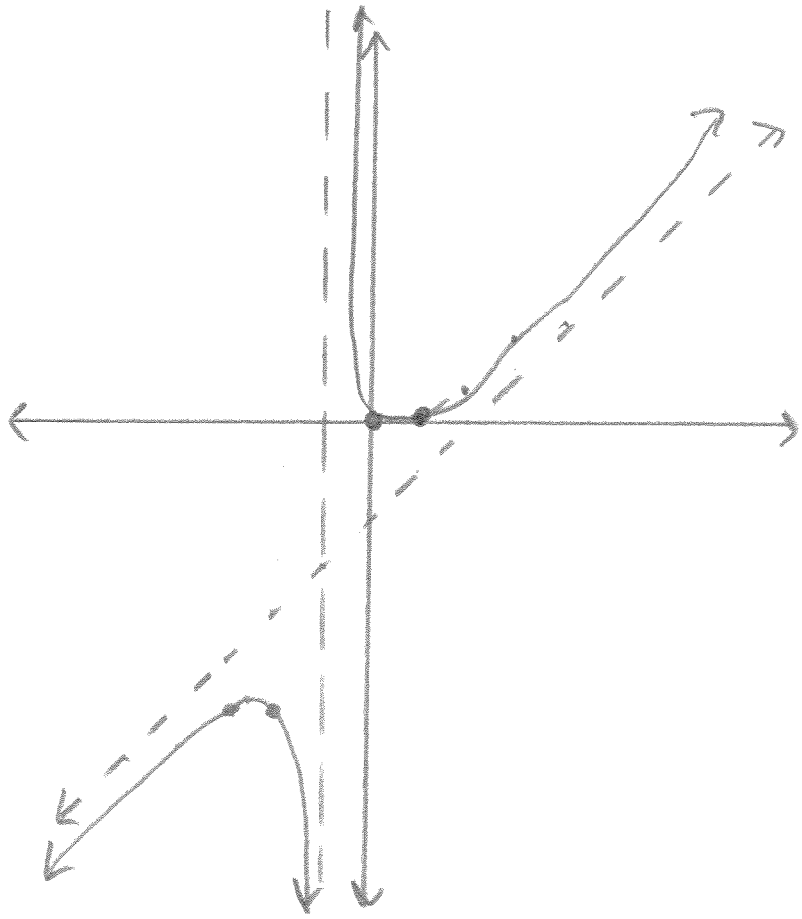
$$\begin{array}{r} x-2 \\ x+1 \overline{\big) x^2 - x} \\ -x^2 - x \\ \hline -2x \\ +2x - 2 \\ \hline 2 \end{array}$$

SA

$x - 2 + \frac{2}{x+1}$

$y = x - 2$

f) NO HOLES



ADDITIONAL POINTS

$f(7) = .\overline{6}$

$f(3) = 1.5$

$f(-2) = -6$

$f(-3) = -6$

6) $f(x) = \dfrac{x^2 - 9}{x^2 - 2x - 3}$

$= \dfrac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)}$

$= \dfrac{x+3}{x+1}, \; x \neq 3$

a) $f(0) = 3$

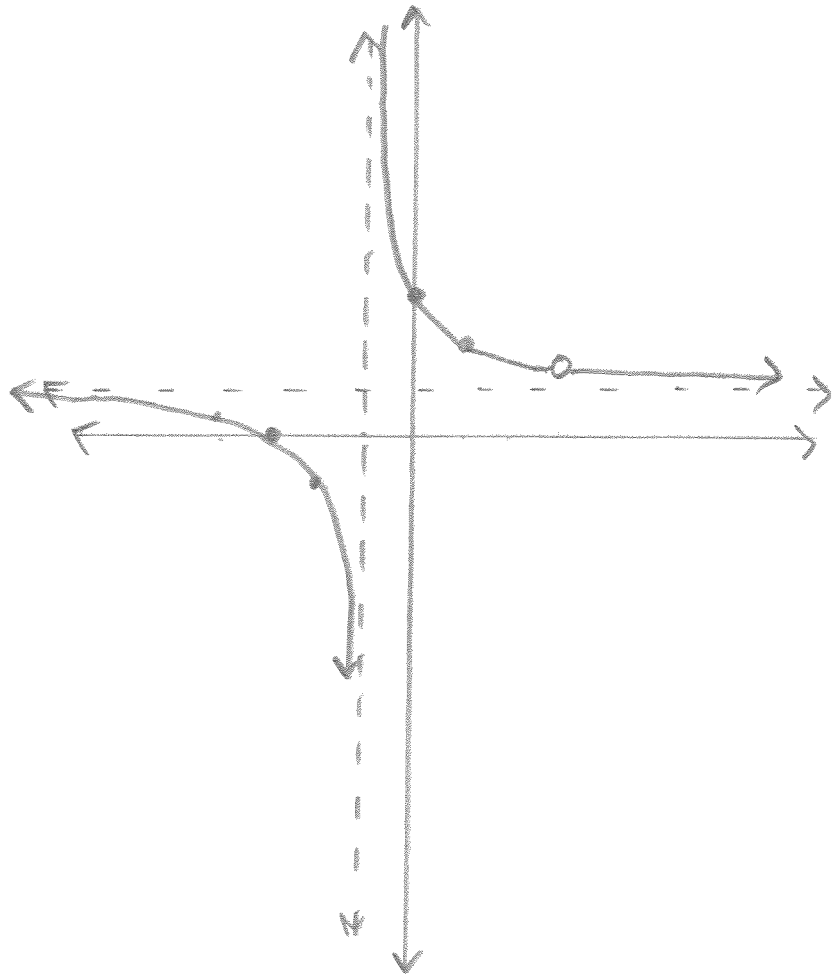
b) $0 = x + 3$
$x = -3$
$(-3, 0)$

c) $x = -1$

d) $y = 1$

e) NONE

f) $x - 3 = 0$ $\quad f(3) = \dfrac{3+3}{3+1}$
$x = 3$
$(3, \frac{3}{2})$



ADDITIONAL POINTS
$f(1) = 2$
$f(2) = 1.6$
$f(-2) = -1$
$f(-4) = .3$