Honors Trigonometry/Pre-Calculus Chapter 2 Practice Exam

Name: ANSWER KEY Date: _____ Period: _____

**Show all work** neatly and clearly in order to receive full credit. Notes and Graphing Utilities (TI-84) are not permitted on this exam. However, scientific calculators (TI-30) are permitted.

1. *True* or *False*? Determine whether the statement is true or false. Justify your answer.

a) The function $f(x) = -12x^2 - 1$ has no $x$-intercepts.

$-12x^2 - 1 = 0$

$-12x^2 = 1$

$12x^2 = -1$

$x^2 = -1/12$

$x = \pm\sqrt{-1/12}$

SINCE THE ROOT IS IMAGINARY, THEREFORE NO X-INTERCEPT. $\boxed{\text{TRUE}}$

b) The graph of the function $f(x) = 2x(x-1)^2(x+3)^3$ crosses the x-axis at $x = 1$.

$f(x)$ HAS A ZERO AT 1 WITH A MULTIPLICITY OF TWO, THEREFORE, THE POLYNOMIAL WILL TOUCH THE ROOT AT 1. $\boxed{\text{FALSE}}$

c) The graph of the function $f(x) = 2x(x-1)^2(x+3)^3$ rises to the left and falls to the right.

THE DEGREE OF $f(x)$ IS 6 AND HAS A LEADING COEFFICIENT OF 2, THEREFORE, THE GRAPH RISES TO THE LEFT & RISES TO THE RIGHT. $\boxed{\text{FALSE}}$

d) If $(7x+4)$ is a factor of some polynomial function $f$, then $\frac{4}{7}$ is a zero of $f$.

$7x + 4 = 0$

$7x = -4$

$x = -4/7$

IF $(7x+4)$ IS A FACTOR, THEN $-4/7$ IS A ZERO. $\boxed{\text{FALSE}}$

2. Find a polynomial function with real coefficients that has the given zeros: 3, 3, $-2i-1$

$(x-3)^2[x-(-1-2i)][x-(-1+2i)]$

$[(x+1)+2i][(x+1)-2i]$

$(x+1)^2-(2i)^2$

$x^2+2x+1-(4i^2)$

$x^2+2x+1-4(-1)$

$x^2-6x+9)(x^2+2x+5)$

$x^4+2x^3+5x^2$

$-6x^3-12x^2-30x$

$9x^2+18x+45$

$\boxed{f(x)=x^4-4x^3+2x^2-12x+45}$

3. Use the given zero and polynomial to find all the zeros of the function

Function: $f(x)=x^3+4x^2+14x+20$ Zero: $-1-3i$

| | | | | |
|---|---|---|---|---|
| $-1-3i$ | 1 | 4 | 14 | 20 |
| | | $-1-3i$ | $-12-6i$ | $-20$ |
| | 1 | $3-3i$ | $2-6i$ | 0 |

| | | | |
|---|---|---|---|
| $-1+3i$ | 1 | $3-3i$ | $2-6i$ |
| | | $-1+3i$ | $-2+6i$ |
| | 1 | 2 | 0 |

$x+2=0$

$x=-2$

$\boxed{\text{ZEROS: } -2, -1\pm 3i}$

4. Find all the zeros of the function and write the polynomial as a product of linear factors.

$f(x)=x^4+6x^3+10x^2+6x+9$

$\frac{p}{q}=\frac{\pm 1, \pm 3, \pm 9}{\pm 1}$

| | | | | | |
|---|---|---|---|---|---|
| $-3$ | 1 | 6 | 10 | 6 | 9 |
| | | $-3$ | $-9$ | $-3$ | $-9$ |
| | 1 | 3 | 1 | 3 | 0 |

| | | | | |
|---|---|---|---|---|
| $-3$ | 1 | 3 | 1 | 3 |
| | | $-3$ | 0 | $-3$ |
| | 1 | 0 | 1 | 0 |

$x^2+1=0$

$x^2=-1$

$x=\sqrt{-1}$

$x=\pm i$

$\boxed{\text{ZEROS: } -3, -3, \pm i}$

$\boxed{\text{FACTORS: } (x+3)^2(x+i)(x-i)}$

5. Answer each part below. Graph the rational function with at least 5 points.

$$f(x)=\frac{x^2+2x+1}{2x^2-x-3}$$

a) $y-$ intercept(s)

$f(0)=\frac{1}{-3}$ $\boxed{(0,-1/3)}$

b) $x-$ intercept(s)

$f(x)=\frac{(x+1)(x+1)}{(2x-3)(x+1)}=\frac{x+1}{2x-3}$

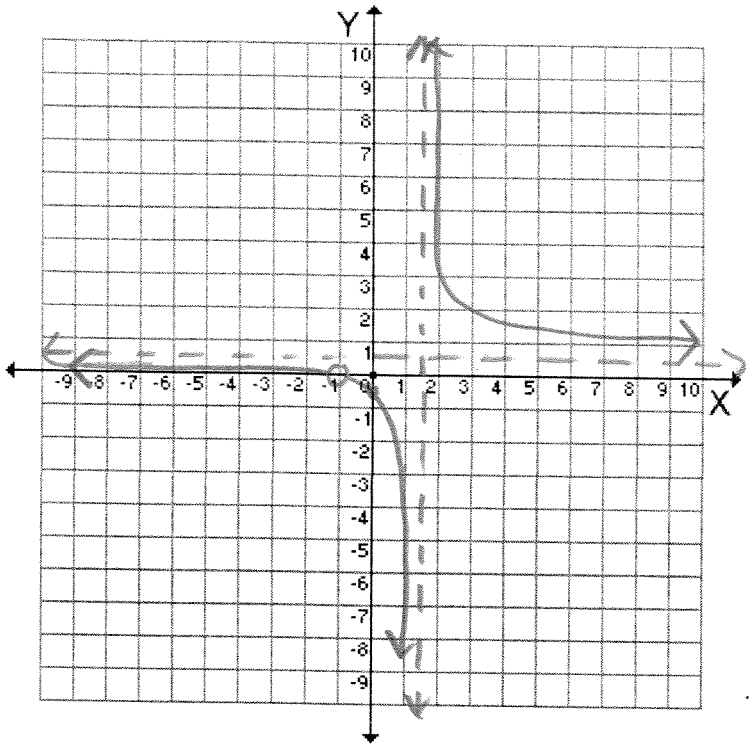
$x+1=0$

$x=-1$ $\boxed{(-1,0)}$

c) Vertical Asymptote(s)

$2x-3=0$

$x=3/2$




d) Horizontal Asymptote(s)

$y=1/2$

e) Slanted Asymptote

NONE

f) Hole(s)

$x+1=0$

$x=-1$

$f(-1)=\frac{-1+1}{2(-1)-3}=0$

$(-1,0)$

g) ~~Write the interval(s) when~~ $\cancel{f'(x)>0}$

~~NONE~~

h) ~~Write the interval(s) when~~ $\cancel{f'(x)<0}$

~~$(-\infty,3/2)\cup(3/2,\infty)$~~

i) ~~Write the interval(s) when~~ $\cancel{f'(x)=0}$

5. Answer each part below. Graph the rational function with at least 5 points.

$$f(x)=\frac{x^2-x}{x+1}$$

a) $y-$intercept(s)

$f(0)=\frac{0}{1}=0$ $\boxed{(0,0)}$

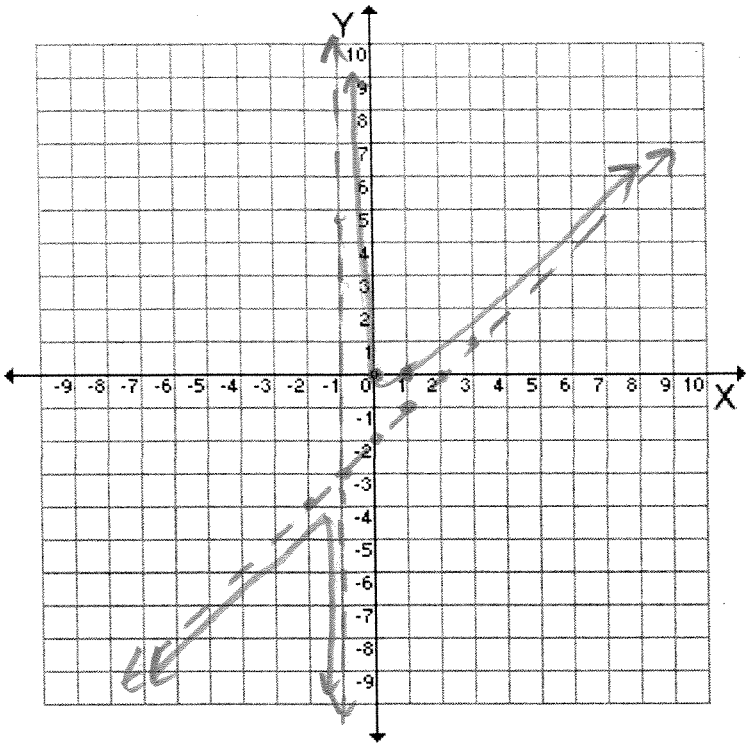
b) $x-$intercept(s)

$\frac{x(x-1)}{x+1}$, $x=0$ $x=1$

$\boxed{(0,0)}$ $\boxed{(1,0)}$

c) Vertical Asymptote(s)

$x=-1$




d) Horizontal Asymptote(s)

NONE

e) Slanted Asymptote

$$\begin{array}{r} x-2 \\ x+1\overline{\big)\,x^2-x+0} \\ \underline{-x^2-x} \\ -2x+0 \end{array}$$

$y=x-2$

f) Hole(s)

NONE

g) Write the interval(s) when $f'(x)>0$

_____

h) Write the interval(s) when $f'(x)<0$

_____

i) Write the interval(s) when $f'(x)=0$

_____

7. The polynomial $f(x) = x^5 - x^4 - 2x^3$ is given.

a) State all the zeros

$x^3(x^2 - x - 2)$

$x^3(x-2)(x+1)$

$x = 0, 0, 0, 2, -1$

b) Write $f(x)$ as a product of linear factors

$x^3(x-2)(x+1)$

c) State the y-intercept

$f(0) = 0^5 - 0^4 - 2(0)^3$

$= 0$

d) Find $f'(x)$

$f'(x) = 5x^4 - 4x^3 - 6x^2$

e) State the critical numbers

$5x^4 - 4x^3 - 6x^2 = 0$

$x^2(5x^2 - 4x - 6) = 0$

$x^2 = 0 \quad 5x^2 - 4x - 6 = 0$

$\cancel{x = 0} \quad x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-6)}}{2(5)}$

$= \dfrac{4 \pm \sqrt{136}}{10}$

$x \approx 1.57 \quad x \approx -0.77$

f) State the maxima(s) and/or minima(s)

$f(1.57) = -4.28$

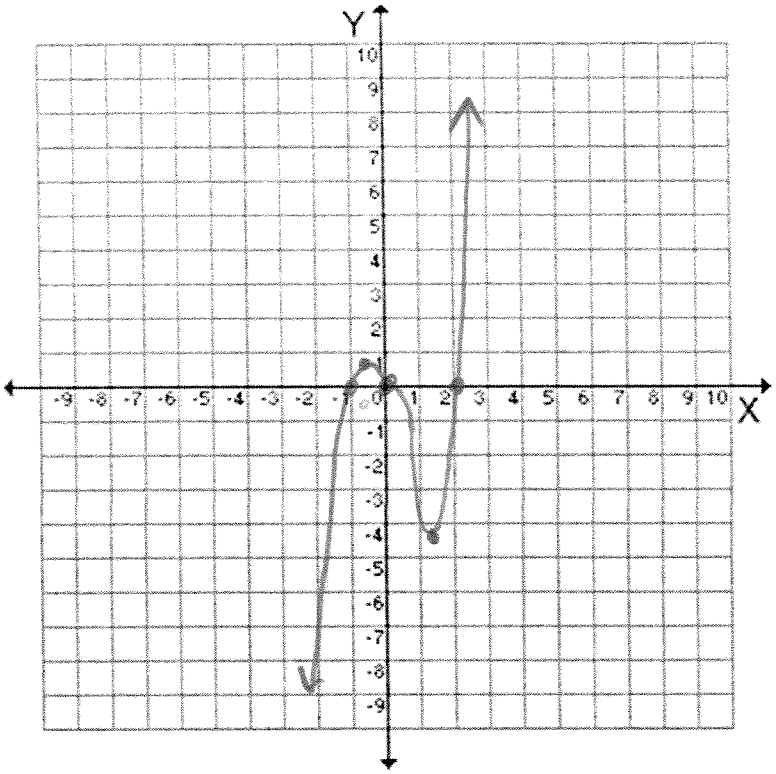
$f(-0.77) \approx 0.29$

g) Find $f(5)$

$f(5) = (5)^5 - (5)^4 - 2(4)^3$

$= 2250$

h) Sketch the polynomial using the zeros, maxima(s), and minima(s). Note: you may need to change the y-values on the y-axis




i) Write the interval(s) when $f'(x) > 0$ $(-\infty, -0.77)$ & $(1.57, \infty)$

j) Write the interval(s) when $f'(x) < 0$ $(-0.77, 1.57)$

k) Write the ~~interval(s)~~ X-VALUE(S) when $f'(x) = 0$ $1.57$ & $-0.77$

l) State the domain: $\mathbb{R}$

m) State the range: $\mathbb{R}$

n) Explain whether the graph is a function or not

f(x) IS A FUNCTION IT PASSES THE VERTICAL LINE TEST

8. The polynomial $f(x) = x^3 + 2x^2 - 5x - 6$ is given.

a) State all the zeros

$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$x^2 - x - 2$

$(x-2)(x+1) = 0$

$\boxed{x = 2 \quad x = -1 \quad x = -3}$

b) Write $f(x)$ as a product of linear factors

$(x-2)(x+1)(x+3)$

c) State the y-intercept

$f(0) = -6$

d) Find $f'(x)$

$f'(x) = 3x^2 + 4x - 5$

e) State the critical numbers

$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(3)(-5)}}{2(3)}$

$= \frac{-4 \pm \sqrt{76}}{6}$

$x \approx 0.79 \quad x \approx -2.12$

f) State the maxima(s) and/or minima(s)

$f(-2.12) \approx 4.06$

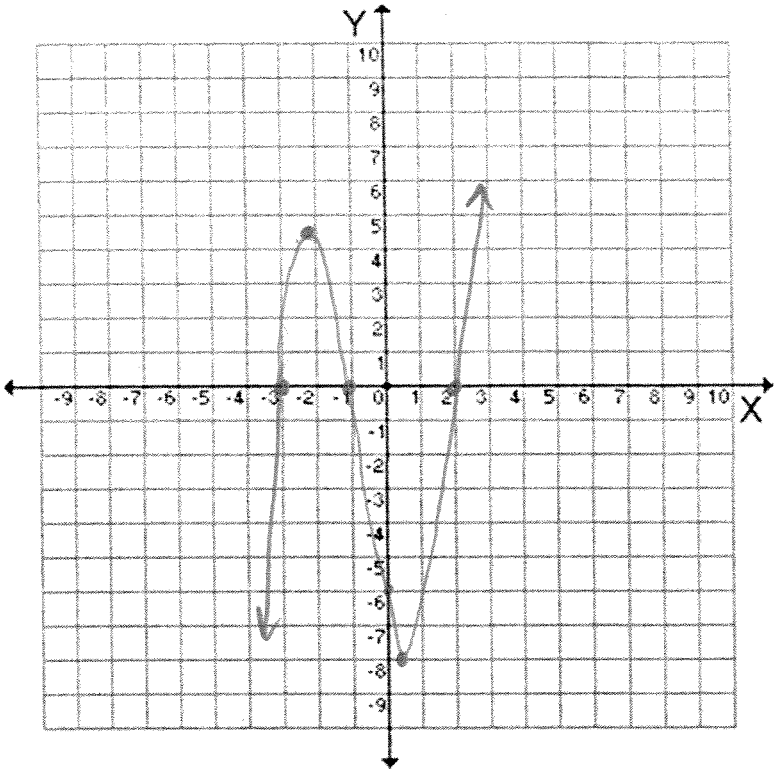
$f(0.79) \approx -8.21$

g) Find $f(5)$

$f(5) = (5)^3 + 2(5)^2 - 5(5) - 6$

$= 144$

h) Sketch the polynomial using the zeros, maxima(s), and minima(s). Note: you may need to change the y-values on the y-axis




i) Write the interval(s) when $f'(x) > 0$ $(-\infty, -2.12)$ & $(0.79, \infty)$

j) Write the interval(s) when $f'(x) < 0$ $(-2.12, 0.79)$

k) Write the ~~interval(s)~~ X-VALUE(S) when $f'(x) = 0$ $-2.12$ & $0.79$

l) State the domain: $\mathbb{R}$

m) State the range: $\mathbb{R}$

n) Explain whether the graph is a function or not

f(x) IS A FUNCTION BECAUSE EACH X-VALUE CORRESPONDS WITH AT MOST ONE Y-VALUE