

Honors Trigonometry/Pre-Calculus Chapter 2 Practice Exam

Name: ANSWER KEY Date: _____ Period: _____

Show all work neatly and clearly in order to receive full credit. Notes and Graphing Utilities (TI-84) are not permitted on this exam. However, scientific calculators (TI-30) are permitted.

1. *True or False?* Determine whether the statement is true or false. Justify your answer.

a) The function $f(x) = -12x^2 - 1$ has no x -intercepts.

$$\begin{array}{ll}
 -12x^2 - 1 = 0 & x = \pm \sqrt{-1/12} \\
 -12x^2 = 1 & \text{SINCE THE ROOT IS IMAGINARY, THEREFORE} \\
 12x^2 = -1 & \text{NO } x\text{-INTERCEPT.} \\
 x^2 = -1/12 & \text{TRUE}
 \end{array}$$

b) The graph of the function $f(x) = 2x(x-1)^2(x+3)^3$ crosses the x -axis at $x = 1$.

$f(x)$ HAS A ZERO AT 1 WITH A MULTIPLICITY OF TWO, THEREFORE, THE POLYNOMIAL WILL TOUCH THE ROOT AT 1. FALSE

c) The graph of the function $f(x) = 2x(x-1)^2(x+3)^3$ rises to the left and falls to the right.

THE DEGREE OF $f(x)$ IS 6 AND HAS A LEADING COEFFICIENT OF 2, THEREFORE, THE GRAPH RISES TO THE LEFT & RISES TO THE RIGHT. FALSE

d) If $(7x+4)$ is a factor of some polynomial function f , then $\frac{4}{7}$ is a zero of f .

$$\begin{array}{ll}
 7x+4=0 & \text{IF } (7x+4) \text{ IS A FACTOR, THEN } -4/7 \text{ IS A ZERO.} \\
 7x=-4 & \text{FALSE} \\
 x=-4/7 &
 \end{array}$$

2. Find a polynomial function with real coefficients that has the given zeros: 3, 3, $-2i-1$

$$(x-3)^2 [x - (-1-2i)] [x - (-1+2i)]$$

$$[(x+1)+2i][(x+1)-2i]$$

$$(x+1)^2 - (2i)^2$$

$$x^2 + 2x + 1 - (4i^2)$$

$$x^2 + 2x + 1 - 4(-1)$$

$$x^4 + 2x^3 + 5x^2$$

$$-6x^3 - 12x^2 - 30x$$

$$9x^2 + 18x + 45$$

$$f(x) = x^4 - 4x^3 + 2x^2 - 12x + 45$$

$$x^2 - 6x + 9)(x^2 + 2x + 5)$$

3. Use the given zero and polynomial to find all the zeros of the function

Function: $f(x) = x^3 + 4x^2 + 14x + 20$

Zero: $-1-3i$

$$\begin{array}{r|rrrr} -1-3i & 1 & 4 & 14 & 20 \\ & & -1-3i & -12-6i & -20 \\ \hline & 1 & 3-3i & 2-6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1+3i & 1 & 3-3i & 2-6i \\ & & -1+3i & -2+6i \\ \hline & 1 & 2 & 0 \end{array}$$

$$x+2=0$$

$$x=-2$$

$$\text{ZEROS: } -2, -1 \pm 3i$$

4. Find all the zeros of the function and write the polynomial as a product of linear factors.

$$f(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$$

$$\frac{p}{q} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = \sqrt{-1}$$

$$x = \pm i$$

$$\text{ZEROS: } -3, -3, \pm i$$

$$\text{FACTORS: } (x+3)^2(x+i)(x-i)$$

5. Answer each part below. Graph the rational function with at least 5 points.

$$f(x) = \frac{x^2 + 2x + 1}{2x^2 - x - 3}$$

a) y - intercept(s)

$$f(0) = \frac{1}{-3} \quad \boxed{(0, -1/3)}$$

b) x - intercept(s)

$$f(x) = \frac{(x+1)(x+1)}{(2x-3)(x+1)} = \frac{x+1}{2x-3}$$

$$x+1=0 \quad \boxed{(-1, 0)}$$

$$x = -1$$

c) Vertical Asymptote(s)

$$2x-3=0$$

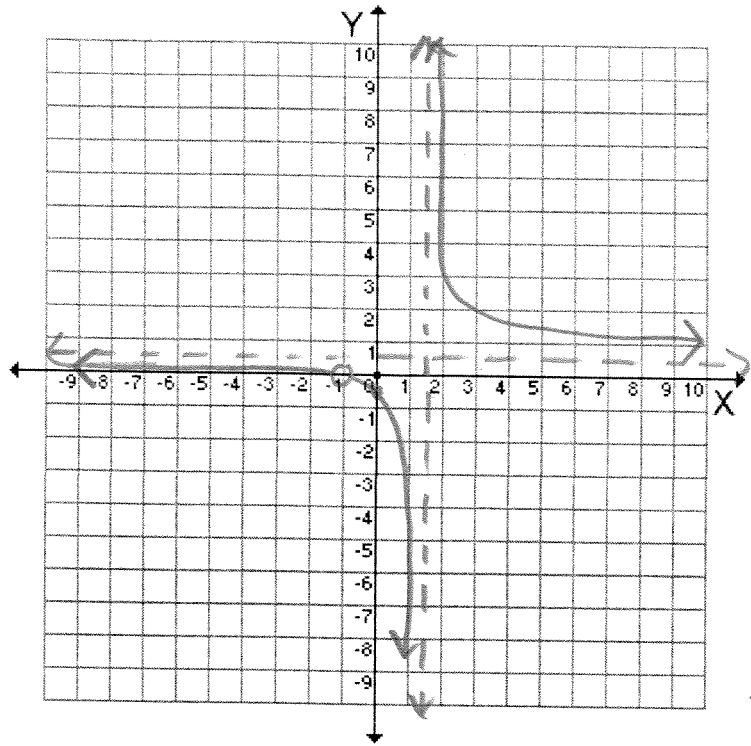
$$x = 3/2$$

d) Horizontal Asymptote(s)

$$y = 1/2$$

e) Slanted Asymptote

NONE



f) Hole(s)

$$x+1=0$$

$$x = -1$$

$$f(-1) = \frac{-1+1}{2(-1)-3} = 0$$

$$(-1, 0)$$

~~g) Write the interval(s) when $f'(x) > 0$~~

~~NONE~~

~~h) Write the interval(s) when $f'(x) < 0$~~

~~$(-\infty, 3/2) \cup (3/2, \infty)$~~

~~i) Write the interval(s) when $f'(x) = 0$~~

5. Answer each part below. Graph the rational function with at least 5 points.

$$f(x) = \frac{x^2 - x}{x + 1}$$

a) y - intercept(s)

$$f(0) = \frac{0}{1} = 0 \quad \boxed{(0, 0)}$$

b) x - intercept(s)

$$\frac{x(x-1)}{x+1}, \quad x=0 \quad x=1$$

$$\boxed{(0, 0)} \quad \boxed{(1, 0)}$$

c) Vertical Asymptote(s)

$$x = -1$$

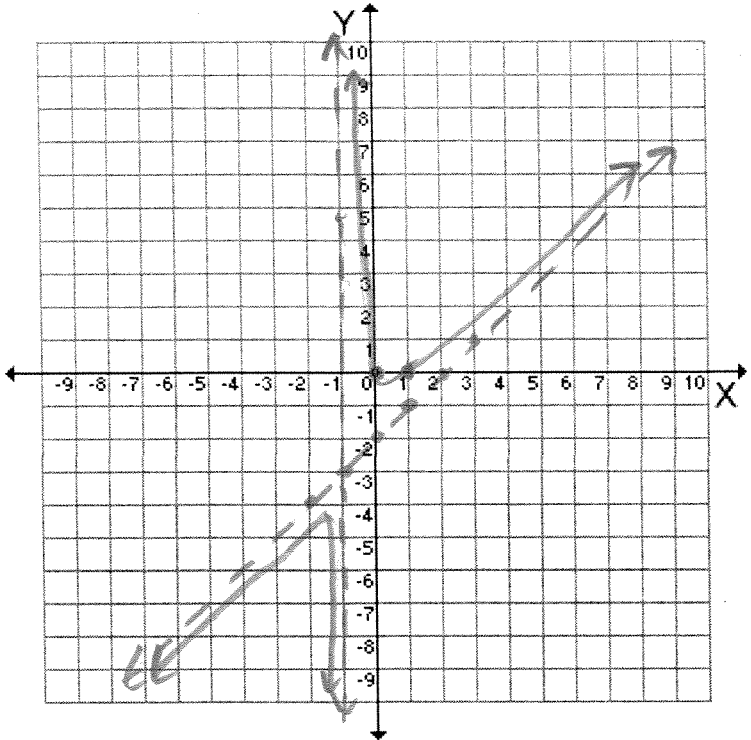
d) Horizontal Asymptote(s)

NONE

e) Slanted Asymptote

$$x+1 \overline{) \begin{array}{r} x^2 - x + 0 \\ -x^2 + x \\ \hline -2x + 0 \end{array}}$$

$$y = x - 2$$



f) Hole(s)

NONE

~~g) Write the interval(s) when $f'(x) > 0$~~

~~h) Write the interval(s) when $f'(x) < 0$~~

~~i) Write the interval(s) when $f'(x) = 0$~~

7. The polynomial $f(x) = x^5 - x^4 - 2x^3$ is given.

a) State all the zeros

$$x^3(x^2 - x - 2)$$

$$x^3(x-2)(x+1)$$

$$x = 0, 0, 0, 2, -1$$

b) Write $f(x)$ as a product of linear factors

$$x^3(x-2)(x+1)$$

c) State the y-intercept

$$f(0) = 0^5 - 0^4 - 2(0)^3 \\ = 0$$

d) Find $f'(x)$

$$f'(x) = 5x^4 - 4x^3 - 6x^2$$

e) State the critical numbers

$$5x^4 - 4x^3 - 6x^2 = 0$$

$$x^2(5x^2 - 4x - 6) = 0$$

$$x^2 = 0 \quad 5x^2 - 4x - 6 = 0$$

$$x \neq 0 \quad x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-6)}}{2(5)}$$

$$= \frac{4 \pm \sqrt{136}}{10}$$

$$x \approx 1.57 \quad x \approx -0.77$$

f) State the maxima(s) and/or minima(s)

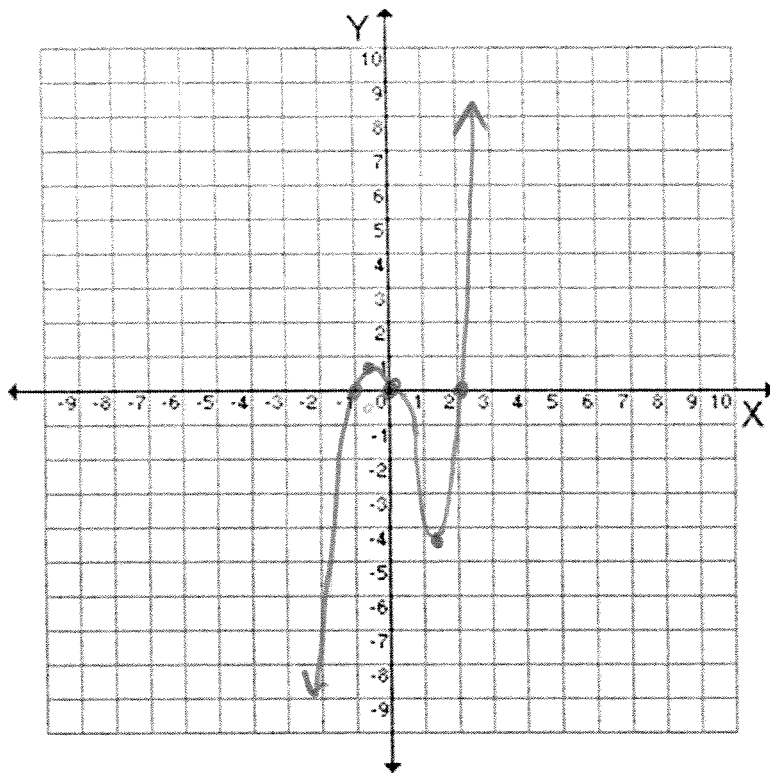
$$f(1.57) = -4.28$$

$$f(-0.77) \approx 0.29$$

g) Find $f(5)$

$$f(5) = (5)^5 - (5)^4 - 2(5)^3 \\ = 2250$$

h) Sketch the polynomial using the zeros, maxima(s), and minima(s). Note: you may need to change the y-values on the y-axis



i) Write the interval(s) when $f'(x) > 0$ $(-\infty, -0.77) \cup (1.57, \infty)$

j) Write the interval(s) when $f'(x) < 0$ $(-0.77, 1.57)$

k) Write the ~~interval(s)~~ ^{X-VALUE(S)} when $f'(x) = 0$ $1.57 \text{ \& } -0.77$

l) State the domain: \mathbb{R}

m) State the range: \mathbb{R}

n) Explain whether the graph is a function or not

$f(x)$ IS A FUNCTION IT PASSES THE VERTICAL LINE TEST

8. The polynomial $f(x) = x^3 + 2x^2 - 5x - 6$ is given.

a) State all the zeros

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$x^2 - x - 2$$

$$(x-2)(x+1) = 0$$

$$\boxed{x=2 \quad x=-1 \quad x=-3}$$

b) Write $f(x)$ as a product of linear factors

$$(x-2)(x+1)(x+3)$$

c) State the y-intercept

$$f(0) = -6$$

d) Find $f'(x)$

$$f'(x) = 3x^2 + 4x - 5$$

e) State the critical numbers

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{76}}{6}$$

$$x \approx 0.79 \quad x \approx -2.12$$

f) State the maxima(s) and/or minima(s)

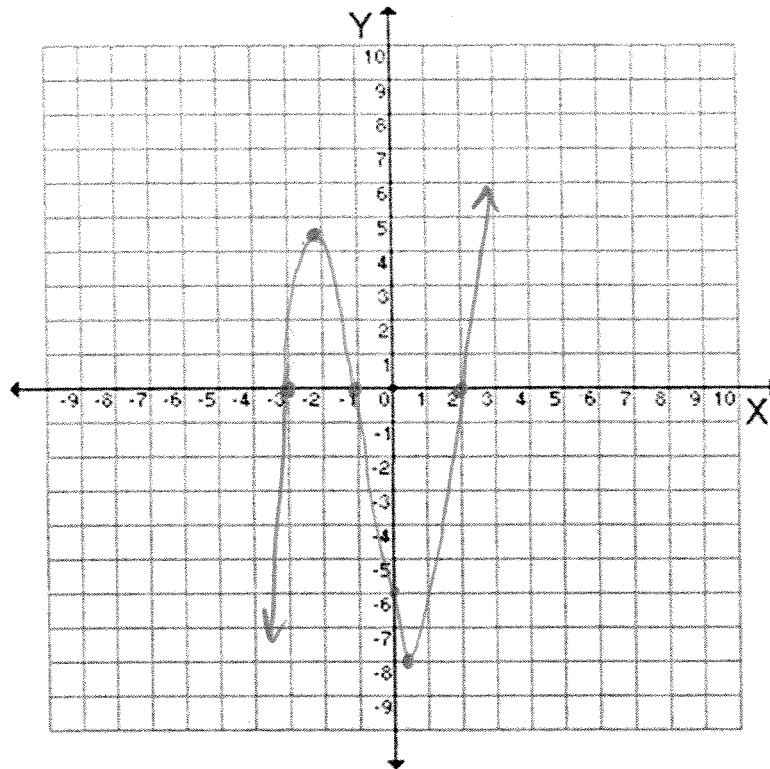
$$f(-2.12) \approx 4.06$$

$$f(0.79) \approx -8.21$$

g) Find $f(5)$

$$\begin{aligned} f(5) &= (5)^3 + 2(5)^2 - 5(5) - 6 \\ &= 144 \end{aligned}$$

h) Sketch the polynomial using the zeros, maxima(s), and minima(s). Note: you may need to change the y-values on the y-axis



i) Write the interval(s) when $f'(x) > 0$ $(-\infty, -2.12) \cup (0.79, \infty)$

j) Write the interval(s) when $f'(x) < 0$ $(-2.12, 0.79)$

k) Write the ~~interval(s)~~ ^{X-VALUE(S)} when $f'(x) = 0$ $-2.12 \text{ \& } 0.79$

l) State the domain: \mathbb{R}

m) State the range: \mathbb{R}

n) Explain whether the graph is a function or not

$f(x)$ IS A FUNCTION BECAUSE EACH X-VALUE CORRESPONDS WITH AT MOST ONE Y-VALUE