CHAPTER 6
Additional Topics in Trigonometry

C H A P T E R 6
Additional Topics in Trigonometry

Section 6.1 Law of Sines

- If ABC is any oblique triangle with sides a, b, and c, then the Law of Sines says

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

- You should be able to use the Law of Sines to solve an oblique triangle for the remaining three parts, given:
  (a) Two angles and any side (AAS or ASA)
  (b) Two sides and an angle opposite one of them (SSA)
    1. If A is acute and $h = b \sin A$:
       (a) $a < h$, no triangle is possible.
       (b) $a = h$ or $a \geq b$, one triangle is possible.
       (c) $h < a < b$, two triangles are possible.
    2. If A is obtuse and $h = b \sin A$:
       (a) $a \leq b$, no triangle is possible.
       (b) $a > b$, one triangle is possible.
- The area of any triangle equals one-half the product of the lengths of two sides times the sine of their included angle.

$$A = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}ac \sin B = \tfrac{1}{2}bc \sin A$$

Vocabulary Check

1. oblique

2. $\dfrac{b}{\sin B}$

3. (a) Two; any; AAS; ASA
(b) Two; an opposite; SSA

4. $\dfrac{1}{2}bc \sin A; \dfrac{1}{2}ab \sin C; \dfrac{1}{2}ac \sin B$

1. Given: $A = 25°, B = 60°, a = 12$

$C = 180° - 25° - 60° = 95°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{12}{\sin 25°}(\sin 60°) \approx 24.59 \text{ in.}$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{12}{\sin 25°}(\sin 95°) \approx 28.29 \text{ in.}$$

2. Given: $A = 35°, B = 55°, a = 18$

$C = 180° - 35° - 55° = 90°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{18}{\sin 35°}(\sin 55°) \approx 25.71 \text{ mm}$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{18}{\sin 35°}(\sin 90°) \approx 31.38 \text{ mm}$$

3. Given: $B = 15°, C = 125°, c = 20$

$A = 180° - 15° - 125° = 40°$

$$a = \frac{c}{\sin C}(\sin A) = \frac{20}{\sin 125°}(\sin 40°) \approx 15.69 \text{ cm}$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{20}{\sin 125°}(\sin 15°) \approx 6.32 \text{ cm}$$

4. Given: $B = 40°, C = 110°, c = 30$

$A = 180° - 40° - 110° = 30°$

$$a = \frac{c}{\sin C}(\sin A) = \frac{30}{\sin 110°}(\sin 30°) \approx 15.96 \text{ ft}$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{30}{\sin 110°}(\sin 40°) \approx 20.52 \text{ ft}$$

5. Given: $A = 80° \, 15', \; B = 25° \, 30', \; b = 2.8$

$C = 180° - 80° \, 15' - 25° \, 30' = 74° \, 15'$

$$a = \frac{b}{\sin B}(\sin A) = \frac{2.8}{\sin 25° \, 30'}(\sin 80° \, 15') \approx 6.41 \text{ km}$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{2.8}{\sin 25° \, 30'}(\sin 74° \, 15') \approx 6.26 \text{ km}$$

6. Given: $A = 88° \, 35', \; B = 22° \, 45', \; b = 50.2$

$C = 180° - 88° \, 35' - 22° \, 45' = 68° \, 40'$

$$a = \frac{b}{\sin B}(\sin A) = \frac{50.2}{\sin 22° \, 45'}(\sin 88° \, 35') \approx 129.77 \text{ yd}$$

$$c = \frac{b}{\sin B}(\sin C) = \frac{50.2}{\sin 22° \, 45'}(\sin 68° \, 40') \approx 120.92 \text{ yd}$$

7. Given: $A = 36°, a = 8, b = 5$

$$\sin B = \frac{b \sin A}{a} = \frac{5 \sin(36°)}{8} \approx 0.3674 \implies B \approx 21.6°$$

$C = 180° - A - B \approx 180° - 36° - 21.6° = 122.4°$

$$c = \frac{a}{\sin A}(\sin C) = \frac{8}{\sin(36°)} \sin(122.4°) \approx 11.49$$

8. Given: $A = 60°, a = 9, c = 10$

$$\sin C = \frac{c \sin A}{a} = \frac{10 \sin 60°}{9} \approx 0.9623 \implies C \approx 74.2° \text{ or } C \approx 105.8°$$

Case 1

$C \approx 74.2°$

$B = 180° - A - C \approx 45.8°$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 45.8°}{\sin 60°} \approx 7.45$$

Case 2

$C \approx 105.8°$

$B = 180° - A - C \approx 14.2°$

$$b = \frac{a}{\sin A}(\sin B) \approx \frac{9 \sin 14.2°}{\sin 60°} \approx 2.55$$

9. Given: $A = 102.4°, C = 16.7°, a = 21.6$

$B = 180° - A - C = 180° - 102.4° - 16.7° = 60.9°$

$$b = \frac{a}{\sin A}(\sin B) = \frac{21.6}{\sin 102.4°}(\sin 60.9°) \approx 19.32$$

$$c = \frac{a}{\sin A}(\sin C) = \frac{21.6}{\sin 102.4°}(\sin 16.7°) \approx 6.36$$

10. Given: $A = 24.3°, C = 54.6°, c = 2.68$

$B = 180° - A - C = 101.1°$

$$a = \frac{c}{\sin C}(\sin A) = \frac{2.68 \sin 24.3°}{\sin 54.6°} \approx 1.35$$

$$b = \frac{c}{\sin C}(\sin B) = \frac{2.68 \sin 101.1°}{\sin 54.6°} \approx 3.23$$

11. Given: $A = 110° 15', a = 48, b = 16$

$$\sin B = \frac{b \sin A}{a} = \frac{16 \sin 110° 15'}{48} \approx 0.31273 \implies B \approx 18° 13'$$

$C = 180° - A - B \approx 180° - 110° 15' - 18° 13' = 51° 32'$

$$c = \frac{a}{\sin A}(\sin C) = \frac{48}{\sin 110° 15'}(\sin 51° 32') \approx 40.06$$

12. Given: $B = 2° 45', b = 6.2, c = 5.8$

$$\sin C = \frac{c \sin B}{b} = \frac{5.8 \sin 2° 45'}{6.2} \approx 0.04488 \implies C \approx 2.57°(2° 34')$$

$A = 180° - B - C \approx 174.68°(174° 41')$

$$a = \frac{b}{\sin B}(\sin A) \approx \frac{6.2 \sin 174° 41'}{\sin 2° 45'} \approx 11.97$$

13. Given: $A = 110°, a = 125, b = 100$

$$\sin B = \frac{b \sin A}{a} = \frac{100 \sin 110°}{125} \approx 0.75175 \implies B \approx 48.74°$$

$C = 180° - A - B \approx 21.26°$

$$c = \frac{a}{\sin A}(\sin C) = \frac{125 \sin 21.26°}{\sin 110°} \approx 48.23$$

14. Given: $A = 110°, a = 125, b = 200$

A obtuse and $a < b \implies$ No solution

15. Given: $A = 76°, a = 18, b = 20$

$$\sin B = \frac{b \sin A}{a} = \frac{20 \sin 76°}{18} \approx 1.078$$

No solution

16. Given: $A = 76°, a = 34, b = 21$

$$\sin B = \frac{b \sin A}{a} = \frac{21 \sin 76°}{34} \approx 0.5993 \implies B \approx 36.8°$$

$C \approx 180° - 76° - 36.8° \approx 67.2°$

$$c = \frac{a}{\sin A} \sin C = \frac{34}{\sin 76°} \sin 67.2° \approx 32.30$$

17. Given: $A = 58°, a = 11.4, b = 12.8$

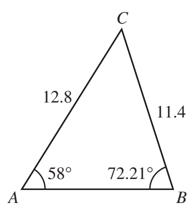
$$\sin B = \frac{b \sin A}{a} = \frac{12.8 \sin 58°}{11.4} \approx 0.9522 \implies B \approx 72.21° \text{ or } 107.79°$$

Case 1

$B \approx 72.21°$

$C = 180° - 58° - 72.21° = 49.79°$

$c = \frac{a}{\sin A}(\sin C) = \frac{11.4}{\sin 58°}(\sin 49.79°) \approx 10.27$

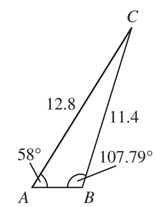


Case 2

$B \approx 107.79°$

$C \approx 180° - 58° - 107.79° = 14.21°$

$c = \frac{a}{\sin A}(\sin C) = \frac{11.4}{\sin 58°}(\sin 14.21°) \approx 3.30$



18. Given: $A = 58°, a = 4.5, b = 12.8$

$a < h = b \sin 58°$

$4.5 < 10.86$

No solution

19. $\text{Area} = \frac{1}{2}ab \sin C$

$= \frac{1}{2}(6)(10) \sin(110°)$

≈ 28.2 square units

20. $\text{Area} = \frac{1}{2}ac \sin B$

$= \frac{1}{2}(92)(30) \sin(130°)$

≈ 1057.1 square units

21. $\text{Area} = \frac{1}{2}bc \sin A$

$= \frac{1}{2}(67)(85) \sin(38° 45')$

≈ 1782.3 square units

22. $A = 5° 15', b = 4.5, c = 22$

$\text{Area} = \frac{1}{2}bc \sin A$

$= \left(\frac{1}{2}\right)(4.5)(22) \sin 5.25°$

≈ 4.529 square units

23. $\text{Area} = \frac{1}{2}ac \sin B$

$= \frac{1}{2}(103)(58) \sin 75° 15'$

≈ 2888.6 square units

24. $\text{Area} = \frac{1}{2}ab \sin C$

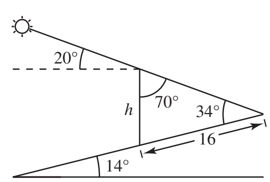
$= \frac{1}{2}(16)(20) \sin 85° 45'$

≈ 159.6 square units

25. Angle $CAB = 70°$

Angle $B = 20° + 14° = 34°$

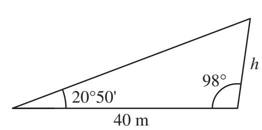
(a)



(b) $\frac{16}{\sin 70°} = \frac{h}{\sin 34°}$

(c) $h = \frac{16 \sin 34°}{\sin 70°} \approx 9.52$ meters

26. (a)



$20° 50' \approx 20.83°$

(b) $A = 180° - 98° - 20.83° = 61.17°$ (or $61° 10'$)

$$\frac{40}{\sin A} = \frac{h}{\sin(20° 50')} \implies h = \frac{40 \sin(20° 50')}{\sin 61.17°}$$

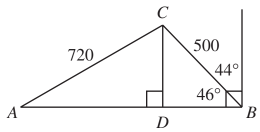
(c) $h \approx 16.2$ m

27. $\sin A = \dfrac{a \sin B}{b} = \dfrac{500 \sin(46°)}{720} \approx 0.4995$

$A \approx 29.97°$

$\angle ACD = 90° - 29.97° \approx 60°$

Bearing: S 60° W or (240° in plane navigation)

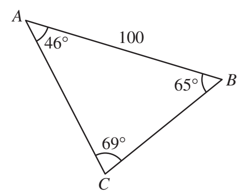


28. Given: $A = 74° - 28° = 46°$,

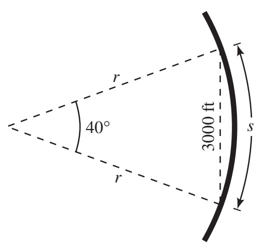
$B = 180° - 41° - 74° = 65°,\ c = 100$

$C = 180° - 46° - 65° = 69°$

$a = \dfrac{c}{\sin C}(\sin A) = \dfrac{100}{\sin 69°}(\sin 46°) \approx 77$ meters



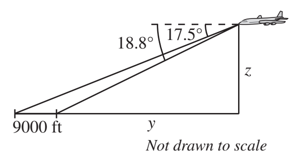
29. (a)



(b) $r = \dfrac{3000 \sin[1/2(180° - 40°)]}{\sin 40°} \approx 4385.71$ feet

(c) $s \approx 40°\left(\dfrac{\pi}{180°}\right)4385.71 \approx 3061.80$ feet

30. (a)



(b) $\dfrac{x}{\sin 17.5°} = \dfrac{9000}{\sin 1.3°}$

$x \approx 119{,}289.1261$ feet ≈ 22.6 miles

(c) $\dfrac{y}{\sin 71.2°} = \dfrac{x}{\sin 90°}$

$y = x \sin 71.2° \approx 119{,}289.1261 \sin 71.2°$

$\approx 112{,}924.963$ feet ≈ 21.4 miles

(d) $z = 119{,}289.1261 \sin 18.8° \approx 38{,}442.8$ feet

31. $\angle ACD = 65°$

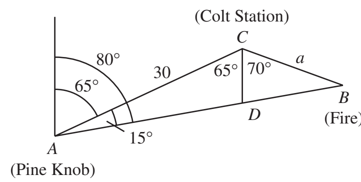
$\angle ADC = 180° - 65° - 15° = 100°$

$\angle CDB = 180° - 100° = 80°$

$\angle B = 180° - 80° - 70° = 30°$

$a = \dfrac{b}{\sin B}(\sin A) = \dfrac{30}{\sin 30°}(\sin 15°) \approx 15.53$ km

$c = \dfrac{b}{\sin B}(\sin C) = \dfrac{30}{\sin 30°}(\sin 135°) \approx 42.43$ km

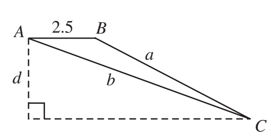


32. $A = 20°, B = 90° + 63° = 153°, c = 10\left(\dfrac{1}{4}\right) = 2.5$

$C = 180° - 20° - 153° = 7°$

$b = \dfrac{c}{\sin C}(\sin B) = \dfrac{2.5 \sin 153°}{\sin 7°} \approx 9.31$

$d \approx b \sin A \approx 9.31 \sin 20° \approx 3.2$ miles



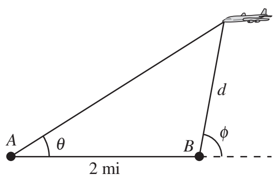
33. $\dfrac{\sin(42^\circ - \theta)}{10} = \dfrac{\sin 48^\circ}{17}$

$$\sin(42^\circ - \theta) = \frac{10}{17}\sin 48^\circ \approx 0.4371$$

$$42^\circ - \theta \approx 25.919$$

$$\theta \approx 16.1^\circ$$

34. (a)



Not drawn to scale

(b) Third angle in triangle $= \alpha$

$$\theta + \alpha + (180^\circ - \phi) = 180^\circ \implies \alpha = \phi - \theta$$

$$\frac{d}{\sin\theta} = \frac{2}{\sin\alpha}$$

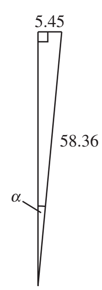
$$d = \frac{2\sin\theta}{\sin\alpha} = \frac{2\sin\theta}{\sin(\phi - \theta)}$$

35. (a) $\sin\alpha = \dfrac{5.45}{58.36} \approx 0.0934$

$$\alpha \approx 5.36^\circ$$

(b) $\dfrac{d}{\sin\beta} = \dfrac{58.36}{\sin\theta} \implies \sin\beta = \dfrac{d\sin\theta}{58.36}$

$$\beta = \sin^{-1}\left[\frac{d\sin\theta}{58.36}\right]$$



(c) $\theta + \beta + 90^\circ + 5.36^\circ = 180^\circ \implies \beta = 84.64^\circ - \theta$

$$d = \sin\beta\left(\frac{58.36}{\sin\theta}\right) = \sin(84.64^\circ - \theta)\frac{58.36}{\sin\theta}$$

(d)

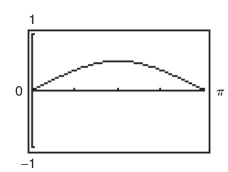
θ	10°	20°	30°	40°	50°	60°
d	324.1	154.2	95.2	63.8	43.3	28.1

36. (a) $\dfrac{\sin\alpha}{9} = \dfrac{\sin\beta}{18}$

$$\sin\alpha = 0.5\sin\beta$$

$$\alpha = \arcsin(0.5\sin\beta)$$

(b)



Domain: $0 < \beta < \pi$

Range: $0 < \alpha \le \pi/6$

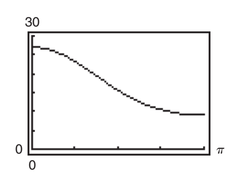
(c) $\gamma = \pi - \alpha - \beta = \pi - \beta - \arcsin(0.5\sin\beta)$

$$\frac{c}{\sin\gamma} = \frac{18}{\sin\beta}$$

$$c = \frac{18\sin\gamma}{\sin\beta}$$

$$= \frac{18\sin[\pi - \beta - \arcsin(0.5\sin\beta)]}{\sin\beta}$$

(d)



Domain: $0 < \beta < \pi$

Range: $9 < c < 27$

(e)

β	0.4	0.8	1.2	1.6	2.0	2.4	2.8
α	0.1960	0.3669	0.4848	0.5234	0.4720	0.3445	0.1683
c	25.95	23.07	19.19	15.33	12.29	10.31	9.27

As $\beta \to 0$, $c \to 27$.

As $\beta \to \pi$, $c \to 9$.

37. False. If just the three angles are known, the triangle cannot be solved.

38. True. No angle could be 90°.

39. Yes, the Law of Sines can be used to solve a right triangle if you are given at least one side and one angle, or two sides.

40. Answers will vary. $A = 36°, a = 5$

(a) $b = 4$ one solution

(b) $b = 7$ two solutions $[h = b \sin A < a < b]$

(c) $b = 10$ no solution $[a < h = b \sin A]$

41. Given: $A = 45°, B = 52°, a = 16$

$C = 180° - 45° - 52° = 83°$

$b = \frac{a}{\sin A}(\sin B) = \frac{16}{\sin 45°}(\sin 52°) \approx 17.83$

$c = \frac{a}{\sin A}(\sin C) = \frac{16}{\sin 45°}(\sin 83°) \approx 22.46$

$(a + b) \sin\left(\frac{C}{2}\right) = c \cos\left(\frac{A - B}{2}\right)$

$(16 + 17.83) \sin\left(\frac{83°}{2}\right) = 22.46 \cos\left(\frac{45° - 52°}{2}\right)$

$22.42 = 22.42$

42. Given: $A = 42°, B = 60°, a = 24$

$C = 180° - 42° - 60° = 78°$

$b = \frac{a}{\sin A}(\sin B) = \frac{24}{\sin 42°}(\sin 60°) \approx 31.06$

$c = \frac{a}{\sin A}(\sin C) = \frac{24}{\sin 42°}(\sin 78°) \approx 35.08$

$(a - b) \cos\left(\frac{C}{2}\right) = c \sin\left(\frac{A - B}{2}\right)$

$(24 - 31.06) \cos\left(\frac{78°}{2}\right) = 35.08 \sin\left(\frac{42° - 60°}{2}\right)$

$-5.49 = -5.49$

43. $\tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{12}{5}$

$\sec \theta = \frac{13}{5}$

$\cot \theta = -\frac{5}{12}$

$\csc \theta = -\frac{13}{12}$

44. $\cot \theta = \frac{9}{2}$

$\sin \theta = -\frac{2}{\sqrt{85}} = -\frac{2\sqrt{85}}{85}$

$\cos \theta = \cot \theta \cdot \sin \theta$

$= \frac{9}{2}\left(-\frac{2}{\sqrt{85}}\right) = -\frac{9}{\sqrt{85}} = -\frac{9\sqrt{85}}{85}$

$\sec \theta = -\frac{\sqrt{85}}{9}$

45. $6 \sin 8\theta \cos 3\theta = 6\left(\frac{1}{2}\right)[\sin(8\theta + 3\theta) + \sin(8\theta - 3\theta)] = 3(\sin 11\theta + \sin 5\theta)$

46. $2 \cos 2\theta \cos 5\theta = 2\left(\frac{1}{2}\right)[\cos(2\theta - 5\theta) + \cos(2\theta + 5\theta)] = \cos 3\theta + \cos 7\theta$

47. $3 \cos \frac{\pi}{6} \sin \frac{5\pi}{3} = 3\left(\frac{1}{2}\right)\left[\sin\left(\frac{\pi}{6} + \frac{5\pi}{3}\right) - \sin\left(\frac{\pi}{6} - \frac{5\pi}{3}\right)\right]$

$= \frac{3}{2}\left[\sin\left(\frac{11\pi}{6}\right) - \sin\left(-\frac{3\pi}{2}\right)\right]$

$= \frac{3}{2}\left[-\frac{1}{2} - 1\right] = -\frac{9}{4}$

48. $\frac{5}{2} \sin \frac{3\pi}{4} \sin \frac{5\pi}{6} = \frac{5}{2} \cdot \frac{1}{2}\left[\cos\left(\frac{3\pi}{4} - \frac{5\pi}{6}\right) - \cos\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)\right]$

$= \frac{5}{4}\left[\cos\left(-\frac{\pi}{12}\right) - \cos\left(\frac{19\pi}{12}\right)\right]$

$= \frac{5}{4}\left[\cos\left(\frac{\pi}{12}\right) - \cos\left(\frac{19\pi}{12}\right)\right]$

Section 6.2 Law of Cosines

- If *ABC* is any oblique triangle with sides *a, b,* and *c*, then the Law of Cosines says:

  (a) $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

  (b) $b^2 = a^2 + c^2 - 2ac\cos B$ or $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

  (c) $c^2 = a^2 + b^2 - 2ab\cos C$ or $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$
- You should be able to use the Law of Cosines to solve an oblique triangle for the remaining three parts, given:

  (a) Three sides (SSS)

  (b) Two sides and their included angle (SAS)
- Given any triangle with sides of lengths *a, b,* and *c*, then the area of the triangle is

  $$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}. \qquad \text{(Heron's Formula)}$$

Vocabulary Check

1. $c^2 = a^2 + b^2 - 2ab\cos C$

2. Heron's Area

3. $\frac{1}{2}bh$, $\sqrt{s(s-a)(s-b)(s-c)}$

1. Given: $a = 12,\ b = 16,\ c = 18$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{16^2 + 18^2 - 12^2}{2(16)(18)} \approx 0.75694 \implies A \approx 40.80^\circ$$

$$\sin B = \frac{b\sin A}{a} \approx 0.8712 \implies B \approx 60.61^\circ$$

$$C \approx 180^\circ - 60.61^\circ - 40.80^\circ = 78.59^\circ$$

2. Given: $a = 8,\ b = 18,\ c = 12$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{18^2 + 12^2 - 8^2}{2(18)(12)} \approx 0.9352 \implies A \approx 20.74^\circ$$

$$\sin C = \frac{c\sin A}{a} \approx 0.5312 \implies C \approx 32.09^\circ$$

$$B \approx 180^\circ - 32.09^\circ - 20.74^\circ = 127.17^\circ$$

3. Given: $a = 8.5,\ b = 9.2,\ c = 10.8$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{9.2^2 + 10.8^2 - 8.5^2}{2(9.2)(10.8)} \approx 0.6493 \implies A \approx 49.51^\circ$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{9.2\sin 49.51^\circ}{8.5} \approx 0.82315 \implies B \approx 55.40^\circ$$

$$C \approx 180^\circ - 55.40^\circ - 49.51^\circ = 75.09^\circ$$

4. Given: $a = 4.2,\ b = 5.4,\ c = 2.1$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{4.2^2 + 5.4^2 - 2.1^2}{2(4.2)(5.4)} \approx 0.9345 \implies C \approx 20.85°$$

$$\sin A = \frac{a \sin C}{c} \approx \frac{4.2 \sin 20.8°}{2.1} \approx 0.7102 \implies A \approx 45.38°$$

$$B \approx 180° - 20.85° - 45.38° = 113.77°$$

5. Given: $a = 10,\ c = 15,\ B = 20°$

$$b^2 = a^2 + c^2 - 2ac \cos B = 100 + 225 - 2(10)(15) \cos 20° \approx 43.0922 \implies b \approx 6.56 \text{ mm}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{43.0922 + 225 - 100}{2(6.56)(15)} \approx 0.8541 \implies A \approx 31.40°$$

$$C \approx 180° - 20° - 31.40° = 128.60°$$

6. Given: $a = 16,\ c = 10,\ B = 40°$

$$b^2 = a^2 + c^2 - 2ac \cos B = 256 + 100 - 2(16)(10) \cos 40° \approx 110.8658 \implies b \approx 10.5293 \approx 10.53 \text{ km}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{256 + 110.8658 - 100}{2(16)(10.53)} \approx 0.7920 \implies C \approx 37.62°$$

$$A \approx 180° - 40° - 37.62° = 102.38°$$

7. Given: $a = 10.4,\ c = 12.5,\ B = 50°\,30' = 50.5°$

$$b^2 = a^2 + c^2 - 2ac \cos B = 10.4^2 + 12.5^2 - 2(10.4)(12.5) \cos 50.5° \approx 99.0297 \implies b \approx 9.95 \text{ ft}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{99.0297 + 12.5^2 - 10.4^2}{2(9.95)(12.5)} \approx 0.5914 \implies A \approx 53.75° = 53°\,45'$$

$$C \approx 180° - 50.5° - 53.75° = 75.75° = 75°\,45'$$

8. Given: $a = 20.2,\ c = 6.2,\ B = 80°\,45' = 80.75°$

$$b^2 = a^2 + c^2 - 2ac \cos B = 20.2^2 + 6.2^2 - 2(20.2)(6.2) \cos 80.75° \approx 406.2172 \implies b \approx 20.15 \text{ miles}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx \frac{20.2^2 + 406.2172 - 6.2^2}{2(20.2)(20.15)} \approx 0.9528 \implies C \approx 17.68° \approx 17°\,41'$$

$$A \approx 180° - 17.68° - 80.75° = 81.57° = 81°\,34'$$

9. Given: $a = 6,\ b = 8,\ c = 12$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{64 + 144 - 36}{2(8)(12)} \approx 0.8958 \implies A \approx 26.4°$$

$$\sin B = \frac{b \sin A}{a} \approx \frac{8 \sin 26.4°}{6} \approx 0.5928 \implies B \approx 36.3°$$

$$C \approx 180° - 26.4° - 36.3° = 117.3°$$

10. Given: $a = 9, b = 3, c = 11$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3^2 + 11^2 - 9^2}{2(3)(11)} \approx 0.7424 \implies A \approx 42.1°$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{9^2 + 3^2 - 11^2}{2(9)(3)} \approx -0.574 \implies C \approx 125.0°$$

$$B = 180° - A - C \approx 12.9°$$

11. Given: $A = 50°, b = 15, c = 30$

$$a^2 = b^2 + c^2 - 2bc\cos A = 225 + 900 - 2(15)(30)\cos 50° \approx 546.49 \implies a \approx 23.38$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{546.49 + 900 - 225}{2(23.4)(30)} \approx 0.8708 \implies B \approx 29.4°$$

$$C = 180° - A - B \approx 180° - 50° - 29.5° = 100.6°$$

12. $C = 108°, a = 10, b = 7$

$$c^2 = a^2 + b^2 - 2ab\cos C = 10^2 + 7^2 - 2(10)(7)\cos 108° \approx 192.2624 \implies c \approx 13.9$$

$$\sin B = \frac{\sin C}{c}b = \frac{\sin 108°}{13.9}(7) \approx 0.4789 \implies B \approx 28.7°$$

$$A = 180° - 108° - 28.7° = 43.3°$$

13. Given: $a = 9, \ b = 12, \ c = 15$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{81 + 144 - 225}{2(9)(12)} = 0 \implies C = 90°$$

$$\sin A = \frac{9}{15} = \frac{3}{5} \implies A \approx 36.9°$$

$$B \approx 180° - 90° - 36.9° = 53.1°$$

14. Given: $a = 45, b = 30, c = 72$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{45^2 + 30^2 - 72^2}{2(45)(30)} \approx -0.8367 \implies C \approx 146.8°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{45^2 + 72^2 - 30^2}{2(45)(72)} \approx 0.9736 \implies B \approx 13.2°$$

$$A = 180° - B - C = 20.0°$$

15. Given: $a = 75.4, b = 48, c = 48$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{48^2 + 48^2 - 75.4^2}{2(48)(48)} \approx -0.2338 \implies A \approx 103.5°$$

$$\sin B = \frac{b\sin A}{a} \approx \frac{48\sin(103.5°)}{75.4} \approx 0.6190 \implies B \approx 38.2°$$

$C = B \approx 38.2°$ (Because of roundoff error, $A + B + C \neq 180°$.)

16. Given: $a = 1.42, b = 0.75, c = 1.25$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(0.75)^2 + (1.25)^2 - (1.42)^2}{2(0.75)(1.25)} = 0.05792 \implies A \approx 86.7^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{(1.42)^2 + (1.25)^2 - (0.75)^2}{2(1.42)(1.25)} \approx 0.8497 \implies B \approx 31.8^\circ$$

$$180^\circ - 86.7^\circ - 31.8^\circ \approx 61.5^\circ$$

17. Given: $B = 8^\circ\, 15' = 8.25^\circ, a = 26, c = 18$

$$b^2 = a^2 + c^2 - 2ac \cos B = 26^2 + 18^2 - 2(26)(18)\cos(8.25^\circ) \approx 73.6863 \implies b \approx 8.58$$

$$\sin C = \frac{c \sin B}{b} \approx \frac{18 \sin(8.25^\circ)}{8.58} \approx 0.3 \implies C \approx 17.51^\circ \approx 17^\circ\, 31'$$

$$A = 180^\circ - B - C \approx 180^\circ - 8.25^\circ - 17.51^\circ = 154.24^\circ \approx 154^\circ\, 14'$$

18. Given: $B = 10^\circ\, 35' \approx 10.583^\circ, a = 40, c = 30$

$$b^2 = a^2 + c^2 - 2ac \cos B \approx 140.8268 \implies b \approx 11.87$$

$$\sin A = \frac{a \sin B}{b} \approx 0.6189 \implies A \approx 141.75^\circ \approx 141^\circ\, 45'$$

$$C = 180 - A - B = 27.67^\circ \text{ or } 27^\circ\, 40'$$

19. Given: $B = 75^\circ\, 20' \approx 75.33^\circ, a = 6.2, c = 9.5$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$= 6.2^2 + 9.5^2 - 2(6.2)(9.5)\cos(75.33^\circ)$$

$$= 98.86$$

$$b \approx 9.94$$

$$\sin C = \frac{c \sin B}{b} = \frac{9.5 \sin(75.33^\circ)}{9.94}$$

$$\approx 0.9246 \implies C \approx 67.6^\circ$$

$$A = 180^\circ - B - C \approx 37.1^\circ$$

20. Given: $C = 15^\circ\, 15' \approx 15.25^\circ, a = 6.25, b = 2.15$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= 6.25^2 + 2.15^2 - 2(6.25)(2.15)\cos 15.25^\circ$$

$$\approx 17.7563 \implies c \approx 4.21$$

$$\sin B = \frac{b \sin C}{c}$$

$$\approx 0.1342 \implies B \approx 7.71^\circ \approx 7^\circ\, 43'$$

$$A = 180^\circ - B - C \approx 157.04^\circ \approx 157^\circ\, 2'$$

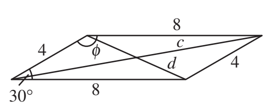
21. $d^2 = 4^2 + 8^2 - 2(4)(8)\cos 30^\circ$

$$\approx 24.57 \implies d \approx 4.96$$

$$2\phi = 360^\circ - 2\theta \implies \phi = 150^\circ$$

$$c^2 = 4^2 + 8^2 - 2(4)(8)\cos 150^\circ \approx 135.43$$

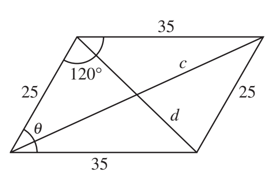
$$c \approx 11.64$$



22. $c^2 = 25^2 + 35^2 - 2(25)(35)\cos 120^\circ = 2725 \implies c \approx 52.2$

$$2\theta = 360^\circ - 2(120^\circ) = 120^\circ \implies \theta = 60^\circ$$

$$d^2 = 25^2 + 35^2 - 2(25)(35)\cos 60^\circ = 975 \implies d \approx 31.22$$



23. $\cos\phi = \dfrac{10^2 + 14^2 - 20^2}{2(10)(14)}$

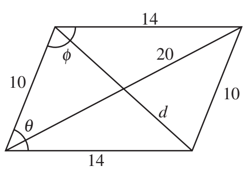
$\phi \approx 111.8°$

$2\theta \approx 360° - 2(111.80°)$

$\theta = 68.2°$

$d^2 = 10^2 + 14^2 - 2(10)(14)\cos 68.2°$

$d \approx 13.86$

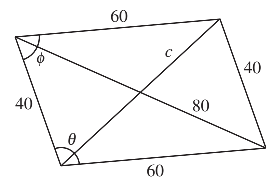


24. $\cos\theta = \dfrac{40^2 + 60^2 - 80^2}{2(40)(60)} \approx -\dfrac{1}{4} \Rightarrow \theta \approx 104.5°$

$2\phi \approx 360° - 2(104.5°) = 151° \Rightarrow \phi = 75.5°$

$c^2 \approx 40^2 + 60^2 - 2(40)(60)\cos 75.5° = 4000$

$c \approx 63.25$



25. $\cos\alpha = \dfrac{15^2 + 12.5^2 - 10^2}{2(15)(12.5)} = 0.75 \Rightarrow \alpha \approx 41.41°$

$\cos\beta = \dfrac{15^2 + 10^2 - 12.5^2}{2(15)(10)} = 0.5625 \Rightarrow \beta \approx 55.77°$

$\delta = 180° - 41.41° - 55.77° \approx 82.82°$

$\mu = 180° - \delta \approx 97.18°$

$b^2 = 12.5^2 + 10^2 - 2(12.5)(10)\cos(97.18°) \approx 287.50$

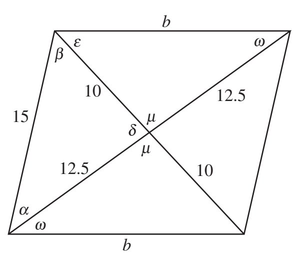
$b \approx 16.96$

$\sin\omega = \dfrac{10}{16.96}\sin\mu \approx 0.585 \Rightarrow \omega \approx 35.8°$

$\sin\epsilon = \dfrac{12.5}{16.96}\sin\mu \approx 0.731 \Rightarrow \epsilon \approx 47°$

$\theta = \alpha + \omega \approx 77.2°$

$\phi = \beta + \epsilon \approx 102.8°$



26. $\cos\alpha = \dfrac{25^2 + 17.5^2 - 25^2}{2(25)(17.5)}$

$\alpha \approx 69.513°$

$\beta \approx 180 - \alpha \approx 110.487°$

$a^2 = 17.5^2 + 25^2 - 2(17.5)(25)\cos 110.487°$

$a \approx 35.18$

$z = 180 - 2\alpha \approx 40.974$

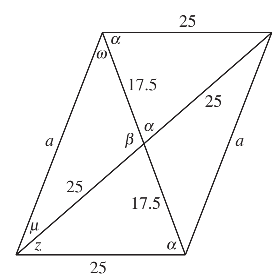
$\cos\mu = \dfrac{25^2 + 35.18^2 - 17.5^2}{2(25)(35.18)}$

$\mu \approx 27.772°$

$\theta = \mu + z \approx 68.7°$

$\omega = 180° - \mu - \beta \approx 41.741°$

$\phi = \omega + \alpha \approx 111.3°$



27. Given: $a = 12, b = 24, c = 18$

$s = \dfrac{a + b + c}{2} = 27$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27(15)(3)(9)}$

≈ 104.57 square inches

28. Given: $a = 25, b = 35, c = 32$

$s = \dfrac{a + b + c}{2} = 46$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{46(21)(11)(14)}$

≈ 385.70 square meters

29. Given: $a = 5, b = 8, c = 10$

$$s = \frac{a+b+c}{2} = \frac{23}{2} = 11.5$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{11.5(6.5)(3.5)(1.5)}\\ &\approx 19.81 \text{ square units}\end{aligned}$$

30. $s = \dfrac{a+b+c}{2} = \dfrac{14+17+7}{2} = 19$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{19(5)(2)(12)}\\ &\approx 47.7 \text{ square units}\end{aligned}$$

31. Given: $a = 1.24, b = 2.45, c = 1.25$

$$s = \frac{a+b+c}{2} = 2.47$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{2.47(1.23)(0.02)(1.22)}\\ &\approx 0.27 \text{ square feet}\end{aligned}$$

32. Given: $a = 2.4, b = 2.75, c = 2.25$

$$s = \frac{a+b+c}{2} = 3.7$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{3.7(1.3)(0.95)(1.45)}\\ &\approx 2.57 \text{ square centimeters}\end{aligned}$$

33. Given: $a = 3.5, b = 10.2, c = 9$

$$s = \frac{a+b+c}{2} = 11.35$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{11.35(7.85)(1.15)(2.35)}\\ &\approx 15.52 \text{ square units}\end{aligned}$$

34. Given: $a = 75.4, b = 52, c = 52$

$$s = \frac{75.4+52+52}{2} = 89.7$$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{89.7(14.3)(37.7)(37.7)}\\ &\approx 1350 \text{ square units}\end{aligned}$$

35. Given: $a = 10.59, b = 6.65, c = 12.31$

$$s = \frac{a+b+c}{2} = 14.775$$

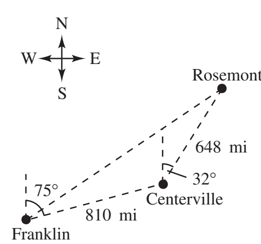
$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{14.775(4.185)(8.125)(2.465)}\\ &\approx 35.19 \text{ square units}\end{aligned}$$

36. $s = \dfrac{a+b+c}{2} = \dfrac{4.45+1.85+3.00}{2} = 4.65$

$$\begin{aligned}\text{Area} &= \sqrt{s(s-a)(s-b)(s-c)}\\ &= \sqrt{4.65(0.2)(2.8)(1.65)}\\ &\approx 2.07 \text{ square units}\end{aligned}$$

37. $B = 105° + 32° = 137°$

$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cdot \cos B\\ &= 648^2 + 810^2 - 2(648)(810)\cos(137°)\\ &= 1{,}843{,}749.862\\ b &= 1357.8 \text{ miles}\end{aligned}$$



From the Law of Sines:

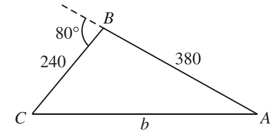
$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin A = \frac{a}{b}\sin B = \frac{648}{1357.8}\sin(137°) \approx 0.32548$$

$$\Rightarrow A \approx 19° \Rightarrow \text{Bearing S } 56° \text{ W (or } 236° \text{ for airplane navigation)}$$

38. Angle at $B = 180° - 80° = 100°$

$b^2 = 240^2 + 380^2 - 2(240)(380)\cos 100°$

$\approx 233{,}673.4 \implies b \approx 483.4$ meters



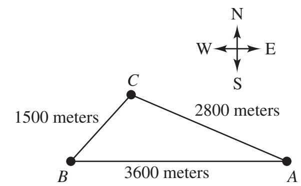
39. $\cos B = \dfrac{1500^2 + 3600^2 - 2800^2}{2(1500)(3600)} \approx 0.6824 \implies B \approx 46.97°$

Bearing at B: $90° - 46.97°$: N 43.03° E

$\cos C = \dfrac{1500^2 + 2800^2 - 3600^2}{2(1500)(2800)} \approx -0.3417 \implies C \approx 109.98°$

Bearing at C: $C - (90° - 46.97°) = 66.95°$

S 66.95° E



40. $\cos\theta = \dfrac{2^2 + 3^2 - (4.5)^2}{2(2)(3)} \approx -0.60417$

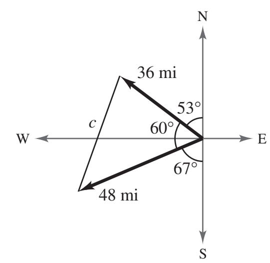
$\theta \approx 127.2°$

41. $C = 180° - 53° - 67° = 60°$

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 36^2 + 48^2 - 2(36)(48)(0.5) = 1872$

$c \approx 43.3$ mi



42. The angles at the base of the tower are 96° and 84°. The longer guy wire g_1 is given by:

${g_1}^2 = 75^2 + 100^2 - 2(75)(100)\cos 96° \approx 17{,}192.9 \implies g_1 \approx 131.1$ feet

The shorter guy wire g_2 is given by: ${g_2}^2 = 75^2 + 100^2 - 2(75)(100)\cos 84° \approx 14{,}057.1 \implies g_2 \approx 118.6$ feet

43. $\overline{RS} = \sqrt{8^2 + 10^2} = \sqrt{164} = 2\sqrt{41} \approx 12.8$ feet

$\overline{PQ} = \dfrac{1}{2}\sqrt{16^2 + 10^2} = \dfrac{1}{2}\sqrt{356} = \sqrt{89} \approx 9.4$ feet

$\tan P = \dfrac{10}{16}$

$P = \arctan\dfrac{5}{8} \approx 32.0°$

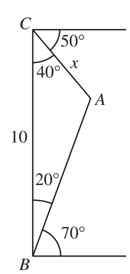
$\overline{QS} = \sqrt{8^2 + 9.4^2 - 2(8)(9.4)\cos 32°}$

$\approx \sqrt{24.81} \approx 5.0$ feet

44. $A = 180° - 40° - 20° = 120°$

$x = \dfrac{(\sin 20°)}{\sin 120°}(10)$

≈ 3.95 feet



45. $s = \dfrac{a + b + c}{2} = \dfrac{145 + 257 + 290}{2} = 346$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{346(201)(89)(56)} \approx 18{,}617.7$ square feet

46. The height is $h = 70\sin 70° \approx 65.778$.

Area = base × height

$= (100)(65.778) \approx 6577.8$ square meters

47. (a) $7^2 = 1.5^2 + x^2 - 2(1.5)(x)\cos\theta$

$49 = 2.25 + x^2 - 3x\cos\theta$

(b) $$x^2 - 3x\cos\theta = 46.75$$

$$x^2 - 3x\cos\theta + \left(\frac{3\cos\theta}{2}\right)^2 = 46.75 + \left(\frac{3\cos\theta}{2}\right)^2$$

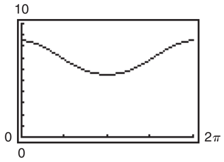
$$\left[x - \frac{3\cos\theta}{2}\right]^2 = \frac{187}{4} + \frac{9\cos^2\theta}{4}$$

$$x - \frac{3\cos\theta}{2} = \pm\sqrt{\frac{187 + 9\cos^2\theta}{4}}$$

Choosing the positive values of x, we have

$$x = \frac{1}{2}\left(3\cos\theta + \sqrt{9\cos^2\theta + 187}\right).$$

(c)



(d) Note that $x = 8.5$ when $\theta = 0$ and $\theta = 2\pi$, and $x = 5.5$ when $\theta = \pi$. Thus, the distance is

$2(8.5 - 5.5) = 2(3) = 6$ inches.

48. (a) $d^2 = 10^2 + 7^2 - 2(10)(7)\cos\theta \implies d = \sqrt{149 - 140\cos\theta}$

(b) $\theta = \arccos\left[\frac{10^2 + 7^2 - d^2}{2(10)(7)}\right] = \arccos\left[\frac{149 - d^2}{140}\right]$

(c) $s = \frac{360° - \theta}{360°}(2\pi r) = \frac{(360° - \theta)\pi}{45°}$

(d)

d (inches)	9	10	12	13	14	15	16
θ (degrees)	60.9°	69.5°	88.0°	98.2°	109.6°	122.9°	139.8°
s (inches)	20.88	20.28	18.99	18.28	17.48	16.55	15.37

49. False. This is not a triangle! $5 + 10 < 16$

50. True. The third side is found by the Law of Cosines. The other angles are determined by the Law of Sines.

51. False. $s = \frac{a + b + c}{2}$, not $\frac{a + b + c}{3}$.

52. $$\frac{1}{2}bc(1 + \cos A) = \frac{1}{2}bc\left[1 + \frac{b^2 + c^2 - a^2}{2bc}\right]$$
$$= \frac{1}{2}bc\left[\frac{2bc + b^2 + c^2 - a^2}{2bc}\right]$$
$$= \frac{1}{4}[(b + c)^2 - a^2]$$
$$= \frac{1}{4}[(b + c) + a][(b + c) - a]$$
$$= \frac{b + c + a}{2} \cdot \frac{b + c - a}{2}$$
$$= \frac{a + b + c}{2} \cdot \frac{-a + b + c}{2}$$

53. $$\frac{1}{2}bc(1 - \cos A) = \frac{1}{2}bc\left[1 - \frac{b^2 + c^2 - a^2}{2bc}\right]$$
$$= \frac{1}{2}bc\left[\frac{2bc - b^2 - c^2 + a^2}{2bc}\right]$$
$$= \frac{1}{4}[a^2 - (b - c)^2]$$
$$= \frac{1}{4}[(a - b + c)(a + b - c)]$$
$$= \left(\frac{a - b + c}{2}\right)\left(\frac{a + b - c}{2}\right)$$

54. $$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{b^2 + c^2 - a^2}{a(2bc)} + \frac{a^2 + c^2 - b^2}{b(2ac)} + \frac{a^2 + b^2 - c^2}{c(2ab)}$$
$$= \frac{a^2 + b^2 + c^2}{2abc}$$

55. Since $0 < C < 180°$, $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{1 + \cos C}{2}}$.

Hence, $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{1 + (a^2 + b^2 - c^2)/(2ab)}{2}} = \sqrt{\frac{2ab + a^2 + b^2 - c^2}{4ab}}$.

On the other hand,

$$\begin{aligned} s(s - c) &= \frac{1}{2}(a + b + c)\left(\frac{1}{2}(a + b + c) - c\right) \\ &= \frac{1}{2}(a + b + c)\frac{1}{2}(a + b - c) \\ &= \frac{1}{4}((a + b)^2 - c^2) \\ &= \frac{1}{4}(a^2 + b^2 + 2ab - c^2). \end{aligned}$$

Thus, $\sqrt{\frac{s(s - c)}{ab}} = \sqrt{\frac{a^2 + b^2 + 2ab - c^2}{4ab}}$ and we have verified that $\cos\left(\frac{C}{2}\right) = \sqrt{\frac{s(s - c)}{ab}}$.

56. Since $0 < C < 180°$, $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{1 - \cos C}{2}}$.

Hence, $\sin\left(\frac{C}{2}\right) = \sqrt{\frac{1 - (a^2 + b^2 - c^2)/(2ab)}{2}} = \sqrt{\frac{2ab - a^2 - b^2 + c^2}{4ab}}$.

On the other hand,

$$\begin{aligned} (s - a)(s - b) &= \left[\frac{1}{2}(a + b + c) - a\right]\left[\frac{1}{2}(a + b + c) - b\right] \\ &= \frac{1}{2}(b + c - a)\frac{1}{2}(a + c - b) \\ &= \frac{1}{4}[c - (a - b)][c + (a - b)] \\ &= \frac{1}{4}[c^2 - (a - b)^2] \\ &= \frac{1}{4}(c^2 - a^2 - b^2 + 2ab). \end{aligned}$$

Thus, $\sqrt{\frac{(s - a)(s - b)}{ab}} = \sqrt{\frac{c^2 - a^2 - b^2 + 2ab}{4ab}} = \sin\left(\frac{C}{2}\right)$.

57. Given: $a = 12,\ b = 30,\ A = 20°$

$a^2 = b^2 + c^2 - 2bc \cos A$

$12^2 = 30^2 + c^2 - 2(30)(c) \cos 20°$

$c^2 - (60 \cos 20°)c + 756 = 0$

Solving this quadratic equation, $c \approx 21.97, 34.41$.

—CONTINUED—

57. —CONTINUED—

For $c = 21.97$,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{12^2 + 21.97^2 - 30^2}{2(12)(21.97)} \approx -0.5184 \implies B \approx 121.2°$$

$$C \approx 180° - 121.2° - 20° = 38.8°$$

For $c = 34.41$,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{12^2 + 34.41^2 - 30^2}{2(12)(34.41)} \approx 0.5183 \implies B \approx 58.8°$$

$$C \approx 180° - 58.8° - 20° = 101.2°.$$

Using the Law of Sines, $\sin B = \dfrac{b \sin A}{a} = \dfrac{30 \sin 20°}{12} \approx 0.8551 \implies B \approx 58.8°$ or $121.2°$.

For $B = 58.8°$, $C = 180° - 58.8° - 20° = 101.2°$ and $c = \dfrac{a \sin C}{\sin A} \approx 34.42$.

For $B = 121.2°$, $C = 180° - 121.2° - 20° = 38.8°$ and $c = \dfrac{a \sin C}{\sin A} \approx 21.98$.

58. Answers will vary. The Law of Cosines can be used to solve the single-solution case of SSA. It cannot be used for the no-solution case.

59. $\arcsin(-1) = -\dfrac{\pi}{2}$ because $\sin\left(-\dfrac{\pi}{2}\right) = -1$.

60. $\cos^{-1} 0 = \dfrac{\pi}{2}$ because $\cos \dfrac{\pi}{2} = 0$.

61. $\tan^{-1}\left(\sqrt{3}\right) = \dfrac{\pi}{3}$ because $\tan\left(\dfrac{\pi}{3}\right) = \sqrt{3}$.

62. $\arcsin\left(\dfrac{-\sqrt{3}}{2}\right) = \dfrac{-\pi}{3}$ because $\sin\left(\dfrac{-\pi}{3}\right) = \dfrac{-\sqrt{3}}{2}$.

Section 6.3 Vectors in the Plane

- A vector $\mathbf{v}$ is the collection of all directed line segments that are equivalent to a given directed line segment $\overrightarrow{PQ}$.
- You should be able to *geometrically* perform the operations of vector addition and scalar multiplication.
- The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

  $$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$
- The magnitude of $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$.
- You should be able to perform the operations of scalar multiplication and vector addition in component form.
- You should know the following properties of vector addition and scalar multiplication.
  - (a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
  - (b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
  - (c) $\mathbf{u} + \mathbf{0} = \mathbf{u}$
  - (d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
  - (e) $c(d\mathbf{u}) = (cd)\mathbf{u}$
  - (f) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
  - (g) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
  - (h) $1(\mathbf{u}) = \mathbf{u}$, $0\mathbf{u} = \mathbf{0}$
  - (i) $\|c\mathbf{v}\| = |c| \, \|\mathbf{v}\|$

—CONTINUED—

Section 6.3 —CONTINUED—

- A unit vector in the direction of $\mathbf{v}$ is given by $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$.
- The standard unit vectors are $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$. $\mathbf{v} = \langle v_1, v_2\rangle$ can be written as $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$.
- A vector $\mathbf{v}$ with magnitude $\|\mathbf{v}\|$ and direction θ can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \|\mathbf{v}\|(\cos\theta)\mathbf{i} + \|\mathbf{v}\|(\sin\theta)\mathbf{j}$ where $\tan\theta = b/a$.

Vocabulary Check

1. directed line segment
2. initial, terminal
3. magnitude
4. vector
5. standard position
6. unit vector
7. multiplication, addition
8. resultant
9. linear combination, horizontal, vertical

1. $\mathbf{u} = \langle 6 - 2, 5 - 4\rangle = \langle 4, 1\rangle = \mathbf{v}$

2. $\mathbf{u} = \langle -3 - 0, -4 - 4\rangle = \langle -3, -8\rangle$

$\mathbf{v} = \langle 0 - 3, -5 - 3\rangle = \langle -3, -8\rangle$

$\mathbf{u} = \mathbf{v}$

3. Initial point: $(0, 0)$

Terminal point: $(4, 3)$

$\mathbf{v} = \langle 4 - 0, 3 - 0\rangle = \langle 4, 3\rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + 3^2} = 5$

4. Initial point: $(0, 0)$

Terminal point: $(4, -2)$

$\mathbf{v} = \langle 4 - 0, -2 - 0\rangle = \langle 4, -2\rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5} \approx 4.47$

5. Initial point: $(2, 2)$

Terminal point: $(-1, 4)$

$\mathbf{v} = \langle -1 - 2, 4 - 2\rangle = \langle -3, 2\rangle$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + 2^2} = \sqrt{13} \approx 3.61$

6. Initial point: $(-1, -1)$

Terminal point: $(3, 5)$

$\mathbf{v} = \langle 3 - (-1), 5 - (-1)\rangle = \langle 4, 6\rangle$

$\|\mathbf{v}\| = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13} \approx 7.21$

7. Initial point: $(3, -2)$

Terminal point: $(3, 3)$

$\mathbf{v} = \langle 3 - 3, 3 - (-2)\rangle = \langle 0, 5\rangle$

$\|\mathbf{v}\| = 5$

8. Initial point: $(-4, -1)$

Terminal point: $(3, -1)$

$\mathbf{v} = \langle 3 - (-4), -1 - (-1)\rangle = \langle 7, 0\rangle$

$\|\mathbf{v}\| = \sqrt{7^2 + 0^2} = 7$

9. Initial point: $\left(\frac{2}{5}, 1\right)$

Terminal point: $\left(1, \frac{2}{5}\right)$

$\mathbf{v} = \left\langle 1 - \frac{2}{5}, \frac{2}{5} - 1\right\rangle = \left\langle \frac{3}{5}, -\frac{3}{5}\right\rangle$

$\|\mathbf{v}\| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{3}{5}\right)^2} = \sqrt{\frac{18}{25}} = \frac{3}{5}\sqrt{2}$

10. Initial point: $\left(\frac{7}{2}, 0\right)$

Terminal point: $\left(0, -\frac{7}{2}\right)$

$\mathbf{v} = \left\langle 0 - \frac{7}{2}, -\frac{7}{2} - 0\right\rangle = \left\langle -\frac{7}{2}, -\frac{7}{2}\right\rangle$

$\|\mathbf{v}\| = \sqrt{\left(-\frac{7}{2}\right)^2 + \left(-\frac{7}{2}\right)^2} = \sqrt{\frac{98}{4}} = \frac{7}{2}\sqrt{2}$

11. Initial point: $\left(\frac{-2}{3}, -1\right)$

Terminal point: $\left(\frac{1}{2}, \frac{4}{5}\right)$

$$\mathbf{v} = \left\langle \frac{1}{2} - \left(-\frac{2}{3}\right), \frac{4}{5} - (-1) \right\rangle = \left\langle \frac{7}{6}, \frac{9}{5} \right\rangle$$

$$\|\mathbf{v}\| = \sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{9}{5}\right)^2} = \frac{\sqrt{4141}}{30} \approx 2.1450$$

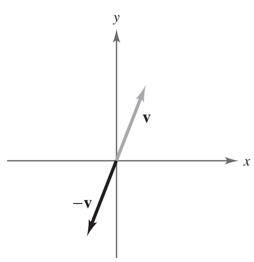
12. Initial point: $\left(\frac{5}{2}, -2\right)$

Terminal point: $\left(1, \frac{2}{5}\right)$

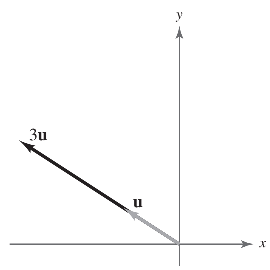
$$\mathbf{v} = \left\langle 1 - \frac{5}{2}, \frac{2}{5} - (-2) \right\rangle = \left\langle -\frac{3}{2}, \frac{12}{5} \right\rangle$$

$$\|\mathbf{v}\| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{12}{5}\right)^2} = \frac{3\sqrt{89}}{10} \approx 2.8302$$

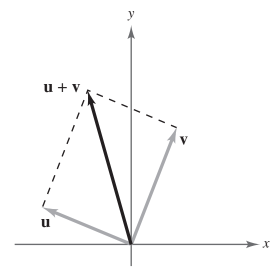
13. $-\mathbf{v}$



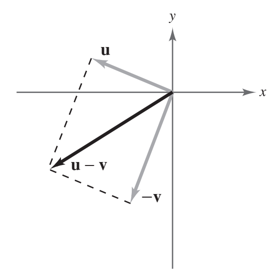
14. $3\mathbf{u}$



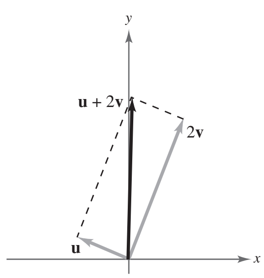
15. $\mathbf{u} + \mathbf{v}$



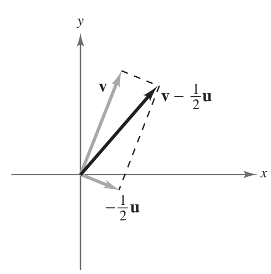
16. $\mathbf{u} - \mathbf{v}$



17. $\mathbf{u} + 2\mathbf{v}$



18. $\mathbf{v} - \frac{1}{2}\mathbf{u}$



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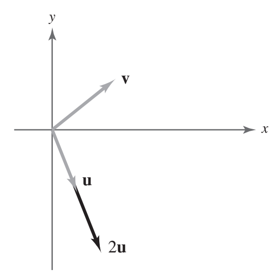

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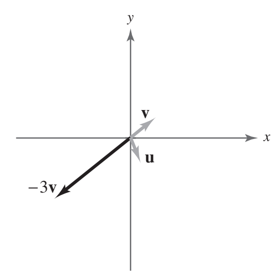

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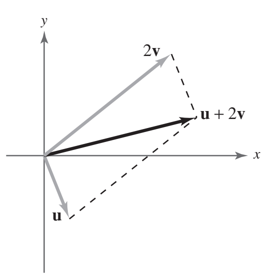

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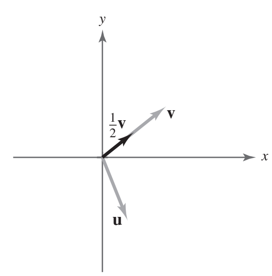
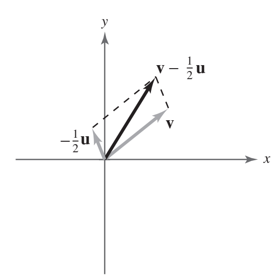


23.



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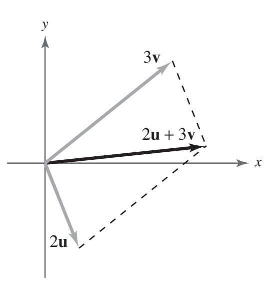


25. $\mathbf{u} = \langle 4, 2 \rangle$, $\mathbf{v} = \langle 7, 1 \rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle 11, 3 \rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -3, 1 \rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle 8, 4 \rangle - \langle 21, 3 \rangle = \langle -13, 1 \rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle 7, 1 \rangle + \langle 16, 8 \rangle = \langle 23, 9 \rangle$

26. (a) $\mathbf{u} + \mathbf{v} = \langle 5, 3\rangle + \langle -4, 0\rangle = \langle 1, 3\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 5, 3\rangle - \langle -4, 0\rangle = \langle 9, 3\rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = 2\langle 5, 3\rangle - 3\langle -4, 0\rangle = \langle 22, 6\rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle -4, 0\rangle + 4\langle 5, 3\rangle = \langle 16, 12\rangle$

27. $\mathbf{u} = \langle -6, -8\rangle, \mathbf{v} = \langle 2, 4\rangle$

(a) $\mathbf{u} + \mathbf{v} = \langle -4, -4\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -8, -12\rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle -12, -16\rangle - \langle 6, 12\rangle = \langle -18, -28\rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle 2, 4\rangle + 4\langle -6, -8\rangle = \langle -22, -28\rangle$

28. (a) $\mathbf{u} + \mathbf{v} = \langle 0, -5\rangle + \langle -3, 9\rangle = \langle -3, 4\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 3, -14\rangle$

(c) $2\mathbf{u} - 3\mathbf{v} = \langle 0, -10\rangle - \langle -9, 27\rangle = \langle 9, -37\rangle$

(d) $\mathbf{v} + 4\mathbf{u} = \langle -3, 9\rangle + \langle 0, -20\rangle = \langle -3, -11\rangle$

29. $\mathbf{u} = \mathbf{i} + \mathbf{j}, \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

(b) $\mathbf{u} - \mathbf{v} = -\mathbf{i} + 4\mathbf{j}$

(c) $2\mathbf{u} - 3\mathbf{v} = (2\mathbf{i} + 2\mathbf{j}) - (6\mathbf{i} - 9\mathbf{j}) = -4\mathbf{i} + 11\mathbf{j}$

(d) $\mathbf{v} + 4\mathbf{u} = (2\mathbf{i} - 3\mathbf{j}) + (4\mathbf{i} + 4\mathbf{j}) = 6\mathbf{i} + \mathbf{j}$

30. $\mathbf{u} = 2\mathbf{i} - \mathbf{j}, \mathbf{v} = -\mathbf{i} + \mathbf{j}$

(a) $\mathbf{u} + \mathbf{v} = \mathbf{i}$

(b) $\mathbf{u} - \mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

(c) $2\mathbf{u} - 3\mathbf{v} = (4\mathbf{i} - 2\mathbf{j}) - (-3\mathbf{i} + 3\mathbf{j}) = 7\mathbf{i} - 5\mathbf{j}$

(d) $\mathbf{v} + 4\mathbf{u} = 7\mathbf{i} - 3\mathbf{j}$

31. $\mathbf{w} = \mathbf{u} + \mathbf{v}$

32. $\mathbf{v} = \mathbf{w} - \mathbf{u}$

33. $\mathbf{u} = \mathbf{w} - \mathbf{v}$

34. $2\mathbf{v} = 2(\mathbf{w} - \mathbf{u})$
$= 2\mathbf{w} - 2\mathbf{u}$

35. $\|\langle 6, 0\rangle\| = 6$

Unit vector: $\frac{1}{6}\langle 6, 0\rangle = \langle 1, 0\rangle$

36. $\mathbf{u} = \langle 0, -2\rangle$

$\|\mathbf{u}\| = 2$

Unit vector:

$\frac{1}{2}\langle 0, -2\rangle = \langle 0, -1\rangle$

37. $\mathbf{v} = \langle -1, 1\rangle$

$\|\mathbf{v}\| = \sqrt{2}$

$$\text{Unit vector} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$$

38. $\mathbf{v} = \langle 3, -4\rangle$

$\|\mathbf{v}\| = \sqrt{3^2 + (-4)^2} = 5$

Unit vector: $\frac{1}{5}\langle 3, -4\rangle = \left\langle \frac{3}{5}, -\frac{4}{5}\right\rangle$

39. $\|\mathbf{v}\| = \|\langle -24, -7\rangle\| = \sqrt{(-24)^2 + (-7)^2} = 25$

Unit vector: $\frac{1}{25}\langle -24, -7\rangle = \left\langle -\frac{24}{25}, -\frac{7}{25}\right\rangle$

40. $\mathbf{v} = \langle 8, -20\rangle$

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{64 + 400}}\langle 8, -20\rangle = \frac{1}{\sqrt{29}}\langle 2, -5\rangle = \left\langle \frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}}\right\rangle = \left\langle \frac{2\sqrt{29}}{29}, \frac{-5\sqrt{29}}{29}\right\rangle$$

41. $$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{16 + 9}}(4\mathbf{i} - 3\mathbf{j}) = \frac{1}{5}(4\mathbf{i} - 3\mathbf{j}) = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$

42. $\mathbf{w} = \mathbf{i} - 2\mathbf{j}$

$$\mathbf{u} = \frac{1}{\|\mathbf{w}\|}\mathbf{w} = \frac{1}{\sqrt{1^2 + (-2)^2}}(\mathbf{i} - 2\mathbf{j}) = \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j}) = \frac{\sqrt{5}}{5}\mathbf{i} - \frac{2\sqrt{5}}{5}\mathbf{j}$$

43. $\mathbf{u} = \frac{1}{2}(2\mathbf{j}) = \mathbf{j}$

44. $\mathbf{w} = -3\mathbf{i}$

$\|\mathbf{w}\| = 3$

$\mathbf{u} = \frac{1}{3}(-3\mathbf{i}) = -\mathbf{i}$

45. $8\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 8\left(\frac{1}{\sqrt{5^2 + 6^2}}\langle 5, 6\rangle\right)$

$= \frac{8}{\sqrt{61}}\langle 5, 6\rangle$

$= \left\langle \frac{40\sqrt{61}}{61}, \frac{48\sqrt{61}}{61}\right\rangle$

46. $\mathbf{v} = 3\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right)$

$= 3\left(\frac{1}{\sqrt{4^2 + (-4)^2}}\langle 4, -4\rangle\right)$

$= 3\left(\frac{1}{4\sqrt{2}}\langle 4, -4\rangle\right)$

$= \left\langle \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right\rangle$

$= \left\langle \frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right\rangle$

47. $7\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 7\left(\frac{1}{\sqrt{3^2 + 4^2}}\langle 3, 4\rangle\right)$

$= \frac{7}{5}\langle 3, 4\rangle$

$= \left\langle \frac{21}{5}, \frac{28}{5}\right\rangle$

$= \frac{21}{5}\mathbf{i} + \frac{28}{5}\mathbf{j}$

48. $\mathbf{v} = 10\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right)$

$= 10\left(\frac{1}{\sqrt{4 + 9}}\langle 2, -3\rangle\right)$

$= \left\langle \frac{20}{\sqrt{13}}, -\frac{30}{\sqrt{13}}\right\rangle$

$= \frac{20}{\sqrt{13}}\mathbf{i} - \frac{30}{\sqrt{13}}\mathbf{j}$

$= \frac{20\sqrt{13}}{13}\mathbf{i} - \frac{30\sqrt{13}}{13}\mathbf{j}$

49. $8\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right) = 8\left(\frac{1}{2}\langle -2, 0\rangle\right)$

$= 4\langle -2, 0\rangle$

$= \langle -8, 0\rangle$

$= -8\mathbf{i}$

50. $\mathbf{v} = 4\left(\frac{1}{\|\mathbf{u}\|}\mathbf{u}\right)$

$= 4\left(\frac{1}{5}\langle 0, 5\rangle\right)$

$= \langle 0, 4\rangle = 4\mathbf{j}$

51. $\mathbf{v} = \langle 4 - (-3), 5 - 1\rangle = \langle 7, 4\rangle = 7\mathbf{i} + 4\mathbf{j}$

52. $\langle 3 - 0, 6 - (-2)\rangle = \langle 3, 8\rangle = 3\mathbf{i} + 8\mathbf{j}$

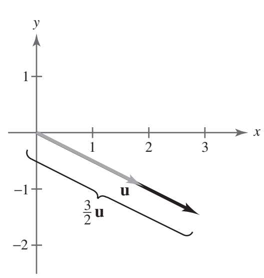
53. $\mathbf{v} = \langle 2 - (-1), 3 - (-5)\rangle = \langle 3, 8\rangle = 3\mathbf{i} + 8\mathbf{j}$

54. $\langle 0 - (-6), 1 - 4\rangle = \langle 6, -3\rangle = 6\mathbf{i} - 3\mathbf{j}$

55. $\mathbf{v} = \frac{3}{2}\mathbf{u}$

$= \frac{3}{2}(2\mathbf{i} - \mathbf{j}) = 3\mathbf{i} - \frac{3}{2}\mathbf{j}$

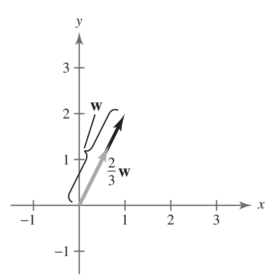
$= \left\langle 3, -\frac{3}{2}\right\rangle$



56. $\mathbf{v} = \frac{2}{3}\mathbf{w}$

$= \frac{2}{3}\langle 1, 2\rangle$

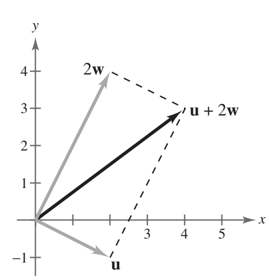
$= \left\langle \frac{2}{3}, \frac{4}{3}\right\rangle$



57. $\mathbf{v} = \mathbf{u} + 2\mathbf{w}$

$= (2\mathbf{i} - \mathbf{j}) + 2(\mathbf{i} + 2\mathbf{j})$

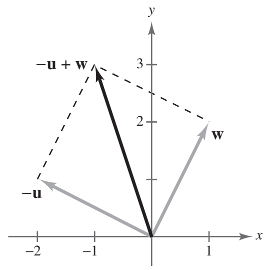
$= 4\mathbf{i} + 3\mathbf{j} = \langle 4, 3\rangle$



58. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$

$= -(2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})$

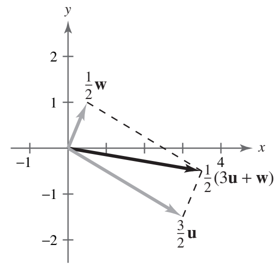
$= -\mathbf{i} + 3\mathbf{j} = \langle -1, 3 \rangle$



59. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$

$= \frac{1}{2}(3\langle 2, -1 \rangle + \langle 1, 2 \rangle)$

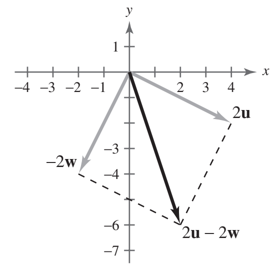
$= \left\langle \frac{7}{2}, -\frac{1}{2} \right\rangle$



60. $\mathbf{v} = 2\mathbf{u} - 2\mathbf{w}$

$= \langle 4, -2 \rangle - \langle 2, 4 \rangle$

$= \langle 2, -6 \rangle$



61. $\mathbf{v} = 5(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

$\|\mathbf{v}\| = 5, \ \theta = 30°$

62. $\mathbf{v} = 8(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

$\|\mathbf{v}\| = 8, \ \theta = 135°$

63. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{6^2 + (-6)^2} = \sqrt{72} = 6\sqrt{2}$

$\tan \theta = -\frac{6}{6} = -1$

Since $\mathbf{v}$ lies in Quadrant IV, $\theta = 315°$.

64. $\mathbf{v} = -4\mathbf{i} - 7\mathbf{j}$, Quadrant III

$\|\mathbf{v}\| = \sqrt{(-4)^2 + (-7)^2} = \sqrt{65}$

$\tan \theta = \frac{-7}{-4} = \frac{7}{4} \implies \theta \approx 240.3°$

65. $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-2)^2 + 5^2} = \sqrt{29}$

$\tan \theta = -\frac{5}{2}$

Since $\mathbf{v}$ lies in Quadrant II, $\theta \approx 111.8°$.

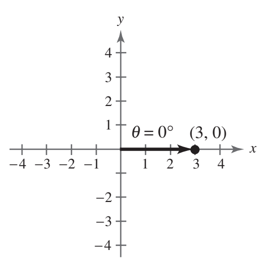
66. $\mathbf{v} = 12\mathbf{i} + 15\mathbf{j}$, Quadrant I

$\|\mathbf{v}\| = \sqrt{12^2 + 15^2} = \sqrt{369} = 3\sqrt{41}$

$\tan \theta = \frac{15}{12} \implies \theta \approx 51.3°$

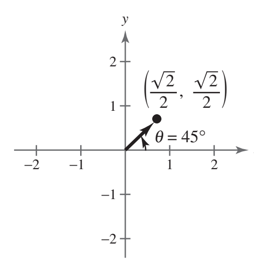
67. $\mathbf{v} = \langle 3 \cos 0°, 3 \sin 0° \rangle$

$= \langle 3, 0 \rangle$



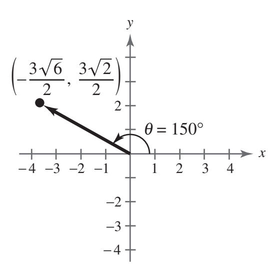
68. $\mathbf{v} = \langle \cos 45°, \sin 45° \rangle$

$= \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

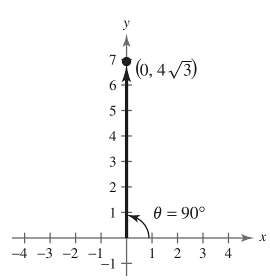


69. $\mathbf{v} = \left\langle 3\sqrt{2} \cos 150°, 3\sqrt{2} \sin 150° \right\rangle$

$= \left\langle -\frac{3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \right\rangle$



70. $\mathbf{v} = 4\sqrt{3}\langle \cos 90°, \sin 90° \rangle = \left\langle 0, 4\sqrt{3} \right\rangle$



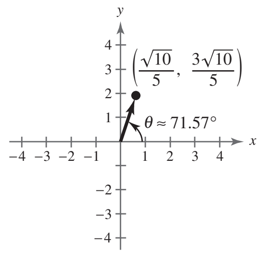
71. $\mathbf{v} = 2\left(\dfrac{1}{\sqrt{3^2 + 1^2}}\right)(\mathbf{i} + 3\mathbf{j})$

$= \dfrac{2}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j})$

$= \dfrac{\sqrt{10}}{5}\mathbf{i} + \dfrac{3\sqrt{10}}{5}\mathbf{j}$

$= \left\langle \dfrac{\sqrt{10}}{5}, \dfrac{3\sqrt{10}}{5} \right\rangle$

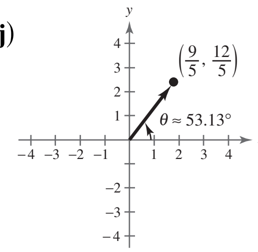
$\tan\theta = \dfrac{3}{1} \implies \theta \approx 71.57°$



72. $\mathbf{v} = 3\left(\dfrac{1}{\sqrt{3^2 + 4^2}}\right)(3\mathbf{i} + 4\mathbf{j})$

$= \dfrac{3}{5}(3\mathbf{i} + 4\mathbf{j})$

$= \dfrac{9}{5}\mathbf{i} + \dfrac{12}{5}\mathbf{j} = \left\langle \dfrac{9}{5}, \dfrac{12}{5} \right\rangle$



73. $\mathbf{u} = \langle 5\cos 60°, 5\sin 60° \rangle = \left\langle \dfrac{5}{2}, \dfrac{5\sqrt{3}}{2} \right\rangle$

$\mathbf{v} = \langle 5\cos 90°, 5\sin 90° \rangle = \langle 0, 5 \rangle$

$\mathbf{u} + \mathbf{v} = \left\langle \dfrac{5}{2}, \dfrac{5\sqrt{3}}{2} \right\rangle + \langle 0, 5 \rangle = \left\langle \dfrac{5}{2}, 5 + \dfrac{5}{2}\sqrt{3} \right\rangle$

74. $\mathbf{u} = \langle 2\cos 30°, 2\sin 30° \rangle = \langle \sqrt{3}, 1 \rangle$

$\mathbf{v} = \langle 2\cos 90°, 2\sin 90° \rangle = \langle 0, 2 \rangle$

$\mathbf{u} + \mathbf{v} = \langle \sqrt{3}, 3 \rangle$

75. $\mathbf{u} = \langle 20\cos 45°, 20\sin 45° \rangle = \langle 10\sqrt{2}, 10\sqrt{2} \rangle$

$\mathbf{v} = \langle 50\cos 150°, 50\sin 150° \rangle = \langle -25\sqrt{3}, 25 \rangle$

$\mathbf{u} + \mathbf{v} = \langle 10\sqrt{2} - 25\sqrt{3}, 10\sqrt{2} + 25 \rangle$

76. $\mathbf{u} = \langle 35\cos 25°, 35\sin 25° \rangle = \langle 31.72, 14.79 \rangle$

$\mathbf{v} = \langle 50\cos 120°, 50\sin 120° \rangle = \langle -25, 25\sqrt{3} \rangle$

$\mathbf{u} + \mathbf{v} \approx \langle 6.72, 58.09 \rangle$

77. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

$\mathbf{w} = 2(\mathbf{i} - \mathbf{j})$

$\mathbf{u} = \mathbf{v} - \mathbf{w} = -\mathbf{i} + 3\mathbf{j}$

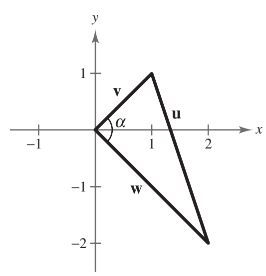
$\|\mathbf{v}\| = \sqrt{2}$

$\|\mathbf{w}\| = 2\sqrt{2}$

$\|\mathbf{v} - \mathbf{w}\| = \sqrt{10}$

$\cos\alpha = \dfrac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\,\|\mathbf{w}\|} = \dfrac{2 + 8 - 10}{2\sqrt{2} \cdot 2\sqrt{2}} = 0$

$\alpha = 90°$



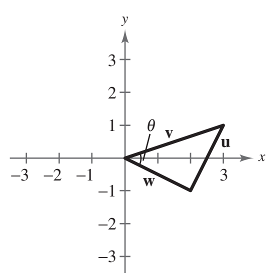
78. $\mathbf{v} = 3\mathbf{i} + \mathbf{j}$

$\mathbf{w} = 2\mathbf{i} - \mathbf{j}$

$\mathbf{u} = \mathbf{v} - \mathbf{w} = \mathbf{i} + 2\mathbf{j}$

$\cos\theta = \dfrac{\|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2}{2\|\mathbf{v}\|\,\|\mathbf{w}\|} = \dfrac{10 + 5 - 5}{2\sqrt{10}\sqrt{5}} = \dfrac{\sqrt{2}}{2}$

$\theta = 45°$



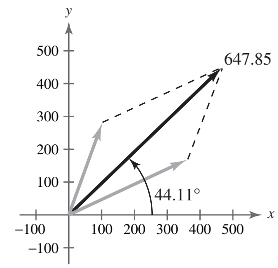
79. $\mathbf{u} = 400 \cos 25°\mathbf{i} + 400 \sin 25°\mathbf{j}$

$\mathbf{v} = 300 \cos 70°\mathbf{i} + 300 \sin 70°\mathbf{j}$

$\mathbf{u} + \mathbf{v} \approx 465.13\mathbf{i} + 450.96\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx \sqrt{(465.13)^2 + (450.96)^2} \approx 647.85$

$\alpha = \arctan\left(\frac{450.96}{465.13}\right) \approx 44.11°$



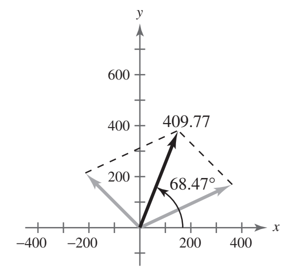
80. Analytically: $\mathbf{v} = 400\langle\cos 25°, \sin 25°\rangle$

$\mathbf{u} = 300\langle\cos 135°, \sin 135°\rangle$

$\mathbf{u} + \mathbf{v} = \langle 150.39, 381.18\rangle$

$\|\mathbf{u} + \mathbf{v}\| = 409.77$

$\theta = \arctan\left(\frac{381.18}{150.39}\right) = 68.47°$



81. Force One: $\mathbf{u} = 45\mathbf{i}$

Force Two: $\mathbf{v} = 60 \cos \theta\mathbf{i} + 60 \sin \theta\mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (45 + 60 \cos \theta)\mathbf{i} + 60 \sin \theta\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(45 + 60 \cos \theta)^2 + (60 \sin \theta)^2} = 90$

$$2025 + 5400 \cos \theta + 3600 = 8100$$

$$5400 \cos \theta = 2475$$

$$\cos \theta = \tfrac{2475}{5400} \approx 0.4583$$

$$\theta \approx 62.7°$$

82. Force One: $\mathbf{u} = 3000\mathbf{i}$

Force Two: $\mathbf{v} = 1000 \cos \theta\mathbf{i} + 1000 \sin \theta\mathbf{j}$

Resultant Force: $\mathbf{u} + \mathbf{v} = (3000 + 1000 \cos \theta)\mathbf{i} + 1000 \sin \theta\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(3000 + 1000 \cos \theta)^2 + (1000 \sin \theta)^2} = 3750$

$$9{,}000{,}000 + 6{,}000{,}000 \cos \theta + 1{,}000{,}000 = 14{,}062, 500$$

$$6{,}000{,}000 \cos \theta = 4{,}062, 500$$

$$\cos \theta = \frac{4{,}062{,}500}{6{,}000{,}000} \approx 0.6771$$

$$\theta \approx 47.4°$$

83. Horizontal component of velocity: $70 \cos 40° \approx 53.62$ ft/sec

Vertical component of velocity: $70 \sin 40° \approx 45.0$ ft/sec

84. Horizontal component of velocity: $1200 \cos 4° \approx 1197.1$ ft/sec

Vertical component of velocity: $1200 \sin 4° \approx 83.7$ ft/sec

85. Rope $\overrightarrow{AC}$: $\mathbf{u} = 10\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant IV and its reference angle is $\arctan\left(\frac{12}{5}\right)$.

$\mathbf{u} = \|\mathbf{u}\|\left[\cos\left(\arctan\frac{12}{5}\right)\mathbf{i} - \sin\left(\arctan\frac{12}{5}\right)\mathbf{j}\right]$

Rope $\overrightarrow{BC}$: $\mathbf{v} = -20\mathbf{i} - 24\mathbf{j}$

The vector lies in Quadrant III and its reference angle is $\arctan\left(\frac{6}{5}\right)$.

$\mathbf{v} = \|\mathbf{v}\|\left[-\cos\left(\arctan\frac{6}{5}\right)\mathbf{i} - \sin\left(\arctan\frac{6}{5}\right)\mathbf{j}\right]$

Resultant: $\mathbf{u} + \mathbf{v} = -5000\mathbf{j}$

$\|\mathbf{u}\|\cos\left(\arctan\frac{12}{5}\right) - \|\mathbf{v}\|\cos\left(\arctan\frac{6}{5}\right) = 0$

$-\|\mathbf{u}\|\sin\left(\arctan\frac{12}{5}\right) - \|\mathbf{v}\|\sin\left(\arctan\frac{6}{5}\right) = -5000$

Solving this system of equations yields:

$T_{AC} = \|\mathbf{u}\| \approx 3611.1$ pounds

$T_{BC} = \|\mathbf{v}\| \approx 2169.5$ pounds

86. Left cable: $\mathbf{u} = \|\mathbf{u}\|(\cos 155.7^\circ\mathbf{i} + \sin 155.7^\circ\mathbf{j})$

Right cable: $\mathbf{v} = \|\mathbf{v}\|(\cos 44.5^\circ\mathbf{i} + \sin 44.5^\circ\mathbf{j})$

$\mathbf{u} + \mathbf{v} = 20{,}240\mathbf{j} \implies \|\mathbf{u}\|\cos 155.7^\circ + \|\mathbf{v}\|\cos 44.5^\circ = 0$

$\|\mathbf{u}\|\sin 155.7^\circ + \|\mathbf{v}\|\sin 44.5^\circ = 20{,}240$

$\implies -0.9114\|\mathbf{u}\| + 0.7133\|\mathbf{v}\| = 0$

$0.4115\|\mathbf{u}\| + 0.7009\|\mathbf{v}\| = 20{,}240$

Solving this system for $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ yields:

Tension of left cable: $\|\mathbf{u}\| = 15{,}485$ pounds

Tension of right cable: $\|\mathbf{v}\| = 19{,}786$ pounds

87. (a) Tow line 1: $\mathbf{u} = \|\mathbf{u}\|(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$

Tow line 2: $\mathbf{v} = \|\mathbf{u}\|(\cos(-\theta)\mathbf{i} + \sin(-\theta)\mathbf{j})$

Resultant: $\mathbf{u} + \mathbf{v} = 6000\mathbf{i} = [\|\mathbf{u}\|\cos\theta + \|\mathbf{u}\|\cos(-\theta)]\mathbf{i} \implies$

$6000 = 2\|\mathbf{u}\|\cos\theta \implies \|\mathbf{u}\| \approx 3000\sec\theta$

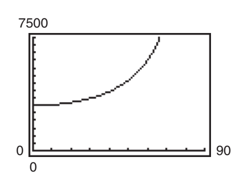
$T = \|\mathbf{u}\| = 3000\sec\theta$

Domain: $0^\circ \le \theta < 90^\circ$

(b)

θ	10°	20°	30°	40°	50°	60°
T	3046.3	3192.5	3464.1	3916.2	4667.2	6000.0

(c)



(d) The tension increases because the component in the direction of the motion of the barge decreases.

88. (a) $\mathbf{u} = \|\mathbf{u}\|(\cos(90° - \theta)\mathbf{i} + \sin(90° - \theta)\mathbf{j})$

$\mathbf{v} = \|\mathbf{v}\|(\cos(90° + \theta)\mathbf{i} + \sin(90° + \theta)\mathbf{j})$

$\mathbf{u} + \mathbf{v} = 100\mathbf{j} \implies \|\mathbf{u}\|\cos(90° - \theta) + \|\mathbf{v}\|\cos(90° + \theta) = 0$

$\implies \|\mathbf{u}\| = \|\mathbf{v}\|$, and

$100 = \|\mathbf{u}\|\sin(90° - \theta) + \|\mathbf{u}\|\sin(90° + \theta)$

$= 2\|\mathbf{u}\|\cos\theta$

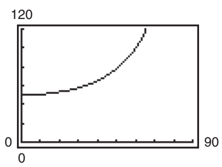
Hence, $\|\mathbf{u}\| = T = \dfrac{50}{\cos\theta} = 50\sec\theta$.

Domain: $0° \le \theta < 90°$

(b)

θ	10°	20°	30°	40°	50°	60°
T	50.8	53.2	57.7	65.3	77.8	100

(c)



(d) The vertical component of the vectors decreases as θ increases.

89. Airspeed: $\mathbf{v} = 860(\cos 302°\mathbf{i} + \sin 302°\mathbf{j})$

Groundspeed: $\mathbf{u} = 800(\cos 310°\mathbf{i} + \sin 310°\mathbf{j})$

$\mathbf{w} + \mathbf{v} = \mathbf{u}$

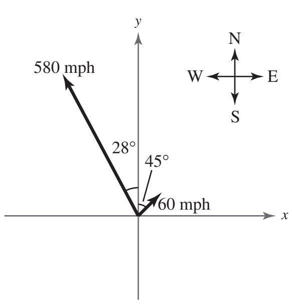
$\mathbf{w} = \mathbf{u} - \mathbf{v} = 800(\cos 310°\mathbf{i} + \sin 310°\mathbf{j}) - 860(\cos 302°\mathbf{i} + \sin 302°\mathbf{j}) \approx 58.50\mathbf{i} + 116.49\mathbf{j}$

$\|\mathbf{w}\| = \sqrt{58.50^2 + 116.49^2} \approx 130.35$ km/hr

$\theta = \arctan\left(\dfrac{116.49}{58.50}\right) \approx 63.3°$

Direction: N 26.7° E

90. (a)



(b) $\mathbf{w} = 60\langle\cos 45°, \sin 45°\rangle = \left\langle 30\sqrt{2}, 30\sqrt{2}\right\rangle$

(c) $\mathbf{v} = 580\langle\cos 118°, \sin 118°\rangle = \langle -272.3, 512.1\rangle$

(d) $\mathbf{w} + \mathbf{v} = \langle -229.9, 554.5\rangle$

$\|\mathbf{w} + \mathbf{v}\| \approx 600.3$ mph

(e) $\tan\theta = \dfrac{554.5}{-229.9} \implies \theta \approx 112.5°$

Direction: N 22.5° W (or 337.5° using airplane navigation)

91. (a) $\mathbf{u} = 220\mathbf{i}, \quad \mathbf{v} = 150\cos 30°\mathbf{i} + 150\sin 30°\mathbf{j}$

$$\mathbf{u} + \mathbf{v} = \left(220 + 75\sqrt{3}\right)\mathbf{i} + 75\mathbf{j}$$

$$\|\mathbf{u} + \mathbf{v}\| = \sqrt{\left(220 + 75\sqrt{3}\right)^2 + 75^2} \approx 357.85 \text{ newtons}$$

$$\tan\theta = \frac{75}{220 + 75\sqrt{3}} \implies \theta \approx 12.1°$$

(b) $\mathbf{u} + \mathbf{v} = 220\mathbf{i} + (150\cos\theta\mathbf{i} + 150\sin\theta\mathbf{j})$

$$M = \|\mathbf{u} + \mathbf{v}\| = \sqrt{220^2 + 150^2(\cos^2\theta + \sin^2\theta) + 2(220)(150)\cos\theta}$$

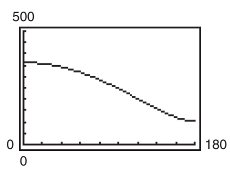
$$= \sqrt{70{,}900 + 66{,}000\cos\theta} = 10\sqrt{709 + 660\cos\theta}$$

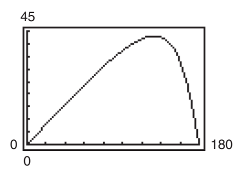
$$\alpha = \arctan\left(\frac{15\sin\theta}{22 + 15\cos\theta}\right)$$

(c)

θ	0°	30°	60°	90°	120°	150°	180°
M	370.0	357.9	322.3	266.3	194.7	117.2	70.0
α	0°	12.1°	23.8°	34.3°	41.9°	39.8°	0°

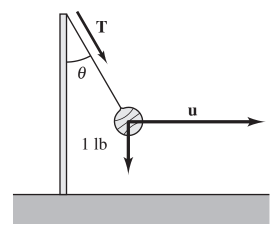
(d)





(e) For increasing θ, the two vectors tend to work against each other resulting in a decrease in the magnitude of the resultant.

92. (a) $\mathbf{T} = \|\mathbf{T}\|(\cos(90° + \theta)\mathbf{i} + \sin(90° + \theta)\mathbf{j})$



$$\mathbf{u} + \mathbf{w} + \mathbf{T} = \mathbf{0} \implies \|\mathbf{u}\| + \|\mathbf{T}\|\cos(90° + \theta) = 0$$

$$\|\mathbf{u}\| - \|\mathbf{T}\|\sin\theta = 0$$

$$-1 + \|\mathbf{T}\|\sin(90° + \theta) = 0$$

$$-1 + \|\mathbf{T}\|\cos\theta = 0 \implies$$

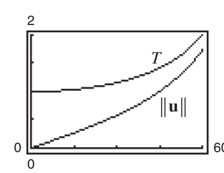
$$\|\mathbf{T}\| = \sec\theta,\ 0 \le \theta < \frac{\pi}{2}$$

Hence, $\|\mathbf{u}\| = \|\mathbf{T}\|\sin\theta = \sec\theta\sin\theta = \tan\theta, 0 \le \theta < \pi/2$.

(b)

θ	0°	10°	20°	30°	40°	50°	60°
$\|\mathbf{T}\|$	1	1.02	1.06	1.15	1.31	1.56	2
$\|\mathbf{u}\|$	0	0.18	0.36	0.58	0.84	1.19	1.73

(c)



(d) Both $\|\mathbf{T}\|$ and $\|\mathbf{u}\|$ increase as θ increases, and approach each other in magnitude.

93. True. See page 424.

94. True, $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$.

95. True. In fact, $a = b = 0$.

96. True

97. True. **a** and **d** are parallel, and pointing in opposite directions.

98. True. **c** and **s** are parallel, and pointing in the same direction.

99. True

100. False. In fact, $\mathbf{v} + \mathbf{s} = \mathbf{w}$.

101. True. $\mathbf{a} + \mathbf{w} = 2\mathbf{a} = -2\mathbf{d}$

102. True. $\mathbf{a} + \mathbf{d} = \mathbf{a} + (-\mathbf{a}) = \mathbf{0}$

103. False. $\mathbf{u} - \mathbf{v} = 2\mathbf{u}$ and

$-2(\mathbf{b} + \mathbf{t}) = -2(-2\mathbf{u}) = 4\mathbf{u}$

104. True

$\mathbf{t} - \mathbf{w} = -\mathbf{s}$ and $\mathbf{b} - \mathbf{a} = -\mathbf{c} = -\mathbf{s}$

105. (a) The angle between them is 0°.

(b) The angle between them is 180°.

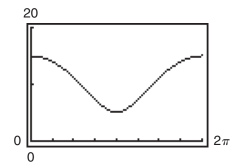
(c) No. At most it can be equal to the sum when the angle between them is 0°.

106. $\mathbf{F}_1 = \langle 10, 0\rangle$, $\mathbf{F}_2 = 5\langle \cos\theta, \sin\theta\rangle$

(a) $\mathbf{F}_1 + \mathbf{F}_2 = \langle 10 + 5\cos\theta, 5\sin\theta\rangle$

$$\begin{aligned}\|\mathbf{F}_1 + \mathbf{F}_2\| &= \sqrt{(10 + 5\cos\theta)^2 + (5\sin\theta)^2}\\ &= \sqrt{100 + 100\cos\theta + 25\cos^2\theta + 25\sin^2\theta}\\ &= 5\sqrt{4 + 4\cos\theta + \cos^2\theta + \sin^2\theta}\\ &= 5\sqrt{4 + 4\cos\theta + 1}\\ &= 5\sqrt{5 + 4\cos\theta}\end{aligned}$$

(b)



(c) Range: $[5, 15]$

Maximum is 15 when $\theta = 0$.

Minimum is 5 when $\theta = \pi$.

(d) The magnitude of the resultant is never 0 because the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are not the same.

107. Let $\mathbf{v} = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j}$.

$\|\mathbf{v}\| = \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$

Therefore, $\mathbf{v}$ is a unit vector for any value of θ.

108. The following program is written for a TI-82 or TI-83 graphing calculator. The program sketches two vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ and $\mathbf{v} = c\mathbf{i} + d\mathbf{j}$ in standard position, and then sketches the vector difference $\mathbf{u} - \mathbf{v}$ using the parallelogram law.

```
PROGRAM: SUBVECT
:Input "ENTER A", A
:Input "ENTER B", B
:Input "ENTER C", C
:Input "ENTER D", D
:Line (0, 0, A, B)
:Line (0, 0, C, D)
:Pause
:A-C→E
:B-D→F
:Line (A, B, C, D)
:Line (A, B, E, F)
:Line (0, 0, E, F)
:Pause
:ClrDraw
:Stop
```


109.

$$\begin{aligned}\mathbf{u} &= \langle 5 - 1, 2 - 6\rangle = \langle 4, -4\rangle\\ \mathbf{v} &= \langle 9 - 4, 4 - 5\rangle = \langle 5, -1\rangle\\ \mathbf{u} - \mathbf{v} &= \langle -1, -3\rangle\\ \mathbf{v} - \mathbf{u} &= \langle 1, 3\rangle\end{aligned}$$

110.

$$\begin{aligned}\mathbf{u} &= \langle 80 - 10, 80 - 60\rangle = \langle 70, 20\rangle\\ \mathbf{v} &= \langle -20 - (-100), 70 - 0\rangle = \langle 80, 70\rangle\\ \mathbf{u} - \mathbf{v} &= \langle 70 - 80, 20 - 70\rangle = \langle -10, -50\rangle\\ \mathbf{v} - \mathbf{u} &= \langle 80 - 70, 70 - 20\rangle = \langle 10, 50\rangle\end{aligned}$$

111. $\left(\dfrac{6x^4}{7y^{-2}}\right)(14x^{-1}y^5) = \dfrac{12x^4y^5y^2}{x}$

$= 12x^3y^7,\ x \neq 0, y \neq 0$

112. $(5s^5t^{-5})\left(\dfrac{3s^{-2}}{50t^{-1}}\right) = \dfrac{3s^3}{10t^4},\ s \neq 0$

113. $(18x)^0(4xy)^2(3x^{-1}) = \dfrac{16x^2y^2(3)}{x}$

$= 48xy^2,\ x \neq 0$

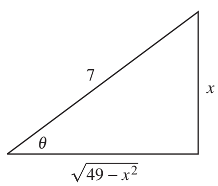
114. $(5ab^2)(a^{-3}b^0)(2a^0b)^{-2} = (5ab^2)\dfrac{1}{a^3}\dfrac{1}{(2b)^2}$

$= \dfrac{5}{4a^2},\ b \neq 0$

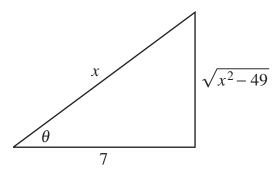
115. $(2.1 \times 10^9)(3.4 \times 10^{-4}) = 7.14 \times 10^5$

116. $(6.5 \times 10^6)(3.8 \times 10^4) = 24.7 \times 10^{10} = 2.47 \times 10^{11}$

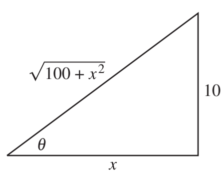
117. $\sin\theta = \dfrac{x}{7} \implies \sqrt{49 - x^2} = 7\cos\theta$



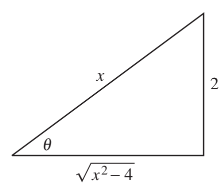
118. $\sqrt{x^2 - 49} = 7\tan\theta$



119. $\cot\theta = \dfrac{x}{10} \implies \sqrt{x^2 + 100} = 10 \cdot \csc\theta$



120. $\sqrt{x^2 - 4} = 2\cot\theta$



121. $\cos x(\cos x + 1) = 0$

$\cos x = 0 \implies x = \dfrac{\pi}{2} + n\pi$

$\cos x = -1 \implies x = \pi + 2n\pi$

122. $\sin x\left(2\sin x + \sqrt{2}\right) = 0$

$\sin x = 0$ or $\sin x = \dfrac{-\sqrt{2}}{2}$

$x = n\pi$ or $x = \dfrac{5\pi}{4} + 2n\pi$

$x = \dfrac{7\pi}{4} + 2n\pi$

123. $3\sec x + 4 = 10$

$\sec x = 2$

$\cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3} + 2n\pi, \dfrac{5\pi}{3} + 2n\pi$

124. $\cos x\cot x - \cos x = 0$

$\cos x(\cot x - 1) = 0$

$\cos x = 0$ or $\cot x = 1$

$x = \dfrac{\pi}{2} + n\pi$ or $x = \dfrac{\pi}{4} + n\pi$

Section 6.4 Vectors and Dot Products

- Know the definition of the dot product of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$.

  $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2$
- Know the following properties of the dot product:
  1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
  2. $\mathbf{0} \cdot \mathbf{v} = 0$
  3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
  4. $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$
  5. $c(\mathbf{u} \cdot \mathbf{v}) = c\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot c\mathbf{v}$
- If θ is the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then

  $$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}.$$
- The vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$.
- Know the definition of vector components. $\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$ where $\mathbf{w}_1$ and $\mathbf{w}_2$ are orthogonal, and $\mathbf{w}_1$ is parallel to $\mathbf{v}$. $\mathbf{w}_1$ is called the projection of $\mathbf{u}$ onto $\mathbf{v}$ and is denoted by

  $$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}.$$

  Then we have $\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1$.
- Know the definition of work.
  1. Projection form: $W = \|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \, \|\overrightarrow{PQ}\|$
  2. Dot product form: $W = \mathbf{F} \cdot \overrightarrow{PQ}$

Vocabulary Check

1. dot product

2. $\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}$

3. orthogonal

4. $\left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

5. $\|\text{proj}_{\overrightarrow{PQ}} \mathbf{F}\| \, \|\overrightarrow{PQ}\|, \mathbf{F} \cdot \overrightarrow{PQ}$

1. $\mathbf{u} \cdot \mathbf{v} = \langle 6, 3 \rangle \cdot \langle 2, -4 \rangle = 6(2) + 3(-4) = 0$

2. $\mathbf{u} \cdot \mathbf{v} = \langle -4, 1 \rangle \cdot \langle 2, -3 \rangle$

$= (-4)(2) + 1(-3) = -11$

3. $\mathbf{u} \cdot \mathbf{v} = \langle 5, 1 \rangle \cdot \langle 3, -1 \rangle = 5(3) + 1(-1) = 14$

4. $\mathbf{u} \cdot \mathbf{v} = \langle 3, 2 \rangle \cdot \langle -2, 1 \rangle = 3(-2) + 2(1) = -4$

5. $\mathbf{u} = \langle 2, 2 \rangle$

$\mathbf{u} \cdot \mathbf{u} = 2(2) + 2(2) = 8$, scalar

6. $\mathbf{v} \cdot \mathbf{w} = \langle -3, 4 \rangle \cdot \langle 1, -4 \rangle$

$= (-3)(1) + 4(-4) = -19$, scalar

7. $\mathbf{u} = \langle 2, 2 \rangle, \mathbf{v} = \langle -3, 4 \rangle$

$\mathbf{u} \cdot 2\mathbf{v} = 2\mathbf{u} \cdot \mathbf{v} = 4(-3) + 4(4) = 4$, scalar

8. $4\mathbf{u} \cdot \mathbf{v} = 4\langle 2, 2 \rangle \cdot \langle -3, 4 \rangle$

$= 4(-6 + 8)$

$= 8$, scalar

9. $(3\mathbf{w}\cdot\mathbf{v})\mathbf{u} = (3\langle 1, -4\rangle \cdot \langle -3, 4\rangle)\langle 2, 2\rangle$
$= (3(-3) + (-12)(4))\langle 2, 2\rangle$
$= -57\langle 2, 2\rangle$
$= \langle -114, -114\rangle$, vector

10. $(\mathbf{u}\cdot 2\mathbf{v})\mathbf{w} = (\langle 2, 2\rangle \cdot 2\langle -3, 4\rangle)\langle 1, -4\rangle$
$= (2(-6) + 2(8))\langle 1, -4\rangle$
$= 4\langle 1, -4\rangle$
$= \langle 4, -16\rangle$, vector

11. $\mathbf{u} = \langle -5, 12\rangle$
$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(-5)^2 + 12^2} = 13$

12. $\mathbf{u} = \langle 2, -4\rangle$
$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}}$
$= \sqrt{2(2) + (-4)(-4)}$
$= \sqrt{20} = 2\sqrt{5}$

13. $\mathbf{u} = 20\mathbf{i} + 25\mathbf{j}$
$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(20)^2 + (25)^2} = \sqrt{1025} = 5\sqrt{41}$

14. $\mathbf{u} = 6\mathbf{i} - 10\mathbf{j}$
$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{6(6) + (-10)(-10)} = \sqrt{136} = 2\sqrt{34}$

15. $\mathbf{u} = -4\mathbf{j}$
$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{(-4)(-4)} = 4$

16. $\mathbf{u} = 9\mathbf{i}$
$\|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{9(9) + 0} = \sqrt{81} = 9$

17. $\mathbf{u} = \langle -1, 0\rangle, \mathbf{v} = \langle 0, 2\rangle$
$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{0}{(1)(2)} = 0 \implies \theta = 90^\circ$$

18. $\mathbf{u} = \langle 4, 4\rangle, \mathbf{v} = \langle -2, 0\rangle$
$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$
$$= \frac{4(-2) + 4(0)}{\left(4\sqrt{2}\right)(2)}$$
$$= -\frac{\sqrt{2}}{2}$$
$\theta = 135^\circ$

19. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}, \mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$
$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-6 + 12}{(5)\left(\sqrt{13}\right)} = \frac{6}{5\sqrt{13}}$$
$$\theta = \arccos\left(\frac{6}{5\sqrt{13}}\right) \approx 70.56^\circ$$

20. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$
$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$
$$= \frac{2(1) + (-3)(-2)}{\sqrt{2^2 + 3^2}\sqrt{1^2 + 2^2}}$$
$$= \frac{8}{\sqrt{65}} \approx 0.992278$$
$\theta \approx 7.13^\circ$

21. $\mathbf{u} = 2\mathbf{i}, \mathbf{v} = -3\mathbf{j}$
$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{0}{(2)(3)} = 0 \implies \theta = 90^\circ$$

22. $\mathbf{u}\cdot\mathbf{v} = \langle 0, 4\rangle \cdot \langle -3, 0\rangle = 0 \implies \theta = 90^\circ$

23. $\mathbf{u} = \left(\cos\frac{\pi}{3}\right)\mathbf{i} + \left(\sin\frac{\pi}{3}\right)\mathbf{j} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$

$$\mathbf{v} = \left(\cos\frac{3\pi}{4}\right)\mathbf{i} + \left(\sin\frac{3\pi}{4}\right)\mathbf{j} = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = 1$$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \mathbf{u}\cdot\mathbf{v} = \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2}+\sqrt{6}}{4}$$

$$\theta = \arccos\left(\frac{-\sqrt{2}+\sqrt{6}}{4}\right) = 75^\circ = \frac{5\pi}{12}$$

24. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$

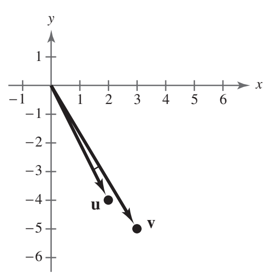
$$\mathbf{v} = \cos\left(\frac{2\pi}{3}\right)\mathbf{i} + \sin\left(\frac{2\pi}{3}\right)\mathbf{j} = -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-\left(\sqrt{2}/4\right) + \left(\sqrt{6}/4\right)}{(1)(1)} = \frac{\sqrt{6}-\sqrt{2}}{4} \implies \theta = 75^\circ = \frac{5\pi}{12}$$

25. $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j},\ \mathbf{v} = 3\mathbf{i} - 5\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{6+20}{\sqrt{20}\sqrt{34}}$$

$$= \frac{13\sqrt{170}}{170} \implies \theta \approx 4.40^\circ$$

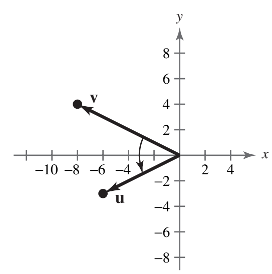


26. $\mathbf{u} = -6\mathbf{i} - 3\mathbf{j},\ \mathbf{v} = -8\mathbf{i} + 4\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{-6(-8)+(-3)(4)}{\sqrt{45}\sqrt{80}}$$

$$= \frac{36}{60} = 0.6$$

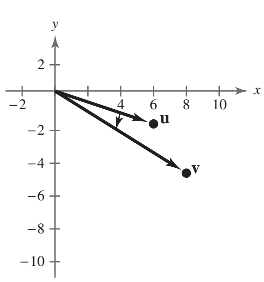
$$\theta \approx 53.13^\circ$$



27. $\mathbf{u} = 6\mathbf{i} - 2\mathbf{j},\ \mathbf{v} = 8\mathbf{i} - 5\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{48+10}{\sqrt{40}\sqrt{89}}$$

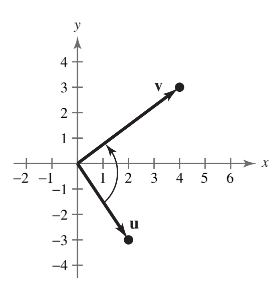
$$= \frac{29\sqrt{890}}{890} \implies \theta \approx 13.57^\circ$$



28. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j},\ \mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{2(4)+(-3)(3)}{\sqrt{13}\sqrt{25}} \approx -0.0555$$

$$\theta \approx 93.18^\circ$$



29. $P = (1, 2),\ Q = (3, 4),\ R = (2, 5)$

$\overrightarrow{PQ} = \langle 2, 2\rangle,\ \overrightarrow{PR} = \langle 1, 3\rangle,\ \overrightarrow{QR} = \langle -1, 1\rangle$

$$\cos\alpha = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{PR}\|} = \frac{8}{(2\sqrt{2})(\sqrt{10})} \implies \alpha = \arccos\frac{2}{\sqrt{5}} \approx 26.6^\circ$$

$$\cos\beta = \frac{\overrightarrow{PQ}\cdot\overrightarrow{QR}}{\|\overrightarrow{PQ}\|\,\|\overrightarrow{QR}\|} = 0 \implies \beta = 90^\circ$$

Thus, $\gamma \approx 180^\circ - 26.6^\circ - 90^\circ = 63.4^\circ$.

30. $P = (-3, 0), Q = (2, 2), R = (0, 6)$

$\overrightarrow{PQ} = \langle 5, 2\rangle, \overrightarrow{QR} = \langle -2, 4\rangle, \overrightarrow{PR} = \langle 3, 6\rangle,$

$\overrightarrow{QP} = \langle -5, -2\rangle$

$$\cos\alpha = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\|\overrightarrow{PR}\|\,\|\overrightarrow{PR}\|} = \frac{27}{(\sqrt{29})(\sqrt{45})} \implies \alpha \approx 41.6^\circ$$

$$\cos\beta = \frac{\overrightarrow{QR}\cdot\overrightarrow{QP}}{\|\overrightarrow{QR}\|\,\|\overrightarrow{QP}\|} = \frac{2}{(\sqrt{20})(\sqrt{29})} \implies \beta \approx 85.2$$

$\phi = 180^\circ - 41.6^\circ - 85.2^\circ \approx 53.2^\circ$

31. $$\mathbf{u}\cdot\mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta = (9)(36)\cos\frac{3\pi}{4} = 324\left(-\frac{\sqrt{2}}{2}\right) = -162\sqrt{2}$$

32. $$\mathbf{u}\cdot\mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\cos\theta = (4)(12)\cos\frac{\pi}{3} = 48\left(\frac{1}{2}\right) = 24$$

33. $\mathbf{u} = \langle -12, 30\rangle,\ \mathbf{v} = \left\langle \frac{1}{2}, -\frac{5}{4}\right\rangle$

$\mathbf{u} = -24\mathbf{v} \implies$ **u** and **v** are parallel.

34. $\mathbf{u} = \langle 15, 45\rangle, \mathbf{v} = \langle -5, 12\rangle$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

35. $\mathbf{u} = \frac{1}{4}(3\mathbf{i} - \mathbf{j}),\ \mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

36. $\mathbf{u} = \mathbf{j}, \mathbf{v} = \mathbf{i} - 2\mathbf{j}$

$\mathbf{u} \neq k\mathbf{v} \implies$ Not parallel

$\mathbf{u}\cdot\mathbf{v} \neq 0 \implies$ Not orthogonal

Neither

37. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j},\ \mathbf{v} = -\mathbf{i} - \mathbf{j}$

$\mathbf{u}\cdot\mathbf{v} = 0 \implies$ **u** and **v** are orthogonal.

38. $-4\mathbf{v} = -4(-2\mathbf{i} - \mathbf{j}) = 8\mathbf{i} + 4\mathbf{j} = \mathbf{u} \implies$ parallel

39. $\mathbf{u}\cdot\mathbf{v} = \langle 2, -k\rangle\cdot\langle 3, 2\rangle = 6 - 2k = 0 \implies k = 3$

40. $\mathbf{u}\cdot\mathbf{v} = \langle 3, 2\rangle\cdot\langle 2, -k\rangle = 6 - 2k = 0 \implies k = 3$

41. $\mathbf{u}\cdot\mathbf{v} = \langle 1, 4\rangle\cdot\langle 2k, -5\rangle = 2k - 20 = 0 \implies k = 10$

42. $\mathbf{u}\cdot\mathbf{v} = \langle -3k, 5\rangle\cdot\langle 2, -4\rangle = -6k - 20 = 0 \implies k = -\frac{10}{3}$

43. $\mathbf{u}\cdot\mathbf{v} = \langle -3k, 2\rangle\cdot\langle -6, 0\rangle = 18k = 0 \implies k = 0$

44. $\mathbf{u}\cdot\mathbf{v} = \langle 4, -4k\rangle\cdot\langle 0, 3\rangle = -12k = 0 \implies k = 0$

45. $\mathbf{u} = \langle 3, 4\rangle,\ \mathbf{v} = \langle 8, 2\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{32}{68}\right)\mathbf{v} = \frac{8}{17}\langle 8, 2\rangle = \frac{16}{17}\langle 4, 1\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 3, 4\rangle - \frac{16}{17}\langle 4, 1\rangle = \frac{13}{17}\langle -1, 4\rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{16}{17}\langle 4, 1\rangle + \frac{13}{17}\langle -1, 4\rangle$$

46. $\mathbf{u} = \langle 4, 2\rangle, \mathbf{v} = \langle 1, -2\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = 0\langle 1, -2\rangle = \langle 0, 0\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 4, 2\rangle - \langle 0, 0\rangle = \langle 4, 2\rangle$$

$$\mathbf{u} = \langle 4, 2\rangle + \langle 0, 0\rangle$$

47. $\mathbf{u} = \langle 0, 3\rangle,\ \mathbf{v} = \langle 2, 15\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{45}{229}\langle 2, 15\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 0, 3\rangle - \frac{45}{229}\langle 2, 15\rangle = \left\langle -\frac{90}{229}, \frac{12}{229}\right\rangle = \frac{6}{229}\langle -15, 2\rangle$$

$$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2 = \frac{45}{229}\langle 2, 15\rangle + \frac{6}{229}\langle -15, 2\rangle$$

48. $\mathbf{u} = \langle -5, -1\rangle, \mathbf{v} = \langle -1, 1\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{4}{2}\langle -1, 1\rangle = 2\langle -1, 1\rangle$$

$$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle -5, -1\rangle - 2\langle -1, 1\rangle = \langle -3, -3\rangle = 3\langle -1, -1\rangle$$

$$\mathbf{u} = \langle -3, -3\rangle + \langle -2, 2\rangle$$

49. $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u}$ since they are parallel.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \frac{18 + 8}{36 + 16}\mathbf{v} = \frac{26}{52}\langle 6, 4\rangle = \langle 3, 2\rangle = \mathbf{u}$$

50. Because **u** and **v** are parallel, the projection of **u** onto **v** is **u**.

51. $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{0}$ since they are perpendicular.

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = \mathbf{0}, \text{ since } \mathbf{u}\cdot\mathbf{v} = 0.$$

52. Because **u** and **v** are orthogonal, the projection of **u** onto **v** is **0**.

53. $\mathbf{u} = \langle 2, 6\rangle$

For **v** to be orthogonal to **u**, $\mathbf{u}\cdot\mathbf{v}$ must equal 0.

Two possibilities: $\langle 6, -2\rangle$ and $\langle -6, 2\rangle$

54. For **v** to be orthogonal to $\mathbf{u} = \langle -7, 5\rangle$, their dot product must be zero.

Two possibilities: $\langle 5, 7\rangle, \langle -5, -7\rangle$

55. $\mathbf{u} = \frac{1}{2}\mathbf{i} - \frac{3}{4}\mathbf{j}$

For **v** to be orthogonal to **u**, $\mathbf{u}\cdot\mathbf{v}$ must equal 0.

Two possibilities: $\left\langle \frac{3}{4}, \frac{1}{2}\right\rangle$ and $\left\langle -\frac{3}{4}, -\frac{1}{2}\right\rangle$

56. $\mathbf{u} = -\frac{5}{2}\mathbf{i} - 3\mathbf{j}$

For **v** to be orthogonal to **u**, $\mathbf{u}\cdot\mathbf{v}$ must be equal to 0.

Two possibilities: $\mathbf{v} = 3\mathbf{i} - \frac{5}{2}\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} + \frac{5}{2}\mathbf{j}$

57. $W = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\|\,\|\overrightarrow{PQ}\|$ where $\overrightarrow{PQ} = \langle 4, 7\rangle$ and $\mathbf{v} = \langle 1, 4\rangle$

$$\text{proj}_{\overrightarrow{PQ}}\mathbf{v} = \left(\frac{\mathbf{v}\cdot\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|^2}\right)\overrightarrow{PQ} = \left(\frac{32}{65}\right)\langle 4, 7\rangle$$

$$W = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\|\,\|\overrightarrow{PQ}\| = \left(\frac{32\sqrt{65}}{65}\right)\left(\sqrt{65}\right) = 32$$

58. $P = (1, 3)$, $Q = (-3, 5)$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$

work $= \|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\|\,\|\overrightarrow{PQ}\|$ where $\overrightarrow{PQ} = \langle -4, 2\rangle$ and $\mathbf{v} = \langle -2, 3\rangle$.

$$\text{proj}_{\overrightarrow{PQ}}\mathbf{v} = \left(\frac{\mathbf{v}\cdot\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|^2}\right)\overrightarrow{PQ} = \left(\frac{14}{20}\right)\langle -4, 2\rangle$$

$$\text{work} = \|\text{proj}_{\overrightarrow{PQ}}\mathbf{v}\|\,\|\overrightarrow{PQ}\| = \left(\frac{14\sqrt{20}}{20}\right)\left(\sqrt{20}\right) = 14$$

59. (a) $\mathbf{u}\cdot\mathbf{v} = \langle 1245, 2600\rangle\cdot\langle 12.20, 8.50\rangle$

$= 1245(12.20) + 2600(8.50) = 37{,}289$

This is the total dollar value of the picture frames produced.

(b) Multiply $\mathbf{v}$ by 1.02.

60. $\mathbf{u} = \langle 3240, 2450\rangle$, $\mathbf{v} = \langle 1.75, 1.25\rangle$

(a) $\mathbf{u}\cdot\mathbf{v} = 3240(1.75) + 2450(1.25) = 8732.50$

This gives the total revenue earned.

(b) To increase prices by $2\frac{1}{2}$ percent, multiply $\mathbf{v}$ by 1.025:

$1.025\langle 1.75, 1.25\rangle$, scalar multiplication

61. (a) $\mathbf{F} = -30{,}000\vec{\mathbf{j}}$, Gravitational force

$\mathbf{v} = \langle\cos(d°), \sin(d°)\rangle$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{F} = \left(\frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v} = -30{,}000\sin(d°)\langle\cos d°, \sin d°\rangle$$

$$= \langle -30{,}000\sin d°\cos d°, -30{,}000\sin^2 d°\rangle$$

Force needed: $30{,}000\sin(d°)$

(b)

d	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
Force	0	523.6	1047.0	1570.1	2092.7	2614.7	3135.9	3656.1	4175.2	4693.0	5209.4

(c) $\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1 = -30{,}000\mathbf{j} + 2614.7(\cos(5°)\mathbf{i} + \sin(5°)\mathbf{j}) \approx 2604.75\mathbf{i} - 29{,}772.11\mathbf{j}$

$\|\mathbf{w}_2\| \approx 29{,}885.8$ pounds

62. $\mathbf{F} = -5400\mathbf{j}$, Gravitational force

$\mathbf{v} = \cos 10°\mathbf{i} + \sin 10°\mathbf{j}$

$$\mathbf{w}_1 = \text{proj}_{\mathbf{v}}\mathbf{F} = \frac{\mathbf{F}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v} = (\mathbf{F}\cdot\mathbf{v})\mathbf{v} \approx -937.7\mathbf{v}$$

Magnitude: 937.7 pounds

$\mathbf{w}_2 = \mathbf{F} - \mathbf{w}_1$

$= -5400\mathbf{j} + 937.7[\cos 10°\mathbf{i} + \sin 10°\mathbf{j}]$

$\approx 923.45\mathbf{i} - 5237.17\mathbf{j}$

$\|\mathbf{w}_2\| \approx 5318$ pounds

63. (a) $\mathbf{F} = 15{,}691\langle\cos 30°, \sin 30°\rangle$

$\overrightarrow{PQ} = d\langle 1, 0\rangle$

$$W = \mathbf{F}\cdot\overrightarrow{PQ} = 15{,}691\frac{\sqrt{3}}{2}d \approx 13{,}588.8d$$

(b)

d	0	200	400	800
Work	0	2,717,761	5,435,522	10,871,044

64. $W = (45)(20)\cos 30° \approx 779.4$ foot-pounds

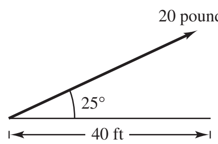
65. $\|\mathbf{F}\| = 250, \|\overrightarrow{PQ}\| = 100, \theta = 30°$

$$\begin{aligned} W &= \|\mathbf{F}\|\,\|\overrightarrow{PQ}\| \cos\theta \\ &= (250)(100)\cos 30° \\ &= 25{,}000\frac{\sqrt{3}}{2} \\ &= 12{,}500\sqrt{3} \text{ foot-pounds} \\ &\approx 21{,}650.64 \text{ foot-pounds} \end{aligned}$$

66. $\|\mathbf{F}\| = 25, \|\overrightarrow{PQ}\| = 50, \theta = 20°$

$$\begin{aligned} W &= \|\mathbf{F}\|\,\|\overrightarrow{PQ}\| \cos\theta \\ &= (25)(50)\cos 20° \\ &\approx 1174.62 \text{ foot-pounds} \end{aligned}$$

67. $W = (\cos 25°)(20)(40) \approx 725.05$ foot-pounds



68. Work $= (\cos 20°)(25)(12) \approx 281.9$ foot-pounds

69. True. $\mathbf{u} \cdot \mathbf{v} = 0$

70. False. Work is a scalar, not a vector.

71. $\mathbf{u} \cdot \mathbf{v} = 0 \implies$ they are orthogonal (unit vectors).

72. (a) $\mathbf{u} \cdot \mathbf{v} = 0 \implies \mathbf{u}$ and $\mathbf{v}$ are orthogonal and $\theta = \frac{\pi}{2}$.

(b) $\mathbf{u} \cdot \mathbf{v} > 0 \implies \cos\theta > 0 \implies 0 \le \theta < \frac{\pi}{2}$

(c) $\mathbf{u} \cdot \mathbf{v} < 0 \implies \cos\theta < 0 \implies \frac{\pi}{2} < \theta \le \pi$

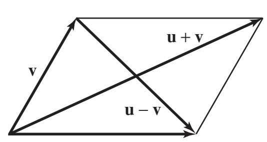
73. (a) $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{u} \implies \mathbf{u}$ and $\mathbf{v}$ are parallel.

(b) $\text{proj}_{\mathbf{v}}\mathbf{u} = \mathbf{0} \implies \mathbf{u}$ and $\mathbf{v}$ are orthogonal.

74. Let $\mathbf{u}$ and $\mathbf{v}$ be two adjacent sides of the rhombus, $\|\mathbf{u}\| = \|\mathbf{v}\|$. The diagonals are $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

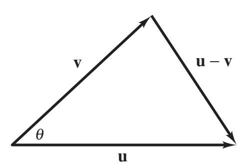
$$\begin{aligned} (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v} \\ &= \|\mathbf{u}\|^2 - \|\mathbf{v}\|^2 = 0 \end{aligned}$$

Hence, the diagonals are perpendicular.



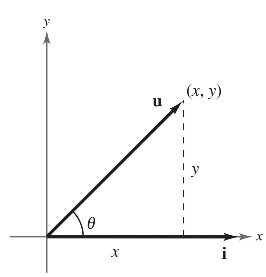
75. Use the Law of Cosines on the triangle:

$$\begin{aligned} \|\mathbf{u} - \mathbf{v}\|^2 &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\,\|\mathbf{v}\| \cos\theta \\ &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v} \end{aligned}$$



76. $\mathbf{u} \cdot (c\mathbf{v} + d\mathbf{w}) = c(\mathbf{u} \cdot \mathbf{v}) + d(\mathbf{u} \cdot \mathbf{w}) = c0 + d0 = 0$

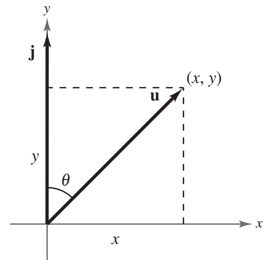
77. From trigonometry, $x = \cos\theta$ and $y = \sin\theta$. Thus, $\mathbf{u} = \langle x, y\rangle = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$.



78. From trigonometry,

$x = \cos\left(\frac{\pi}{2} - \theta\right)$ and $y = \sin\left(\frac{\pi}{2} - \theta\right)$.

Thus, $\mathbf{u} = \langle x, y \rangle = \cos\left(\frac{\pi}{2} - \theta\right)\mathbf{i} + \sin\left(\frac{\pi}{2} - \theta\right)\mathbf{j}$.



79. $g(x) = f(x - 4)$ is a horizontal shift of f four units to the right.

80. g is a reflection of f with respect to the x-axis.

81. $g(x) = f(x) + 6$ is a vertical shift of f six units upward.

82. g is a horizontal shrink of f.

83. $\sqrt{-4} - 1 = 2i - 1 = -1 + 2i$

84. $\sqrt{-8} + 5 = 2\sqrt{2}i + 5 = 5 + 2\sqrt{2}i$

85. $3i(4 - 5i) = 12i + 15 = 15 + 12i$

86. $-2i(1 + 6i) = -2i + 12 = 12 - 2i$

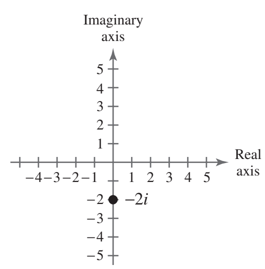
87. $(1 + 3i)(1 - 3i) = 1 - (3i)^2 = 1 + 9 = 10$

88. $(7 - 4i)(7 + 4i) = 49 + 16 = 65$

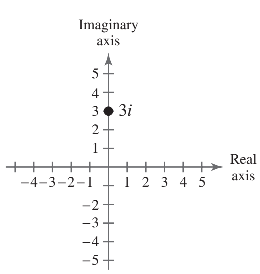
89.
$$\begin{aligned}\frac{3}{1+i} + \frac{2}{2-3i} &= \frac{3}{1+i}\cdot\frac{1-i}{1-i} + \frac{2}{2-3i}\cdot\frac{2+3i}{2+3i}\\ &= \frac{3-3i}{2} + \frac{4+6i}{13}\\ &= \frac{39 - 39i + 8 + 12i}{26}\\ &= \frac{47}{26} - \frac{27}{26}i\end{aligned}$$

90.
$$\begin{aligned}\frac{6}{4-i}\frac{4+i}{4+i} - \frac{3}{1+i}\cdot\frac{1-i}{1-i} &= \frac{24+6i}{17} - \frac{3-3i}{2}\\ &= \frac{48 + 12i - 51 + 51i}{34}\\ &= \frac{-3}{34} + \frac{63}{34}i\end{aligned}$$

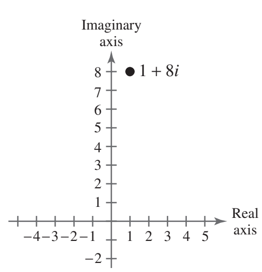
91. $-2i$



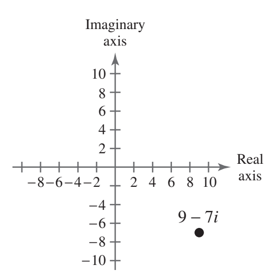
92. $3i$



93. $1 + 8i$



94. $9 - 7i$



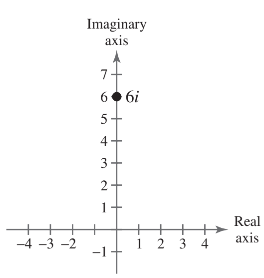
Section 6.5 Trigonometric Form of a Complex Number

- You should be able to graphically represent complex numbers.
- The absolute value of the complex numbers $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$.
- The trigonometric form of the complex number $z = a + bi$ is $z = r(\cos\theta + i\sin\theta)$ where
  (a) $a = r\cos\theta$
  (b) $b = r\sin\theta$
  (c) $r = \sqrt{a^2 + b^2}$; r is called the modulus of z.
  (d) $\tan\theta = b/a$; θ is called the argument of z.
- Given $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$:
  (a) $z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
  (b) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)],\ z_2 \neq 0$
- You should know DeMoivre's Theorem: If $z = r(\cos\theta + i\sin\theta)$, then for any positive integer n,
  $z^n = r^n(\cos n\theta + i\sin n\theta)$.
- You should know that for any positive integer n, $z = r(\cos\theta + i\sin\theta)$ has n distinct nth roots given by
  $$\sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right]$$
  where $k = 0, 1, 2, \ldots, n - 1$.

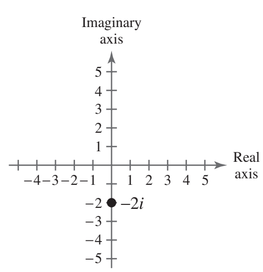
Vocabulary Check

1. absolute value
2. trigonometric form, modulus, argument
3. DeMoivre's
4. nth root

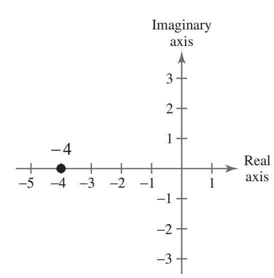
1. $|6i| = 6$



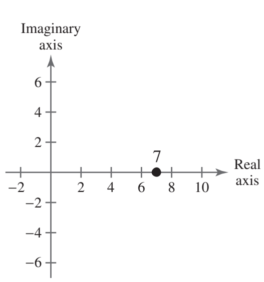
2. $|-2i| = 2$



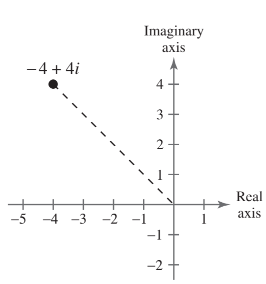
3. $|-4| = \sqrt{(-4)^2 + 0^2}$
$= \sqrt{16} = 4$



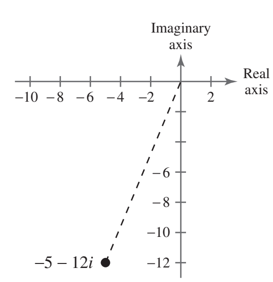
4. $|7| = 7$



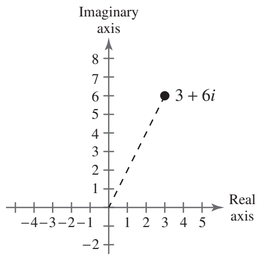
5. $|-4 + 4i| = \sqrt{(-4)^2 + (4)^2}$
$= \sqrt{32} = 4\sqrt{2}$



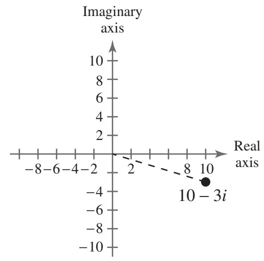
6. $|-5 - 12i| = \sqrt{5^2 + 12^2}$
$= \sqrt{169} = 13$



7. $|3 + 6i| = \sqrt{9 + 36}$
$= \sqrt{45} = 3\sqrt{5}$



8. $|10 - 3i| = \sqrt{10^2 + (-3)^2}$
$= \sqrt{109}$



9. $z = 3i$

$r = \sqrt{0^2 + 3^2} = \sqrt{9} = 3$

$\tan\theta = \dfrac{3}{0}$, undefined $\Rightarrow \theta = \dfrac{\pi}{2}$

$z = 3\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

10. $z = 4$

$r = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$

$\tan\theta = \dfrac{0}{4} = 0 \implies \theta = 0$

$z = 4(\cos 0 + i\sin 0)$

11. $z = -2$

$r = \sqrt{(-2)^2 + 0^2} = 2$

$\tan\theta = \pi \implies \theta = \pi$

$z = 2(\cos\pi + i\sin\pi)$

12. $z = -i$

$r = \sqrt{0^2 + (-1)^2} = 1$

$\tan\theta$ undefined $\implies \theta = \dfrac{3\pi}{2}$

$z = \cos\dfrac{3\pi}{2} + i\sin\dfrac{3\pi}{2}$

13. $z = -2 - 2i$

$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \dfrac{-2}{-2} = 1$, θ is in Quadrant III.

$\theta = \dfrac{5\pi}{4}$

$z = 2\sqrt{2}\left(\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right)$

14. $z = 3 + 3i$

$r = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$\tan\theta = \dfrac{3}{3} = 1 \implies \theta = \dfrac{\pi}{4}$

$z = 3\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

15. $z = \sqrt{3} - i$

$r = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2} = 2$

$\tan\theta = \dfrac{-1}{\sqrt{3}} \implies \theta = \dfrac{11\pi}{6}$

$z = 2\left(\cos\dfrac{11\pi}{6} + i\sin\dfrac{11\pi}{6}\right)$

16. $z = -1 + \sqrt{3}i$

$r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{4} = 2$

$\tan\theta = \dfrac{\sqrt{3}}{-1} = -\sqrt{3} \implies \theta = \dfrac{2\pi}{3}$

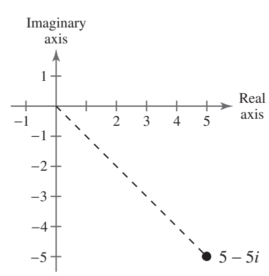
$z = 2\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

17. $z = 5 - 5i$

$r = \sqrt{5^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$

$\tan\theta = -\dfrac{5}{5} = -1 \implies \theta = \dfrac{7\pi}{4}$

$z = 5\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)$

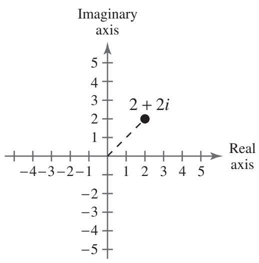


18. $z = 2 + 2i$

$r = \sqrt{4 + 4} = 2\sqrt{2}$

$\tan \theta = \dfrac{2}{2} = 1 \implies \theta = \dfrac{\pi}{4}$

$z = 2\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)$

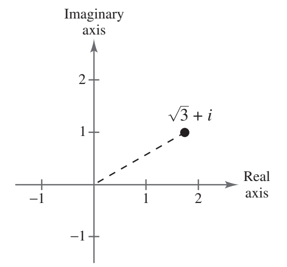


19. $z = \sqrt{3} + i$

$r = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$

$\tan \theta = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} \implies \theta = \dfrac{\pi}{6}$

$z = 2\left(\cos \dfrac{\pi}{6} + i \sin \dfrac{\pi}{6}\right)$

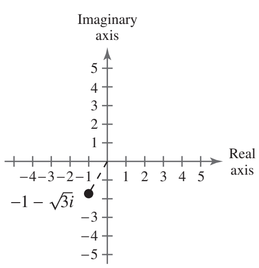


20. $z = -1 - \sqrt{3}i$

$r = \sqrt{1 + 3} = 2$

$\tan \theta = \dfrac{\sqrt{3}}{1} \implies \theta = 240° = \dfrac{4\pi}{3}$

$z = 2\left(\cos \dfrac{4\pi}{3} + i \sin \dfrac{4\pi}{3}\right)$

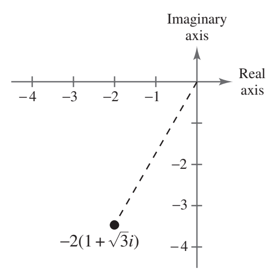


21. $z = -2(1 + \sqrt{3}i)$

$r = \sqrt{(-2)^2 + \left(-2\sqrt{3}\right)^2} = \sqrt{16} = 4$

$\tan \theta = \dfrac{\sqrt{3}}{1} = \sqrt{3} \implies \theta = \dfrac{4\pi}{3}$

$z = 4\left(\cos \dfrac{4\pi}{3} + i \sin \dfrac{4\pi}{3}\right)$



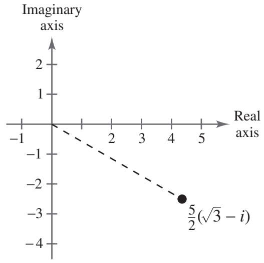
22. $z = \dfrac{5}{2}\left(\sqrt{3} - i\right)$

$r = \sqrt{\left(\dfrac{5}{2}\sqrt{3}\right)^2 + \left(\dfrac{5}{2}(-1)\right)^2} = \sqrt{\dfrac{100}{4}}$

$= \sqrt{25} = 5$

$\tan \theta = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3} \implies \theta = \dfrac{11\pi}{6}$

$z = 5\left(\cos \dfrac{11\pi}{6} + i \sin \dfrac{11\pi}{6}\right)$

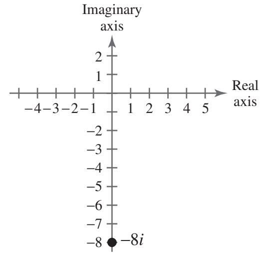


23. $z = -8i$

$r = \sqrt{0 + (-8)^2} = \sqrt{64} = 8$

$\tan \theta = -\dfrac{8}{0}$, undefined $\implies \theta = \dfrac{3\pi}{2}$

$z = 8\left(\cos \dfrac{3\pi}{2} + i \sin \dfrac{3\pi}{2}\right)$

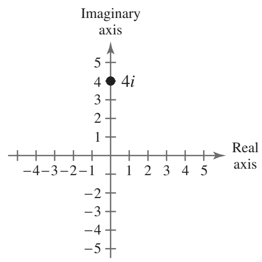


24. $z = 4i$

$r = 4$

$\tan\theta = \dfrac{4}{0}$ undefined $\Rightarrow \theta = \dfrac{\pi}{2}$

$z = 4\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

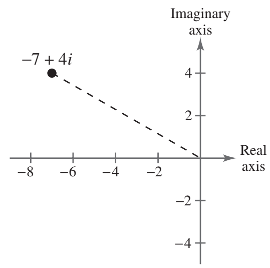


25. $z = -7 + 4i$

$r = \sqrt{49 + 16} = \sqrt{65}$

$\tan\theta = \dfrac{4}{-7} \Rightarrow \theta \approx 2.62$ radians or $150.26°$

$z = \sqrt{65}(\cos 150.26° + i\sin 150.26°)$

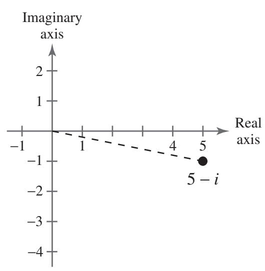


26. $z = 5 - i$

$r = \sqrt{5^2 + (-1)^2} = \sqrt{26}$

$\tan\theta = -\dfrac{1}{5} \Rightarrow \theta \approx -11.3°$ or $348.7°$

$z \approx \sqrt{26}(\cos(-11.3°) + i\sin(-11.3°))$

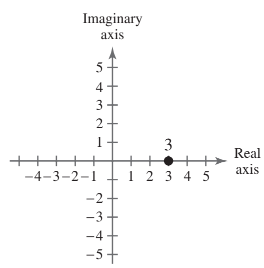


27. $z = 3$

$r = \sqrt{3^2 + 0^2} = 3$

$\tan\theta = \dfrac{0}{3} = 0 \Rightarrow \theta = 0°$

$z = 3(\cos 0° + i\sin 0°)$

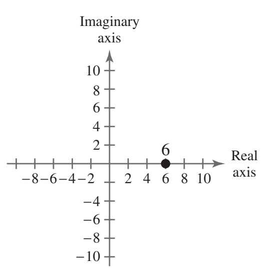


28. $z = 6$

$r = 6$

$\theta = 0$

$z = 6(\cos 0 + i\sin 0)$

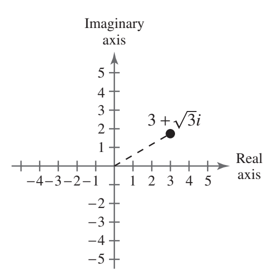


29. $z = 3 + \sqrt{3}i$

$r = \sqrt{9 + 3} = \sqrt{12} = 2\sqrt{3}$

$\tan\theta = \dfrac{\sqrt{3}}{3} \Rightarrow \theta = \dfrac{\pi}{6}$ or $30°$

$z = 2\sqrt{3}\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

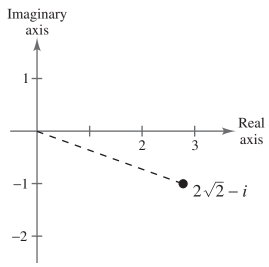


30. $z = 2\sqrt{2} - i$

$r = \sqrt{(2\sqrt{2})^2 + (-1)^2} = \sqrt{9} = 3$

$\tan\theta = \dfrac{-1}{2\sqrt{2}} = -\dfrac{\sqrt{2}}{4} \implies \theta \approx -19.5° \text{ or } 340.5°$

$z = 3(\cos(-19.5°) + i\sin(-19.5°))$

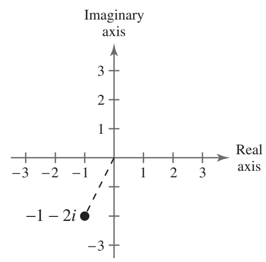


31. $z = -1 - 2i$

$r = \sqrt{1^2 + 2^2} = \sqrt{5}$

$\tan\theta = \dfrac{-2}{-1} = 2 \implies \theta \approx 243.4°$

$z = \sqrt{5}(\cos 243.4° + i\sin 243.4°)$

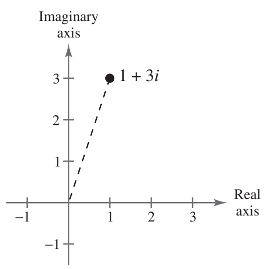


32. $z = 1 + 3i$

$r = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\tan\theta = \frac{3}{1} = 3 \implies \theta \approx 71.6°$

$z \approx \sqrt{10}(\cos 71.6° + i\sin 71.6°)$

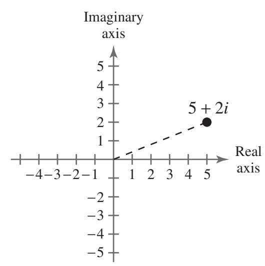


33. $z = 5 + 2i$

$r = \sqrt{25 + 4} = \sqrt{29} \approx 5.385$

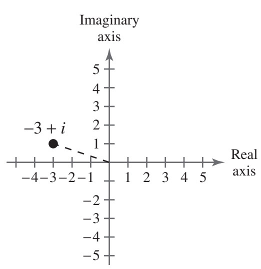
$\tan\theta = \frac{2}{5} \implies \theta \approx 21.80°$

$z = \sqrt{29}(\cos 21.80° + i\sin 21.80°)$



34. $-3 + i \approx 3.16(\cos 161.6° + i\sin 161.6°)$

$= 3.16(\cos 2.82 + i\sin 2.82)$

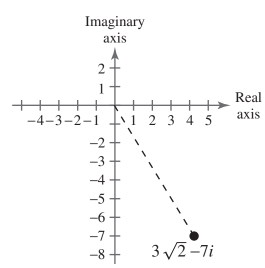


35. $z = 3\sqrt{2} - 7i$

$r = \sqrt{18 + 49} = \sqrt{67} \approx 8.185$

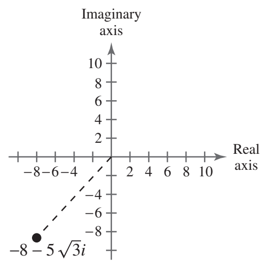
$\tan\theta = \dfrac{-7}{3\sqrt{2}} \approx -1.6499 \implies \theta \approx 301.22°$

$z = \sqrt{67}(\cos 301.22° + i\sin 301.22°)$



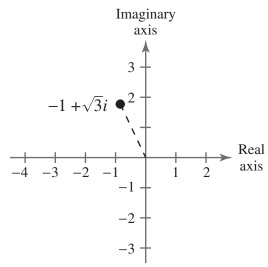
36. $-8 - 5\sqrt{3}i \approx 11.79(\cos 227.3° + i \sin 227.3°)$

$-8 - 5\sqrt{3}i = 11.79(\cos 3.97 + i \sin 3.97)$



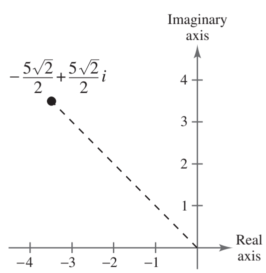
37. $2(\cos 120° + i \sin 120°) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= -1 + \sqrt{3}i$



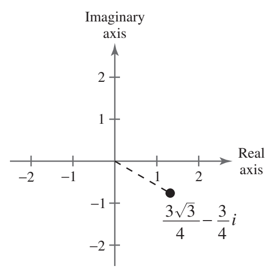
38. $5(\cos 135° + i \sin 135°) = 5\left[-\frac{\sqrt{2}}{2} + i\left(\frac{\sqrt{2}}{2}\right)\right]$

$= -\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$



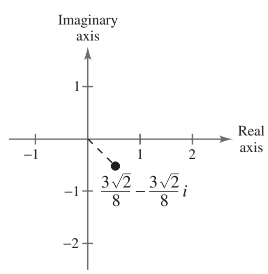
39. $\frac{3}{2}(\cos 330° + i \sin 330°) = \frac{3}{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= \frac{3\sqrt{3}}{4} - \frac{3}{4}i$

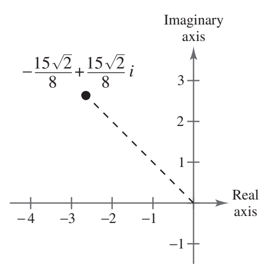


40. $\frac{3}{4}(\cos 315° + i \sin 315°) = \frac{3}{4}\left[\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right]$

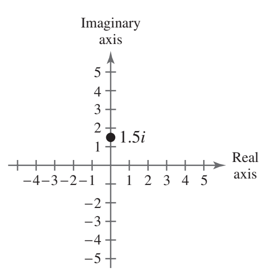
$= \frac{3\sqrt{2}}{8} - \frac{3\sqrt{2}}{8}i$



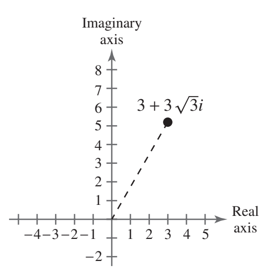
41. $3.75\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\frac{15\sqrt{2}}{8} + \frac{15\sqrt{2}}{8}i$



42. $1.5\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 1.5(0 + i) = 1.5i$

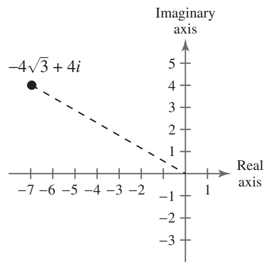


43. $6\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = 6\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 3 + 3\sqrt{3}i$

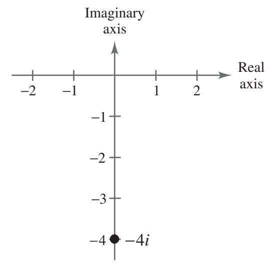


44. $8\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 8\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

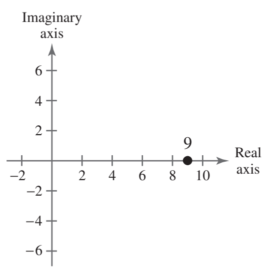
$= -4\sqrt{3} + 4i$



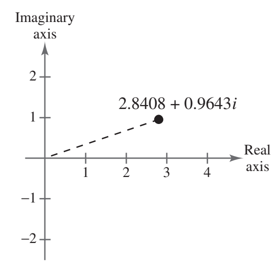
45. $4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = 4(0 - i) = -4i$



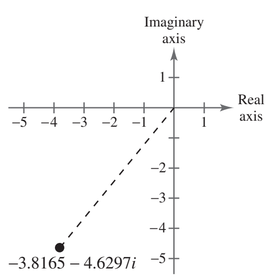
46. $9(\cos 0 + i\sin 0) = 9$



47. $3[\cos(18° 45') + i\sin(18° 45')] \approx 2.8408 + 0.9643i$



48. $6[\cos(230° 30') + i\sin(230° 30')] \approx -3.8165 - 4.6297i$



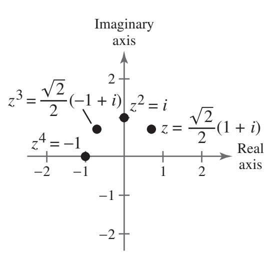
49. $5\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right) \approx 4.6985 + 1.7101i$

50. $12\left(\cos\frac{3\pi}{5} + \text{i}\sin\frac{3\pi}{5}\right) \approx -3.71 + 11.41i$

51. $9(\cos 58° + i\sin 58°) \approx 4.7693 + 7.6324i$

52. $4(\cos 216.5° + i\sin 216.5°) = -3.22 - 2.38i$

53.



The absolute value of each power is 1.

54. $z = \frac{1}{2}(1 + \sqrt{3}i)$

$z^2 = \frac{1}{2}(1 + \sqrt{3}i)\frac{1}{2}(1 + \sqrt{3}i) = \frac{1}{2}(-1 + \sqrt{3}i)$

$z^3 = z^2 z = \frac{1}{2}(-1 + \sqrt{3}i)\frac{1}{2}(1 + \sqrt{3}i)$

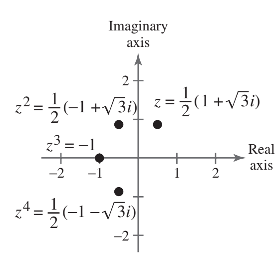
$= -1$

$z^4 = z^3 z$

$= (-1)\frac{1}{2}(1 + \sqrt{3}i)$

$= \frac{1}{2}(-1 - \sqrt{3}i)$

The absolute value of each is 1.



55. $\left[3\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]=(3)(4)\left[\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\right]=12\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

56. $\left[\frac{3}{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]\left[6\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]=\frac{3}{2}(6)\left[\cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{6}+\frac{\pi}{4}\right)\right]$

$=9\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

57. $\left[\frac{5}{3}(\cos 140^\circ+i\sin 140^\circ)\right]\left[\frac{2}{3}(\cos 60^\circ+i\sin 60^\circ)\right]=\left(\frac{5}{3}\right)\left(\frac{2}{3}\right)[\cos(140^\circ+60^\circ)+i\sin(140^\circ+60^\circ)]$

$=\frac{10}{9}(\cos 200^\circ+i\sin 200^\circ)$

58. $\left[\frac{1}{2}(\cos 115^\circ+i\sin 115^\circ)\right]\left[\frac{4}{5}(\cos 300^\circ+i\sin 300^\circ)\right]=\frac{1}{2}\left(\frac{4}{5}\right)[\cos(115^\circ+300^\circ)+i\sin(115^\circ+300^\circ)]$

$=\frac{2}{5}(\cos 415^\circ+i\sin 415^\circ)=\frac{2}{5}(\cos 55^\circ+i\sin 55^\circ)$

59. $\left[\frac{11}{20}(\cos 290^\circ+i\sin 290^\circ)\right]\left[\frac{2}{5}(\cos 200^\circ+i\sin 200^\circ)\right]=\left(\frac{11}{20}\right)\left(\frac{2}{5}\right)[\cos(290^\circ+200^\circ)+i\sin(290^\circ+200^\circ)]$

$=\frac{11}{50}(\cos 490^\circ+i\sin 490^\circ)$

$=\frac{11}{50}(\cos 130^\circ+i\sin 130^\circ)$

60. $(\cos 5^\circ+i\sin 5^\circ)(\cos 20^\circ+i\sin 20^\circ)=\cos(5^\circ+20^\circ)+i\sin(5^\circ+20^\circ)$

$=\cos 25^\circ+i\sin 25^\circ$

61. $\dfrac{\cos 50^\circ+i\sin 50^\circ}{\cos 20^\circ+i\sin 20^\circ}=\cos(50^\circ-20^\circ)+i\sin(50^\circ-20^\circ)=\cos 30^\circ+i\sin 30^\circ$

62. $\dfrac{5[\cos(4.3)+i\sin(4.3)]}{4[\cos(2.1)+i\sin(2.1)]}=\frac{5}{4}[\cos(4.3-2.1)+i\sin(4.3-2.1)=\frac{5}{4}[\cos(2.2)+i\sin(2.2)]$

63. $\dfrac{2(\cos 120^\circ+i\sin 120^\circ)}{4(\cos 40^\circ+i\sin 40^\circ)}=\frac{1}{2}[\cos(120^\circ-40^\circ)+i\sin(120^\circ-40^\circ)]=\frac{1}{2}(\cos 80^\circ+i\sin 80^\circ)$

64. $\dfrac{\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}}{\cos\pi+i\sin\pi}=\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}$

65. $\dfrac{18(\cos 54^\circ+i\sin 54^\circ)}{3(\cos 102^\circ+i\sin 102^\circ)}=6(\cos(54^\circ-102^\circ)+i\sin(54^\circ-102^\circ))$

$=6(\cos(-48^\circ)+i\sin(-48^\circ))=6(\cos 312^\circ+i\sin 312^\circ)$

66. $\dfrac{9(\cos 20^\circ+i\sin 20^\circ)}{5(\cos 75^\circ+i\sin 75^\circ)}=\frac{9}{5}[\cos(20^\circ-75^\circ)+i\sin(20^\circ-75^\circ)]$

$=\frac{9}{5}[\cos(-55^\circ)+i\sin(-55^\circ)]$

$=\frac{9}{5}[\cos 305^\circ+i\sin 305^\circ]$

67. (a) $2 - 2i = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b) $(2 - 2i)(1 + i) = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 4(\cos 2\pi + i\sin 2\pi) = 4$

(c) $(2 - 2i)(1 + i) = 2 + 2 = 4$

68. (a) $3 - 3i = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

(b) $(3 - 3i)(1 - i) = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = 6\left(\cos\frac{7\pi}{2} + i\sin\frac{7\pi}{2}\right) = -6i$

(c) $(3 - 3i)(1 - i) = 3 - 3 - 6i = -6i$

69. (a) $2 + 2i = 2\sqrt{2}(\cos 45° + i\sin 45°)$

$1 - i = \sqrt{2}[\cos(-45°) + i\sin(-45°)]$

(b) $(2 + 2i)(1 - i) = \left[2\sqrt{2}(\cos 45° + i\sin 45°)\right]\left[\sqrt{2}(\cos(-45°) + i\sin(-45°))\right] = 4(\cos 0° + i\sin 0°) = 4$

(c) $(2 + 2i)(1 - i) = 2 - 2i + 2i - 2i^2 = 2 + 2 = 4$

70. (a) $\sqrt{3} + i = 2(\cos 30° + i\sin 30°)$

$1 + i = \sqrt{2}(\cos 45° + i\sin 45°)$

(b)
$$\begin{aligned}(\sqrt{3} + i)(1 + i) &= [2(\cos 30° + i\sin 30°)]\left[\sqrt{2}(\cos 45° + i\sin 45°)\right] \\ &= 2\sqrt{2}(\cos 75° + i\sin 75°) \\ &= 2\sqrt{2}\left[\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) + \left(\frac{\sqrt{6} + \sqrt{2}}{4}\right)i\right] \\ &= (\sqrt{3} - 1) + (\sqrt{3} + 1)i\end{aligned}$$

(c) $(\sqrt{3} + i)(1 + i) = \sqrt{3} + (\sqrt{3} + 1)i + i^2 = (\sqrt{3} - 1) + (\sqrt{3} + 1)i$

71. (a) $-2i = 2[\cos(-90°) + i\sin(-90°)]$

$1 + i = \sqrt{2}(\cos 45° + i\sin 45°)$

(b)
$$\begin{aligned}-2i(1 + i) &= 2[\cos(-90°) + i\sin(-90°)]\left[\sqrt{2}(\cos 45° + i\sin 45°)\right] \\ &= 2\sqrt{2}[\cos(-45°) + i\sin(-45°)] \\ &= 2\sqrt{2}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right] = 2 - 2i\end{aligned}$$

(c) $-2i(1 + i) = -2i - 2i^2 = -2i + 2 = 2 - 2i$

72. (a) $3i = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b) $$3i(1 + i) = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$
$$= 3\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$
$$= 3\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -3 + 3i$$

(c) $3i(1 + i) = 3i - 3 = -3 + 3i$

73. (a) $-2i = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$\sqrt{3} - i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

(b) $$-2i\left(\sqrt{3} - i\right) = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$
$$= 4\left(\cos\frac{20\pi}{6} + i\sin\frac{20\pi}{6}\right)$$
$$= 4\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$
$$= -2 - 2\sqrt{3}i$$

(c) $-2i\left(\sqrt{3} - i\right) = -2\sqrt{3}i - 2$

74. (a) $-i = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$

$1 + \sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(b) $$-i\left(1 + \sqrt{3}i\right) = \left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$
$$= 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$
$$= 2\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \sqrt{3} - i$$

(c) $(-i)\left(1 + \sqrt{3}i\right) = -i + \sqrt{3}$

75. (a) $2 = 2(\cos 0 + i\sin 0)$

$1 - i = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

(b) $$2(1 - i) = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$
$$= 2\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$
$$= 2(1 - i) = 2 - 2i$$

(c) $2(1 - i) = 2 - 2i$

76. (a) $-4 = 4(\cos\pi + i\sin\pi)$

$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b) $$-4\left(1 + i\right) = 4\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$
$$= 4\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$
$$= -4 - 4i$$

(c) $-4(1 + i) = -4 - 4i$

77. (a) $3 + 3i = 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$1 - \sqrt{3}i = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

(b) $\frac{3 + 3i}{1 - \sqrt{3}i} = \frac{3\sqrt{2}}{2}\left(\cos\left(\frac{\pi}{4} - \frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{4} - \frac{5\pi}{3}\right)\right)$

$= \frac{3\sqrt{2}}{2}\left(\cos\left(-\frac{17\pi}{12}\right) + i\sin\left(-\frac{17\pi}{12}\right)\right)$

$\approx -0.549 + 2.049i$

(c) $\frac{3 + 3i}{1 - \sqrt{3}i} \cdot \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i} = \frac{3 - 3\sqrt{3} + \left(3 + 3\sqrt{3}\right)i}{4}$

$\approx -0.549 + 2.049i$

78. (a) $2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$1 + \sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(b) $\frac{2 + 2i}{1 + \sqrt{3}i} = \frac{2\sqrt{2}}{2}\left(\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)\right)$

$= \sqrt{2}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$

$\approx 1.366 - 0.366i$

(c) $\frac{2 + 2i}{1 + \sqrt{3}i} \cdot \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i} = \frac{2 + 2\sqrt{3} + \left(2 - 2\sqrt{3}\right)i}{4} = \frac{1 + \sqrt{3}}{2} + \frac{1 - \sqrt{3}}{2}i$

$\approx 1.366 - 0.366i$

79. (a) $5 = 5(\cos 0 + i\sin 0)$

$2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b) $\frac{5}{2 + 2i} = \frac{5(\cos 0 + i\sin 0)}{2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}$

$= \frac{5}{2\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$= \frac{5}{2\sqrt{2}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = \frac{5}{4} - \frac{5}{4}i$

(c) $\frac{5}{2 + 2i} \cdot \frac{2 - 2i}{2 - 2i} = \frac{5(2 - 2i)}{4 + 4} = \frac{5}{4} - \frac{5}{4}i$

80. (a) $2 = 2(\cos 0 + i\sin 0)$

$\sqrt{3} - i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

(b) $\frac{2}{\sqrt{3} - i} = \frac{2}{2}\left(\cos\left(-\frac{11\pi}{6}\right) + i\sin\left(-\frac{11\pi}{6}\right)\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}i$

(c) $\frac{2}{\sqrt{3} - i} \cdot \frac{\sqrt{3} + i}{\sqrt{3} + i} = \frac{2\left(\sqrt{3} + i\right)}{4} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

81. (a) $4i = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$-1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(b) $\frac{4i}{-1+i} = \frac{4}{\sqrt{2}}\left(\cos\left(\frac{\pi}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)\right)$

$= \frac{4}{\sqrt{2}}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

$= \frac{4}{\sqrt{2}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) = 2 - 2i$

(c) $\frac{4i}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{-4i+4}{2} = 2 - 2i$

82. (a) $2i = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$1 - \sqrt{3}i = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

(b) $\frac{2i}{1-\sqrt{3}i} = \frac{2}{2}\left(\cos\left(\frac{\pi}{2} - \frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{2} - \frac{5\pi}{3}\right)\right)$

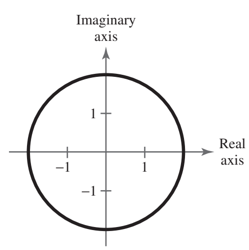
$= \cos\left(-\frac{7\pi}{6}\right) + i\sin\left(-\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{1}{2}i$

(c) $\frac{2i}{1-\sqrt{3}i} \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i} = \frac{2i - 2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2} + \frac{1}{2}i$

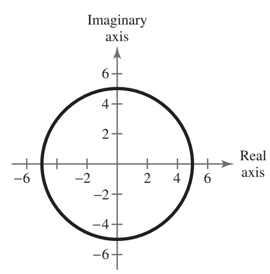
83. Let $z = x + iy$ such that:

$|z| = 2 \implies 2 = \sqrt{x^2 + y^2} \implies 4 = x^2 + y^2$

Circle with radius of 2



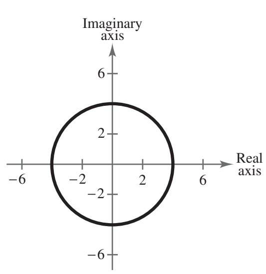
84. $|z| = 5$, Circle of radius 5



85. Let $z = x + iy$.

$|z| = 4 \implies 4 = \sqrt{x^2 + y^2} \implies x^2 + y^2 = 16$

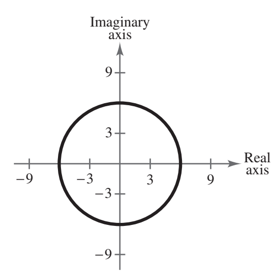
Circle of radius 4



86. Let $z = x + iy$.

$|z| = 6 \implies 6 = \sqrt{x^2 + y^2} \implies x^2 + y^2 = 36$

Circle of radius 6



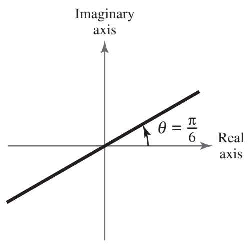
87. $\theta = \dfrac{\pi}{6}$

Let $z = x + iy$ such that:

$\tan \dfrac{\pi}{6} = \dfrac{y}{x} \implies$

$\dfrac{y}{x} = \dfrac{1}{\sqrt{3}} \implies y = \dfrac{1}{\sqrt{3}}x$

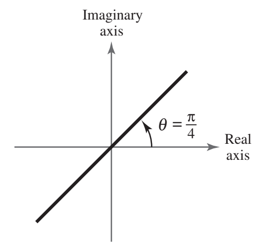
Line



88. $\theta = \dfrac{\pi}{4} = \arctan 1$

Line

$y = x$



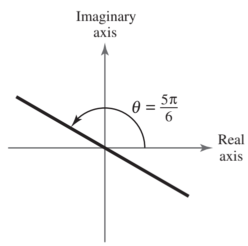
89. $\theta = \dfrac{5\pi}{6}$

Let $z = x + iy$ such that:

$\tan \dfrac{5\pi}{6} = \dfrac{y}{x} \implies$

$\dfrac{y}{x} = -\dfrac{\sqrt{3}}{3} \implies y = -\dfrac{\sqrt{3}}{3}x$

Line



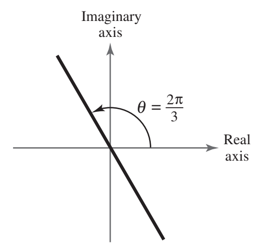
90. $\theta = \dfrac{2\pi}{3}$

Let $z = x + iy$ such that:

$\tan \dfrac{2\pi}{3} = \dfrac{y}{x} \implies$

$-\sqrt{3} = \dfrac{y}{x} \implies y = -\sqrt{3}x$

Line



91. $$\begin{aligned}(1 + i)^3 &= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^3 \\ &= \left(\sqrt{2}\right)^3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) \\ &= 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) \\ &= -2 + 2i\end{aligned}$$

92. $$\begin{aligned}(2 + 2i)^6 &= \left[2\sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]^6 \\ &= \left(2\sqrt{2}\right)^6\left(\cos \frac{6\pi}{4} + i \sin \frac{6\pi}{4}\right) \\ &= 512\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) \\ &= -512i\end{aligned}$$

93. $$\begin{aligned}(-1 + i)^{10} &= \left[\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)\right]^{10} \\ &= \left(\sqrt{2}\right)^{10}\left(\cos \frac{30\pi}{4} + i \sin \frac{30\pi}{4}\right) \\ &= 32\left[\cos\left(\frac{3\pi}{2} + 6\pi\right) + i \sin\left(\frac{3\pi}{2} + 6\pi\right)\right] \\ &= 32\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) \\ &= 32[0 + i(-1)] = -32i\end{aligned}$$

94. $$\begin{aligned}(1 - i)^8 &= \left[\sqrt{2}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)\right]^8 \\ &= \left(\sqrt{2}\right)^8(\cos 14\pi + i \sin 14\pi) \\ &= 16\end{aligned}$$

95. $$\begin{aligned}2\left(\sqrt{3} + i\right)^5 &= 2\left[2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)\right]^5 \\ &= 2\left[2^5\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)\right] \\ &= 64\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \\ &= -32\sqrt{3} + 32i\end{aligned}$$

96. $$\begin{aligned}4\left(1 - \sqrt{3}i\right)^3 &= 4\left[2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)\right]^3 \\ &= 4[2^3(\cos 5\pi + i \sin 5\pi)] \\ &= 32(-1) \\ &= -32\end{aligned}$$

97. $[5(\cos 20° + i \sin 20°)]^3 = 5^3(\cos 60° + i \sin 60°) = \dfrac{125}{2} + \dfrac{125\sqrt{3}}{2}i$

98.
$$\begin{aligned}[3(\cos 150° + i \sin 150°)]^4 &= 3^4(\cos 600° + i \sin 600°)\\ &= 81(\cos 240° + i \sin 240°)\\ &= 81(-\cos 60° - i \sin 60°)\\ &= -\frac{81}{2} - \frac{81\sqrt{3}}{2}i\end{aligned}$$

99.
$$\begin{aligned}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)^{10} &= \cos\frac{25\pi}{2} + i\sin\frac{25\pi}{2}\\ &= \cos\left(12\pi + \frac{\pi}{2}\right) + i\sin\left(12\pi + \frac{\pi}{2}\right) = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i\end{aligned}$$

100. $\left[2\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)\right]^{12} = 2^{12}(\cos 6\pi + i\sin 6\pi) = 2^{12} = 4096$

101.
$$\begin{aligned}[2(\cos 1.25 + i\sin 1.25)]^4 &= 2^4(\cos 5 + i\sin 5)\\ &\approx 4.5386 - 15.3428i\end{aligned}$$

102.
$$\begin{aligned}[4(\cos 2.8 + i\sin 2.8)]^5 &= 4^5(\cos 14 + i\sin 14)\\ &\approx 140.02 + 1014.38i\end{aligned}$$

103.
$$\begin{aligned}[2(\cos\pi + i\sin\pi)]^8 &= 2^8(\cos 8\pi + i\sin 8\pi)\\ &= 256(1) = 256\end{aligned}$$

104.
$$\begin{aligned}(\cos 0 + i\sin 0)^{20} &= \cos 0 + i\sin 0\\ &= 1\end{aligned}$$

105. $(3 - 2i)^5 = -597 - 122i$

106.
$$\begin{aligned}\left(\sqrt{5} - 4i\right)^4 &= \left[\sqrt{21}(\cos(5.2221) + i\sin(5.2221))\right]^4\\ &= 441[\cos(20.8884) + i\sin(20.8884)]\\ &= -199 + 393.55i\end{aligned}$$

107.
$$\begin{aligned}[4(\cos 10° + i\sin 10°)]^6 &= 4^6(\cos 60° + i\sin 60°)\\ &= 4096\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\\ &= 2048 + 2048\sqrt{3}i\end{aligned}$$

108.
$$\begin{aligned}[3(\cos 15° + i\sin 15°)]^4 &= 81(\cos 60° + i\sin 60°)\\ &= \frac{81}{2} + \frac{81\sqrt{3}}{2}i\end{aligned}$$

109.
$$\begin{aligned}\left[3\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)\right]^2 &= 3^2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\\ &= 9\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\\ &= \frac{9}{2}\sqrt{2} + \frac{9}{2}\sqrt{2}i\end{aligned}$$

110.
$$\begin{aligned}\left[2\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)\right]^5 &= 32\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\\ &= 32i\end{aligned}$$

111.
$$\begin{aligned}\left[-\frac{1}{2}(1 + \sqrt{3}i)\right]^6 &= \left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]^6\\ &= \cos 8\pi + i\sin 8\pi\\ &= 1\end{aligned}$$

112. $2^{-1/4}(1 - i)$ is a fourth root of -2 if $-2 = [2^{-1/4}(1 - i)]^4$.

$$\begin{aligned}[2^{-1/4}(1 - i)]^4 &= (2^{-1/4})^4(1 - i)^4\\ &= 2^{-1}(1 - i)^4\\ &= \tfrac{1}{2}(1 - i)^2(1 - i)^2\\ &= \tfrac{1}{2}(-2i)(-2i)\\ &= \tfrac{1}{2}(4i^2)\\ &= \tfrac{1}{2}(-4) = -2\end{aligned}$$

113. (a) In trigonometric form we have:

$2(\cos 30° + i \sin 30°)$

$2(\cos 150° + i \sin 150°)$

$2(\cos 270° + i \sin 270°)$

(b) There are three roots evenly spaced around a circle of radius 2. Therefore, they represent the cube roots of some number of modulus 8. Cubing them shows that they are all cube roots of $8i$.

(c) $[2(\cos 30° + i \sin 30°)]^3 = 8i$

$[2(\cos 150° + i \sin 150°)]^3 = 8i$

$[2(\cos 270° + i \sin 270°)]^3 = 8i$

114. (a) In trigonometric form we have:

$3(\cos 45° + i \sin 45°)$

$3(\cos 135° + i \sin 135°)$

$3(\cos 225° + i \sin 225°)$

$3(\cos 315° + i \sin 315°)$

(b) There are four roots evenly spaced around a circle of radius 3. Therefore, they represent the fourth roots of some number of modulus 81. Raising them to the fourth power shows that they are all fourth roots of -81.

(c) $[3(\cos 45° + i \sin 45°)]^4 = -81$

$[3(\cos 135° + i \sin 135°)]^4 = -81$

$[3(\cos 225° + i \sin 225°)]^4 = -81$

$[3(\cos 315° + i \sin 315°)]^4 = -81$

115. (a) In trigonometric form we have:

$\cos 120° + i \sin 120°$

$\cos 240° + i \sin 240°$

$\cos 0° + i \sin 0°$

(b) These are the three cube roots of 1.

(c) $(\cos 120° + i \sin 120°)^3 = 1$

$(\cos 240° + i \sin 240°)^3 = 1$

$(\cos 0° + i \sin 0°)^3 = 1$

116. (a) In trigonometric form we have:

$2(\cos 30° + i \sin 30°)$

$2(\cos 90° + i \sin 90°)$

$2(\cos 150° + i \sin 150°)$

$2(\cos 210° + i \sin 210°)$

$2(\cos 270° + i \sin 270°)$

$2(\cos 330° + i \sin 330°)$

(b) There are six roots evenly spaced around a circle of radius 2. Raising them to the sixth power shows that they are the six sixth roots of -64.

(c) $[2(\cos 30° + i \sin 30°)]^6 = -64$

$[2(\cos 90° + i \sin 90°)]^6 = -64$

$[2(\cos 150° + i \sin 150°)]^6 = -64$

$[2(\cos 210° + i \sin 210°)]^6 = -64$

$[2(\cos 270° + i \sin 270°)]^6 = -64$

$[2(\cos 330° + i \sin 330°)]^6 = -64$

117. $2i = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Square roots:

$$\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 1 + i$$

$$\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = -1 - i$$

118. $5i = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

Square roots:

$$\sqrt{5}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{2}i$$

$$\sqrt{5}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = -\frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2}i$$

119. $-3i = 3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

Square roots:

$$\sqrt{3}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = -\frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2}i$$

$$\sqrt{3}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{2}i$$

120. $-6i = 6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

Square roots:

$$\sqrt{6}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = -\sqrt{3} + \sqrt{3}i$$

$$\sqrt{6}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \sqrt{3} - \sqrt{3}i$$

121. $2 - 2i = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

Square roots:

$$8^{1/4}\left(\cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}\right) \approx -1.554 + 0.644i$$

$$8^{1/4}\left(\cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}\right) \approx 1.554 - 0.644i$$

122. $2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Square roots:

$$8^{1/4}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right) \approx 1.554 + 0.644i$$

$$8^{1/4}\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right) \approx -1.554 - 0.644i$$

123. $1 + \sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Square roots:

$$\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$$

$$\sqrt{2}\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$$

124. $1 - \sqrt{3}i = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

Square roots:

$$\sqrt{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = \sqrt{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$= -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$$

$$\sqrt{2}\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$$

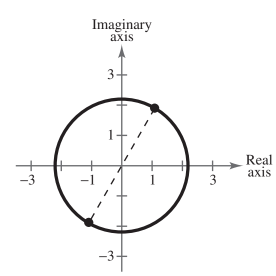
125. (a) Square roots of $5(\cos 120° + i\sin 120°)$:

$$\sqrt{5}\left[\cos\left(\frac{120° + 360°k}{2}\right) + i\sin\left(\frac{120° + 360°k}{2}\right)\right],\ k = 0, 1$$

$$\sqrt{5}(\cos 60° + i\sin 60°)$$

$$\sqrt{5}(\cos 240° + i\sin 240°)$$

(b)



(c) $\frac{\sqrt{5}}{2} + \frac{\sqrt{15}}{2}i,\ -\frac{\sqrt{5}}{2} - \frac{\sqrt{15}}{2}i$

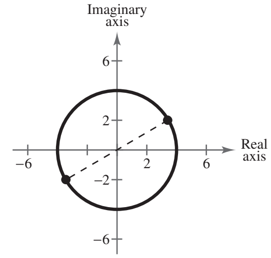
126. (a) Square roots of $16(\cos 60° + i \sin 60°)$:

$$\sqrt{16}\left[\cos\left(\frac{60° + 360° k}{2}\right) + i \sin\left(\frac{60° + 360° k}{2}\right),\ k = 0, 1\right.$$

$$4(\cos 30° + i \sin 30°)$$

$$4(\cos 210° + i \sin 210°)$$

(b)



(c) $4\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2\sqrt{3} + 2i$

$$4\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = -2\sqrt{3} - 2i$$

127. (a) Fourth roots of $16\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$:

$$\sqrt[4]{16}\left[\cos\left(\frac{(4\pi/3) + 2k\pi}{4}\right) + i \sin\left(\frac{(4\pi/3) + 2k\pi}{4}\right)\right],\ k = 0, 1, 2, 3$$

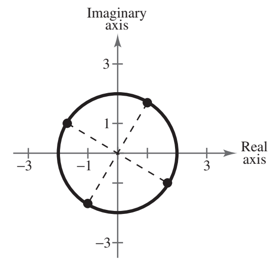
$$2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$$

$$2\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$$

$$2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$$

$$2\left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)$$

(b)



(c) $1 + \sqrt{3}i,\ -\sqrt{3} + i,\ -1 - \sqrt{3}i,\ \sqrt{3} - i$

128. (a) Fifth roots of $32\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$:

$$\sqrt[5]{32}\left[\cos\left(\frac{(5\pi/6) + 2k\pi}{5}\right) + i \sin\left(\frac{(5\pi/6) + 2k\pi}{5}\right)\right.$$

$k = 0, 1, 2, 3, 4$

$k = 0$: $2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

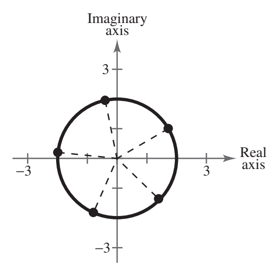
$k = 1$: $2\left(\cos \frac{17\pi}{30} + i \sin \frac{17\pi}{30}\right)$

$k = 2$: $2\left(\cos \frac{29\pi}{30} + i \sin \frac{29\pi}{30}\right)$

$k = 3$: $2\left(\cos \frac{41\pi}{30} + i \sin \frac{41\pi}{30}\right)$

$k = 4$: $2\left(\cos \frac{53\pi}{30} + i \sin \frac{53\pi}{30}\right)$

(b)



(c) $1.732 + i,\ -0.4158 + 1.956i,\ -1.989 + 0.2091i,$
$-0.8135 - 1.827i,\ 1.486 - 1.338i$

129. (a) Cube roots of $-27i = 27\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$:

$$(27)^{1/3}\left[\cos\left(\frac{(3\pi/2) + 2k\pi}{3}\right) + i\sin\left(\frac{(3\pi/2) + 2k\pi}{3}\right)\right], \quad k = 0, 1, 2$$

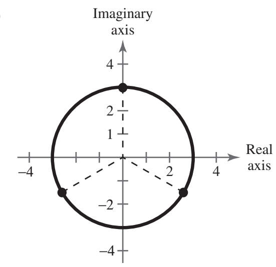
$$3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$3\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

$$3\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$$

(c) $3i, -\frac{3\sqrt{3}}{2} - \frac{3}{2}i, \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

(b)



130. (a) Fourth roots of $625i = 625\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$:

$$\sqrt[4]{625}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{4}\right) + i\sin\left(\frac{(\pi/2) + 2k\pi}{4}\right)\right]$$

$k = 0, 1, 2, 3$

$$k = 0: \ 5\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$$

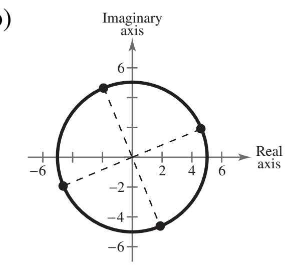
$$k = 1: \ 5\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$$

$$k = 2: \ 5\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$$

$$k = 3: \ 5\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$$

(c) $4.619 + 1.913i, -1.913 + 4.619i, -4.619 - 1.913i, 1.913 - 4.619i$

(b)



131. (a) Cube roots of $-\frac{125}{2}(1 + \sqrt{3}i) = 125\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$:

$$\sqrt[3]{125}\left[\cos\left(\frac{(4\pi/3) + 2k\pi}{3}\right) + i\sin\left(\frac{(4\pi/3) + 2k\pi}{3}\right)\right], \quad k = 0, 1, 2$$

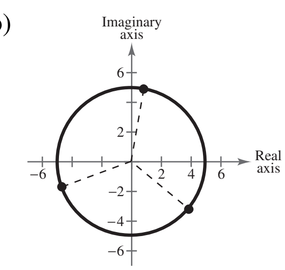
$$5\left(\cos\frac{4\pi}{9} + i\sin\frac{4\pi}{9}\right)$$

$$5\left(\cos\frac{10\pi}{9} + i\sin\frac{10\pi}{9}\right)$$

$$5\left(\cos\frac{16\pi}{9} + i\sin\frac{16\pi}{9}\right)$$

(c) $0.8682 + 4.9240i, \ -4.6985 - 1.7101i, \ 3.8302 - 3.2139i$

(b)



132. (a) Cube roots of $-4\sqrt{2}(1 - i) = 8\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$:

$$\sqrt[3]{8}\left[\cos\left(\frac{(3\pi/4) + 2k\pi}{3}\right) + i \sin\left(\frac{(3\pi/4) + 2k\pi}{3}\right)\right]$$

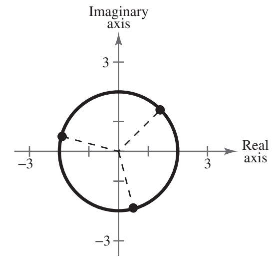
$k = 0, 1, 2$

$k = 0$: $2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

$k = 1$: $2\left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12}\right)$

$k = 2$: $2\left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12}\right)$

(b)



(c) $1.414 + 1.414i, -1.932 + 0.5176i, 0.5176 - 1.9319i$

133. (a) Cube roots of $64i = 64\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

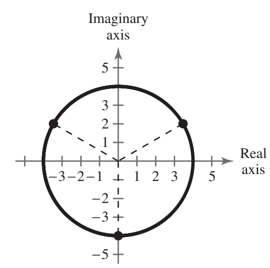
$$(64)^{1/3}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{3}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{3}\right)\right], \quad k = 0, 1, 2$$

$4\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

$4\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$

$4\left(\cos \frac{9\pi}{6} + i \sin \frac{9\pi}{6}\right) = 4\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$

(b)



(c) $2\sqrt{3} + 2i, -2\sqrt{3} + 2i, -4i$

134. (a) Fourth roots of $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$:

$$\sqrt[4]{1}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{4}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{4}\right)\right]$$

$k = 0, 1, 2, 3$

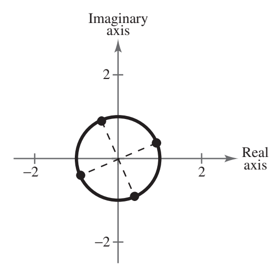
$k = 0$: $\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$

$k = 1$: $\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}$

$k = 2$: $\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}$

$k = 3$: $\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}$

(b)



(c) $0.9239 + 0.3827i, -0.3827 + 0.9239i, -0.9239 - 0.3827i, 0.3827 - 0.9239i$

135. (a) Fifth roots of $1 = \cos 0 + i \sin 0$:

$$\cos \frac{2k\pi}{5} + i \sin \frac{2k\pi}{5}, \quad k = 0, 1, 2, 3, 4$$

$$\cos 0 + i \sin 0$$

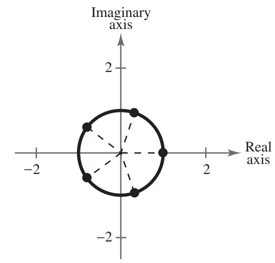
$$\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$$

$$\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$$

$$\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$$

$$\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$$

(b)



(c) $1, 0.3090 + 0.9511i, -0.8090 + 0.5878i, -0.8090 - 0.5878i, 0.3090 - 0.9511i$

136. (a) Cube roots of $1000 = 1000(\cos 0 + i \sin 0)$:

$$\sqrt[3]{1000}\left(\cos \frac{2k\pi}{3} + i \sin \frac{2k\pi}{3}\right)$$

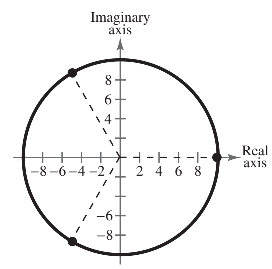
$k = 0, 1, 2$

$k = 0$: $10(\cos 0 + i \sin 0) = 10$

$k = 1$: $10\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

$k = 2$: $10\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

(b)



(c) $10, -5 + 5\sqrt{3}i, -5 - 5\sqrt{3}i$

137. (a) Cube roots of $-125 = 125(\cos 180° + i \sin 180°)$ are:

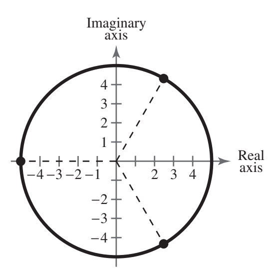
$$\sqrt[3]{125}\left[\cos\left(\frac{180 + 360k}{3}\right) + i \sin\left(\frac{180 + 360k}{3}\right)\right], \quad k = 0, 1, 2$$

$5(\cos 60° + i \sin 60°)$

$5(\cos 180° + i \sin 180°)$

$5(\cos 300° + i \sin 300°)$

(b)



(c) $\frac{5}{2} + \frac{5\sqrt{3}}{2}i, -5, \frac{5}{2} - \frac{5\sqrt{3}}{2}i$

138. (a) Fourth roots of $-4 = 4(\cos 180° + i \sin 180°)$:

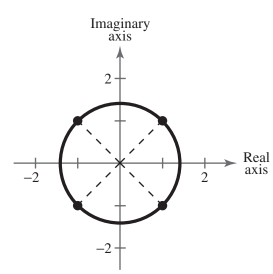
$\sqrt{2}(\cos 45° + i \sin 45°)$

$\sqrt{2}(\cos 135° + i \sin 135°)$

$\sqrt{2}(\cos 225° + i \sin 225°)$

$\sqrt{2}(\cos 315° + i \sin 315°)$

(b)



(c) $1 + i, -1 + i, -1 - i, 1 - i$

139. (a) Fifth roots of $128(-1 + i) = 128\sqrt{2}(\cos 135° + i \sin 135°)$ are:

$$2\sqrt{2}(\cos 27° + i \sin 27°) = 2\sqrt{2}\left(\cos \frac{3\pi}{20} + i \sin \frac{3\pi}{20}\right)$$

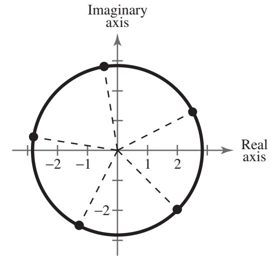
$$2\sqrt{2}(\cos 99° + i \sin 99°)$$

$$2\sqrt{2}(\cos 171° + i \sin 171°)$$

$$2\sqrt{2}(\cos 243° + i \sin 243°)$$

$$2\sqrt{2}(\cos 315° + i \sin 315°)$$

(b)



(c) $2.52 + 1.28i, -0.44 + 2.79i, -2.79 + 0.44i, -1.28 - 2.52i, 2 - 2i$

140. (a) Sixth roots of $729i = 729\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$:

$$\sqrt[6]{729}\left[\cos\left(\frac{\pi/2 + 2k\pi}{6}\right) + i \sin\left(\frac{\pi/2 + 2k\pi}{6}\right)\right], k = 0, 1, 2, 3, 4, 5$$

$$3\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$$

$$3\left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right)$$

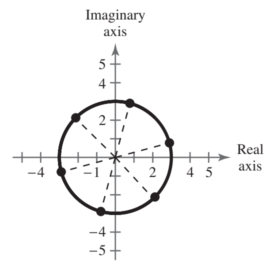
$$3\left(\cos \frac{9\pi}{12} + i \sin \frac{9\pi}{12}\right)$$

$$3\left(\cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12}\right)$$

$$3\left(\cos \frac{17\pi}{12} + i \sin \frac{17\pi}{12}\right)$$

$$3\left(\cos \frac{21\pi}{12} + i \sin \frac{21\pi}{12}\right)$$

(b)



(c) $2.898 + 0.776i, 0.776 + 2.898i, -2.121 + 2.121i,$
$-2.898 - 0.776i, -0.776 - 2.898i, 2.121 - 2.121i$

141. $x^4 - i = 0$

$x^4 = i$

The solutions are the fourth roots of $i = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$:

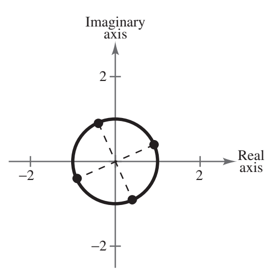
$$\sqrt[4]{1}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{4}\right) + i \sin\left(\frac{(\pi/2) + 2k\pi}{4}\right)\right], \quad k = 0, 1, 2, 3$$

$$\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$$

$$\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}$$

$$\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}$$

$$\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}$$



142. $x^3 + 27 = 0$

$$x^3 = -27$$

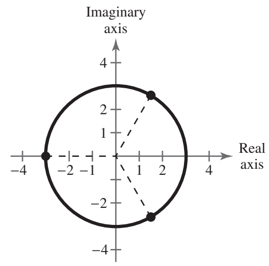
Solutions are cube roots of $-27 = 27(\cos \pi + i \sin \pi)$:

$$3\left(\cos \frac{\pi + 2\pi k}{3} + i \sin \frac{\pi + 2\pi k}{3}\right), k = 0, 1, 2$$

$$3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

$$3(\cos \pi + i \sin \pi) = -3$$

$$3\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right) = \frac{3}{2} - \frac{3\sqrt{3}}{2}i$$



143. $x^5 = -243$

The solutions are the fifth roots of $-243 = 243[\cos \pi + i \sin \pi]$:

$$\sqrt[5]{243}\left[\cos\left(\frac{\pi + 2k\pi}{5}\right) + i \sin\left(\frac{\pi + 2k\pi}{5}\right)\right], \quad k = 0, 1, 2, 3, 4$$

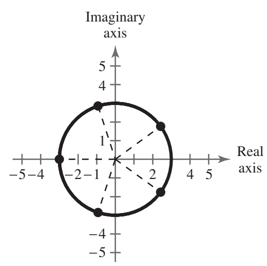
$$3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$$

$$3\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$$

$$3\left(\cos \frac{5\pi}{5} + i \sin \frac{5\pi}{5}\right) = -3$$

$$3\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$$

$$3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$$



144. $x^4 = 81 = 81(\cos 0 + i \sin 0)$

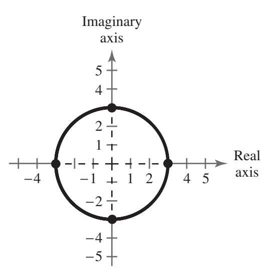
$$3\left[\cos \frac{2\pi k}{4} + i \sin \frac{2\pi k}{4}\right], k = 0, 1, 2, 3$$

$$3(\cos 0 + i \sin 0) = 3$$

$$3\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = 3i$$

$$3(\cos \pi + i \sin \pi) = -3$$

$$3\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right) = -3i$$



145. $x^4 = -16i$

The solutions are the fourth roots of $-16i = 16\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$:

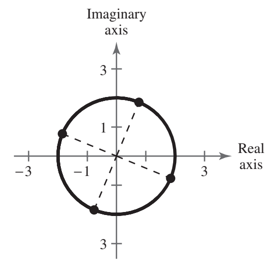
$$\sqrt[4]{16}\left[\cos\left(\frac{(3\pi/2) + 2k\pi}{4}\right) + i\sin\left(\frac{(3\pi/2) + 2k\pi}{4}\right)\right], \quad k = 0, 1, 2, 3$$

$$2\left[\cos\left(\frac{3\pi}{8}\right) + i\sin\left(\frac{3\pi}{8}\right)\right]$$

$$2\left[\cos\left(\frac{7\pi}{8}\right) + i\sin\left(\frac{7\pi}{8}\right)\right]$$

$$2\left[\cos\left(\frac{11\pi}{8}\right) + i\sin\left(\frac{11\pi}{8}\right)\right]$$

$$2\left[\cos\left(\frac{15\pi}{8}\right) + i\sin\left(\frac{15\pi}{8}\right)\right]$$



146. $x^6 - 64i = 0$

$x^6 = 64i$

The solutions are the sixth roots of $64i$:

$$\sqrt[6]{64}\left[\cos\left(\frac{(\pi/2) + 2k\pi}{6}\right) + i\sin\left(\frac{(\pi/2) + 2k\pi}{6}\right)\right), \; k = 0, 1, 2, 3, 4, 5$$

$$k = 0: 2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) \approx 1.932 + 0.5176i$$

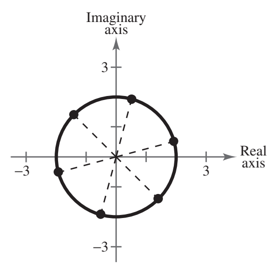
$$k = 1: 2\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) \approx 0.5176 + 1.932i$$

$$k = 2: 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \approx -1.414 + 1.414i$$

$$k = 3: 2\left(\cos\frac{13\pi}{12} + i\sin\frac{13\pi}{12}\right) \approx -1.932 - 0.5176i$$

$$k = 4: 2\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right) \approx -0.5176 - 1.932i$$

$$k = 5: 2\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) \approx 1.414 - 1.414i$$



147. $x^3 - (1 - i) = 0$

$x^3 = 1 - i = \sqrt{2}(\cos 315° + i\sin 315°)$

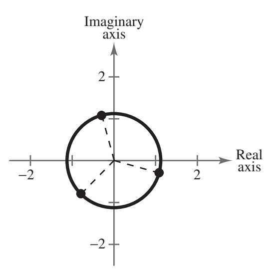
The solutions are the cube roots of $1 - i$:

$$\sqrt[3]{\sqrt{2}}\left[\cos\left(\frac{315° + 360°k}{3}\right) + i\sin\left(\frac{315° + 360°k}{3}\right)\right], \quad k = 0, 1, 2$$

$\sqrt[6]{2}(\cos 105° + i\sin 105°)$

$\sqrt[6]{2}(\cos 225° + i\sin 225°)$

$\sqrt[6]{2}(\cos 345° + i\sin 345°)$



148. $x^4 + (1 + i) = 0$

$$x^4 = -1 - i = \sqrt{2}(\cos 225° + i \sin 225°)$$

The solutions are the fourth roots of $-1 - i$:

$$\sqrt[4]{\sqrt{2}}\left[\cos\left(\frac{225° + 360°k}{4}\right) + i \sin\left(\frac{225° + 360°k}{4}\right)\right]$$

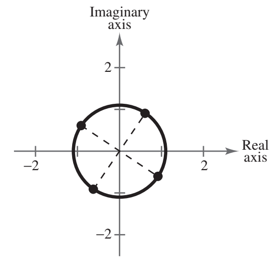
$k = 0, 1, 2, 3$

$k = 0$: $\sqrt[8]{2}(\cos 56.25° + i \sin 56.25°) \approx 0.6059 + 0.9067i$

$k = 1$: $\sqrt[8]{2}(\cos 146.25° + i \sin 146.25°) \approx -0.9067 + 0.6059i$

$k = 2$: $\sqrt[8]{2}(\cos 236.25° + i \sin 236.25°) \approx -0.6059 - 0.9067i$

$k = 3$: $\sqrt[8]{2}(\cos 326.25° + i \sin 326.25°) \approx 0.9067 - 0.6059i$



149. $E = I \cdot Z$
$= (10 + 2i)(4 + 3i)$
$= (40 - 6) + (30 + 8)i$
$= 34 + 38i$

150. $E = I \cdot Z$
$= (12 + 2i)(3 + 5i)$
$= (36 - 10) + (60 + 6)i$
$= 26 + 66i$

151. $Z = \dfrac{E}{I}$
$= \dfrac{5 + 5i}{2 + 4i} \cdot \dfrac{2 - 4i}{2 - 4i}$
$= \dfrac{(10 + 20) + (10 - 20)i}{4 + 16}$
$= \dfrac{30 - 10i}{20}$
$= \dfrac{3}{2} - \dfrac{1}{2}i$

152. $Z = \dfrac{E}{I}$
$= \dfrac{4 + 5i}{10 + 2i} \cdot \dfrac{10 - 2i}{10 - 2i}$
$= \dfrac{(40 + 10) + (50 - 8)i}{100 + 4}$
$= \dfrac{50 + 42i}{104}$
$= \dfrac{25}{52} + \dfrac{21}{52}i$

153. $I = \dfrac{E}{Z}$
$= \dfrac{12 + 24i}{12 + 20i} \cdot \dfrac{12 - 20i}{12 - 20i}$
$= \dfrac{(144 + 480) + (288 - 240)i}{144 + 400}$
$= \dfrac{624 + 48i}{544}$
$= \dfrac{39}{34} + \dfrac{3}{34}i$

154. $I = \dfrac{E}{Z}$
$= \dfrac{15 + 12i}{25 + 24i} \cdot \dfrac{25 - 24i}{25 - 24i}$
$= \dfrac{(375 + 288) + (300 - 360)i}{625 + 576}$
$= \dfrac{663 - 60i}{1201}$
$= \dfrac{663}{1201} - \dfrac{60}{1201}i$

155. True. $\left[\frac{1}{2}\left(1 - \sqrt{3}i\right)\right]^9 = \left[\frac{1}{2} - \frac{\sqrt{3}}{2}i\right]^9 = -1$

156. False. $\left(\sqrt{3} + i\right)^2 = 2 + 2\sqrt{3}i \neq 8i$

157. True

158. $\dfrac{z_1}{z_2} = \dfrac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} \cdot \dfrac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$

$= \dfrac{r_1}{r_2(\cos^2\theta_2 + \sin^2\theta_2)}[\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1)]$

$= \dfrac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

159. $z = r(\cos\theta + i\sin\theta)$

$\bar{z} = r(\cos\theta - i\sin\theta)$

$= r(\cos(-\theta) + i\sin(-\theta))$

160. (a) $z\bar{z} = [r(\cos\theta + i\sin\theta)][r(\cos(-\theta) + i\sin(-\theta)]$

$= r^2[\cos(\theta - \theta) + i\sin(\theta - \theta)]$

$= r^2[\cos 0 + i\sin 0]$

$= r^2$

(b) $\dfrac{z}{\bar{z}} = \dfrac{r(\cos\theta + i\sin\theta)}{r[\cos(-\theta) + i\sin(-\theta)]}$

$= \dfrac{r}{r}[\cos(\theta - (-\theta)) + i\sin(\theta - (-\theta))]$

$= \cos 2\theta + i\sin 2\theta$

161. $z = r(\cos\theta + i\sin\theta)$

$-z = -r(\cos\theta + i\sin\theta)$

$= r(-\cos\theta - i\sin\theta)$

$= r(\cos(\theta + \pi) + i\sin(\theta + \pi))$

162. Let $a = 0$ and $b = \pi$ in Euler's Formula:

$e^{a+bi} = e^a(\cos b + i\sin b)$

$e^{0+\pi i} = e^0(\cos\pi + i\sin\pi)$

$e^{\pi i} = -1$

$e^{\pi i} + 1 = 0$

163. $d = 16\cos\left(\dfrac{\pi}{4}t\right)$

Maximum displacement: 16

Lowest possible t-value: $\dfrac{\pi}{4}t = \dfrac{\pi}{2} \Rightarrow t = 2$

164. Maximum displacement: $\dfrac{1}{16}$

Lowest positive value: $t = \dfrac{4}{5}$

165. $d = \dfrac{1}{8}\cos(12\pi t)$

Maximum displacement: $\dfrac{1}{8}$

Lowest possible t-value: $12\pi t = \dfrac{\pi}{2} \Rightarrow t = \dfrac{1}{24}$

166. Maximum displacement: $\dfrac{1}{12}$

Lowest positive value: $t = \dfrac{1}{60}$

167. $$\begin{aligned} 2\cos(x+\pi) + 2\cos(x-\pi) &= 0 \\ 4\cos x\cos\pi &= 0 \\ \cos x &= 0 \\ x &= \frac{\pi}{2}, \frac{3\pi}{2} \end{aligned}$$

168. $$\begin{aligned} \sin\left(x+\frac{3\pi}{2}\right) - \sin\left(x-\frac{3\pi}{2}\right) &= 0 \\ -\cos x - \cos x &= 0 \\ 2\cos x &= 0 \\ x &= \frac{\pi}{2}, \frac{3\pi}{2} \end{aligned}$$

169. $$\begin{aligned} \sin\left(x-\frac{\pi}{3}\right) - \sin\left(x+\frac{\pi}{3}\right) &= \frac{3}{2} \\ \frac{1}{2}\left[\sin\left(x+\frac{\pi}{3}\right) - \sin\left(x-\frac{\pi}{3}\right)\right] &= \left(-\frac{3}{2}\right)\frac{1}{2} \\ \cos x\sin\frac{\pi}{3} &= -\frac{3}{4} \\ \cos x\left(\frac{\sqrt{3}}{2}\right) &= -\frac{3}{4} \\ \cos x &= -\frac{\sqrt{3}}{2} \\ x &= \frac{5\pi}{6}, \frac{7\pi}{6} \end{aligned}$$

170. $$\begin{aligned} \tan(x+\pi) - \cos\left(x+\frac{5\pi}{2}\right) &= 0 \\ \tan x + \sin x &= 0 \\ \sin x\left(\frac{1}{\cos x} + 1\right) &= 0 \\ x &= 0, \pi \end{aligned}$$

Review Exercises for Chapter 6

1. Given: $A = 32°, B = 50°, a = 16$

$C = 180° - 32° - 50° = 98°$

$$b = \frac{a\sin B}{\sin A} = \frac{16\sin 50°}{\sin 32°} \approx 23.13$$

$$c = \frac{a\sin C}{\sin A} = \frac{16\sin 98°}{\sin 32°} \approx 29.90$$

2. Given: $A = 38°, B = 58°, a = 12$

$C = 180° - 38° - 58° = 84°$

$$b = \frac{a\sin B}{\sin A} = \frac{12\sin 58°}{\sin 38°} \approx 16.53$$

$$c = \frac{a\sin C}{\sin A} = \frac{12\sin 84°}{\sin 38°} \approx 19.38$$

3. Given: $B = 25°, C = 105°, c = 25$

$A = 180° - 25° - 105° = 50°$

$$b = \frac{c\sin B}{\sin C} = \frac{25\sin 25°}{\sin 105°} \approx 10.94$$

$$a = \frac{c\sin A}{\sin C} = \frac{25\sin 50°}{\sin 105°} \approx 19.83$$

4. Given: $B = 20°, C = 115°, c = 30$

$A = 180° - 20° - 115° = 45°$

$$b = \frac{c\sin B}{\sin C} = \frac{30\sin 20°}{\sin 115°} \approx 11.32$$

$$a = \frac{c\sin A}{\sin C} = \frac{30\sin 45°}{\sin 115°} \approx 23.41$$

5. Given: $A = 60°\,15' = 60.25°,\ B = 45°\,30' = 45.5°,\ b = 4.8$

$C = 180° - 60.25° - 45.5° = 74.25° = 74°\,15'$

$$a = \frac{b\sin A}{\sin B} = \frac{4.8\sin 60.25°}{\sin 45.5°} \approx 5.84$$

$$c = \frac{b\sin C}{\sin B} = \frac{4.8\sin 74.25°}{\sin 45.5°} \approx 6.48$$

6. Given: $A = 82° 45' = 82.75°$, $B = 28° 45' = 28.75°$, $b = 40.2$

$C = 180° - 82.75° - 28.75° = 68.5° = 68° 30'$

$$a = \frac{b \sin A}{\sin B} = \frac{40.2 \sin 82.75°}{\sin 28.75°} \approx 82.91$$

$$c = \frac{b \sin C}{\sin B} = \frac{40.2 \sin 68.5°}{\sin 28.75°} \approx 77.76$$

7. Given: $A = 75°$, $a = 2.5$, $b = 16.5$

$$\sin B = \frac{b \sin A}{a} = \frac{16.5 \sin 75°}{2.5} \approx 6.375 \implies \text{no triangle formed}$$

No solution

8. Given: $A = 15°$, $a = 5$, $b = 10$

$$\sin B = \frac{b \sin A}{a} = \frac{10 \sin 15°}{5} \approx 0.5176 \implies B \approx 31.2° \text{ or } 148.8°$$

Case 1: $B \approx 31.2°$

$C \approx 180° - 15° - 31.2° = 133.8°$

$$c = \frac{a \sin C}{\sin A} \approx \frac{5 \sin 133.8°}{\sin 15°} \approx 13.94$$

Case 2: $B \approx 148.8°$

$C \approx 180° - 15° - 148.8° = 16.2°$

$$c = \frac{a \sin C}{\sin A} \approx \frac{5 \sin 16.2°}{\sin 15°} \approx 5.4$$

9. Given: $B = 115°, a = 9, b = 14.5$

$$\sin A = \frac{a \sin B}{b} = \frac{9 \sin 115°}{14.5} \approx 0.5625 \implies A \approx 34.2°$$

$C \approx 180° - 115° - 34.2° = 30.8°$

$$c = \frac{b}{\sin B}(\sin C) \approx \frac{14.5}{\sin 115°}(\sin 30.8°) \approx 8.18$$

10. Given: $B = 150°, a = 64, b = 10$

$$\sin A = \frac{a \sin B}{b} = \frac{64 \sin 150°}{10} \approx 3.2 \implies \text{no triangle formed}$$

No solution

11. Given: $C = 50°, a = 25, c = 22$

$$\sin A = \frac{a \sin C}{c} = \frac{25 \sin 50°}{22} \approx \frac{25(0.7660)}{22} \approx 0.8705 \implies A \approx 60.5° \text{ or } 119.5°$$

Case 1:

$A \approx 60.5°$

$B \approx 180° - 50° - 60.5° = 69.5°$

$$b = \frac{c \sin B}{\sin C} \approx \frac{22(0.9367)}{0.7660} \approx 26.90$$

Case 2:

$A \approx 119.5°$

$B \approx 180° - 50° - 119.5° = 10.5°$

$$b = \frac{c \sin B}{\sin C} \approx 5.24$$

12. Given: $B = 25°,\ a = 6.2,\ b = 4$

$$\sin A = \frac{a \sin B}{b} \approx \frac{6.2 \sin 25°}{4} \approx 0.6551 \implies A \approx 40.9° \text{ or } 139.1°$$

Case 1: $A \approx 40.9°$

$C \approx 180° - 25° - 40.9° = 114.1°$

$$c = \frac{b \sin C}{\sin B} \approx \frac{4 \sin 114.1°}{\sin 25°} \approx 8.64$$

Case 2: $A \approx 139.1°$

$C \approx 180° - 25° - 139.1° = 15.9°$

$$c = \frac{b \sin C}{\sin B} \approx \frac{4 \sin 15.9°}{\sin 25°} \approx 2.60$$

13. $A = 27°,\ b = 5,\ c = 8$

$$\text{Area} = \tfrac{1}{2}bc \sin A = \tfrac{1}{2}(5)(8)(\sin 27°) \approx 9.08 \text{ square units}$$

14. $B = 80°, a = 4, c = 8$

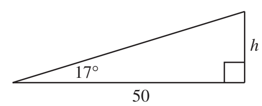
$$\text{Area} = \tfrac{1}{2}ac \sin B = \tfrac{1}{2}(4)(8)(0.9848) = 15.76 \text{ square units}$$

15. $C = 122°, b = 18, a = 29$

$$\text{Area} = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}(29)(18) \sin 122° \approx 221.34 \text{ square units}$$

16. $$\text{Area} = \tfrac{1}{2}ab \sin C = \tfrac{1}{2}(120)(74) \sin 100° \approx 4372.5 \text{ square units}$$

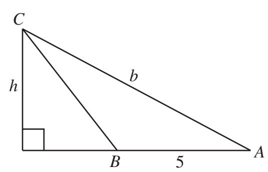
17. $h = 50 \tan 17° \approx 15.3$ meters



18. In triangle ABC, $A = 90° - 62° = 28°$, $B = 90° + 38° = 128°$, and $C = 180° - A - B = 24°$.

$$b = \frac{c \sin B}{\sin C} = \frac{5 \sin 128°}{\sin 24°} \approx 9.6870$$

$h = b \sin A = b \sin 28° \approx 4.548 \approx 4.5$ miles



19. $\sin 28° = \dfrac{h}{75}$

$h = 75 \sin 28° \approx 35.21$ feet

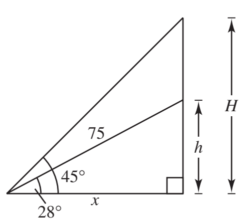
$\cos 28° = \dfrac{x}{75}$

$x = 75 \cos 28° \approx 66.22$ feet

$\tan 45° = \dfrac{H}{x}$

$H = x \tan 45° \approx 66.22$ feet

Height of tree: $H - h \approx 31$ feet

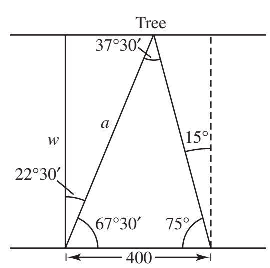


20. $\dfrac{a}{\sin 75°} = \dfrac{400}{\sin 37.5°}$

$$a = \frac{400 \sin 75°}{\sin 37.5°} \approx 634.7 \text{ feet}$$

$\sin 67.5° = \dfrac{w}{a}$

$w = 634.7 \sin 67.5° \approx 586.4$ feet



21. Given: $a = 18, b = 12, c = 15$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{12^2 + 15^2 - 18^2}{2(12)(15)} = 0.125 \implies A \approx 82.82°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{18^2 + 15^2 - 12^2}{2(18)(15)} = 0.75 \implies B \approx 41.41°$$

$$C = 180° - A - B \approx 55.77°$$

22. Given: $a = 10, b = 12, c = 16$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{10^2 + 12^2 - 16^2}{2(10)(12)} = -0.05 \implies C \approx 92.87°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{10^2 + 16^2 - 12^2}{2(10)(16)} = 0.6625 \implies B \approx 48.51°$$

$$A = 180° - B - C \approx 38.62°$$

23. Given: $a = 9, b = 12, c = 20$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{81 + 144 - 400}{2(9)(12)} \approx -0.8102 \implies C \approx 144.1°$$

$$\sin A = \frac{a \sin C}{c} = \frac{9 \sin(144.1°)}{20} \approx 0.264 \implies A \approx 15.3$$

$$B = 180° - 144.1° - 15.3° = 20.6°$$

24. $a = 7, b = 15, c = 19$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx -0.4143 \implies C \approx 114.5°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx 0.6955 \implies B \approx 45.9°$$

$$A = 180° - B - C = 19.6°$$

25. Given: $a = 6.5, b = 10.2, c = 16$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{10.2^2 + 16^2 - 6.5^2}{2(10.2)(16)} \approx 0.97 \implies A \approx 13.19°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6.5^2 + 16^2 - 10.2^2}{2(6.5)(16)} \approx 0.93 \implies B \approx 20.98°$$

$$C = 180° - A - B \approx 145.83°$$

26. Given: $a = 6.2, b = 6.4, c = 2.1$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6.4^2 + 2.1^2 - 6.2^2}{2(6.4)(2.1)} \approx 0.26 \implies A \approx 75.06°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6.2^2 + 2.1^2 - 6.4^2}{2(6.2)(2.1)} \approx 0.07 \implies B \approx 85.84°$$

$$C = 180° - A - B \approx 19.10°$$

27. Given: $C = 65°, a = 25, b = 12$

$c^2 = a^2 + b^2 - 2ab \cos C = 25^2 + 12^2 - 2(25)(12) \cos 65° \approx 515.4290 \implies c \approx 22.70$

$\sin A = \dfrac{a \sin C}{c} = \dfrac{25 \sin 65°}{22.70} \approx 0.998 \implies A \approx 86.38°$

$B = 180° - A - C \approx 28.62°$

28. Given: $B = 48°, a = 18, c = 12$

$b^2 = a^2 + c^2 - 2ac \cos B = 18^2 + 12^2 - 2(18)(12) \cos 48° \approx 178.9356 \implies b \approx 13.38$

$\sin C = \dfrac{c \sin B}{b} = \dfrac{12 \sin 48°}{13.38} \approx 0.666663 \implies C \approx 41.81°$

$A = 180° - B - C \approx 90.19°$

(Answers may vary.)

29. Given: $B = 110°, a = 4, c = 4$

$b^2 = a^2 + c^2 - 2ac \cos B = 16 + 16 - 2(4)(4)(\cos 110) \approx 42.94 \implies b \approx 6.55$

$\sin A = \dfrac{a \sin B}{b} \approx \dfrac{4 \sin 110°}{6.55} \approx 0.5739 \implies A \approx 35°$

$c = a \implies C = A \approx 35°$

30. Given: $B = 150°,\ a = 10,\ c = 20$

$b^2 = a^2 + c^2 - 2ac \cos B \approx 100 + 400 - 400(-0.8660) \approx 846.4 \implies b \approx 29.09$

$\sin C = \dfrac{c \sin B}{b} \approx \dfrac{20(0.5)}{29.09} \approx 0.3437 \implies C \approx 20.1°$

$\sin A = \dfrac{a \sin B}{b} \approx \dfrac{10(0.5)}{29.09} \approx 0.1719 \implies A \approx 9.9°$

31. Given: $B = 55° 30' = 55.5°,\ a = 12.4,\ c = 18.5$

$b^2 = a^2 + c^2 - 2ac \cos B = 12.4^2 + 18.5^2 - 2(12.4)(18.5) \cos 55.5° \approx 236.1428 \implies b \approx 15.37$

$\sin A = \dfrac{a \sin B}{b} \approx \dfrac{12.4 \sin 55.5°}{15.37} \approx 0.665 \implies A \approx 41.68° \approx 41° 41'$

$C = 180° - A - B = 82.82° \approx 82° 49'$

32. Given: $B = 85° 15' = 85.25°,\ a = 24.2,\ c = 28.2$

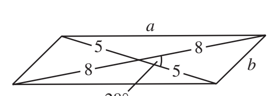
$b^2 = a^2 + c^2 - 2ac \cos B = 24.2^2 + 28.2^2 - 2(24.2)(28.2) \cos 85.25° \approx 1267.8567 \implies b \approx 35.61$

$\sin A = \dfrac{a \sin B}{b} \approx \dfrac{24.2 \sin 85.25°}{35.61} \approx 0.677 \implies A \approx 42.63° \approx 42° 38'$

$C = 180° - A - B = 52.12° \approx 52° 7'$

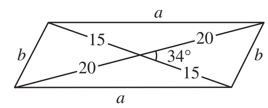
33. $a^2 = 5^2 + 8^2 - 2(5)(8) \cos 152° \approx 159.6 \implies a \approx 12.63$ ft

$b^2 = 5^2 + 8^2 - 2(5)(8) \cos 28° \approx 18.36 \implies b \approx 4.285$ ft



34. $a^2 = 15^2 + 20^2 - 2(15)(20)\cos(146°) \approx 1122.42 \implies a \approx 33.5$ m

$b^2 = 15^2 + 20^2 - 2(15)(20)\cos(34°) \approx 127.58 \implies b \approx 11.3$ m



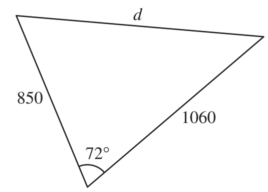
35. Angle between planes is $5° + 67° = 72°$.

In two hours, distances from airport are 850 miles and 1060 miles.

By the Law of Cosines,

$d^2 = 850^2 + 1060^2 - 2(850)(1060)\cos(72°) \approx 1{,}289{,}251.376$

$d \approx 1135.5$ miles.



36. $b^2 = a^2 + c^2 - 2ac\cos B$

$= 300^2 + 425^2 - 2(300)(425)\cos(180° - 65°)$

$\approx 378{,}392.66$

$b \approx 615.1$ meters

37. $a = 4,\ b = 5,\ c = 7$

$s = \dfrac{a+b+c}{2} = \dfrac{4+5+7}{2} = 8$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(4)(3)(1)}$

≈ 9.798 square units

38. $a = 15, b = 8, c = 10$

$s = \dfrac{15+8+10}{2} = 16.5$

Area $= \sqrt{16.5(1.5)(8.5)(6.5)}$

≈ 36.98 square units

39. $a = 64.8, b = 49.2, c = 24.1$

$s = \dfrac{a+b+c}{2} = \dfrac{64.8+49.2+24.1}{2} = 69.05$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{69.05(4.25)(19.85)(44.95)}$

≈ 511.7 square units

40. $s = \dfrac{8.55+5.14+12.73}{2} = 13.21$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{13.21(4.66)(8.07)(0.48)}$

≈ 15.4 square units

41. Initial point: $(-5, 4)$

Terminal point: $(2, -1)$

$\mathbf{v} = \langle 2-(-5), -1-4\rangle = \langle 7, -5\rangle$

42. Initial point: $(0, 1)$

Terminal point: $\left(6, \frac{7}{2}\right)$

$\mathbf{v} = \left\langle 6-0, \frac{7}{2}-1\right\rangle = \left\langle 6, \frac{5}{2}\right\rangle$

43. Initial point: $(0, 10)$

Terminal point: $(7, 3)$

$\mathbf{v} = \langle 7-0, 3-10\rangle = \langle 7, -7\rangle$

44. Initial point: $(1, 5)$

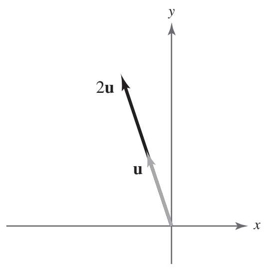
Terminal point: $(15, 9)$

$\mathbf{v} = \langle 15-1, 9-5\rangle = \langle 14, 4\rangle$

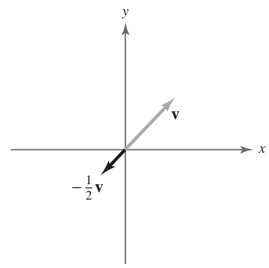
45. $8\cos 120°\mathbf{i} + 8\sin 120°\mathbf{j} = \left\langle -4, 4\sqrt{3}\right\rangle$

46. $\left\langle \frac{1}{2}\cos 225°, \frac{1}{2}\sin 225°\right\rangle = \left\langle -\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4}\right\rangle$

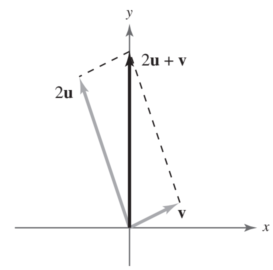
47. $2\mathbf{u}$



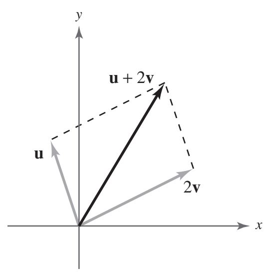
48. $-\frac{1}{2}\mathbf{v}$



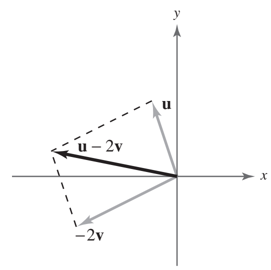
49. $2\mathbf{u} + \mathbf{v}$



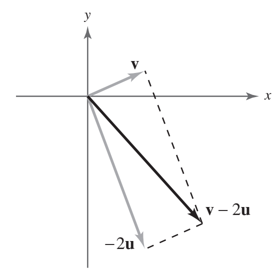
50. $\mathbf{u} + 2\mathbf{v}$



51. $\mathbf{u} - 2\mathbf{v}$



52. $\mathbf{v} - 2\mathbf{u}$



53. (a) $\mathbf{u} + \mathbf{v} = \langle -1, -3\rangle + \langle -3, 6\rangle = \langle -4, 3\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 2, -9\rangle$

(c) $3\mathbf{u} = \langle -3, -9\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle -6, 12\rangle + \langle -5, -15\rangle = \langle -11, -3\rangle$

54. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 5\rangle + \langle 0, -1\rangle = \langle 4, 4\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 4, 6\rangle$

(c) $3\mathbf{u} = \langle 12, 15\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 0, -2\rangle + \langle 20, 25\rangle = \langle 20, 23\rangle$

55. (a) $\mathbf{u} + \mathbf{v} = \langle -5, 2\rangle + \langle 4, 4\rangle = \langle -1, 6\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -9, -2\rangle$

(c) $3\mathbf{u} = \langle -15, 6\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 8, 8\rangle + \langle -25, 10\rangle = \langle -17, 18\rangle$

56. (a) $\mathbf{u} + \mathbf{v} = \langle 1, -8\rangle + \langle 3, -2\rangle = \langle 4, -10\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -2, -6\rangle$

(c) $3\mathbf{u} = \langle 3, -24\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 6, -4\rangle + \langle 5, -40\rangle = \langle 11, -44\rangle$

57. (a) $\mathbf{u} + \mathbf{v} = \langle 2, -1\rangle + \langle 5, 3\rangle = \langle 7, 2\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -3, -4\rangle$

(c) $3\mathbf{u} = \langle 6, -3\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 10, 6\rangle + \langle 10, -5\rangle = \langle 20, 1\rangle$

58. (a) $\mathbf{u} + \mathbf{v} = \langle 0, -6\rangle + \langle 1, 1\rangle = \langle 1, -5\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -1, -7\rangle$

(c) $3\mathbf{u} = \langle 0, -18\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle 2, 2\rangle + \langle 0, -30\rangle = \langle 2, -28\rangle$

59. (a) $\mathbf{u} + \mathbf{v} = \langle 4, 0\rangle + \langle -1, 6\rangle = \langle 3, 6\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle 5, -6\rangle$

(c) $3\mathbf{u} = \langle 12, 0\rangle$

(d) $2\mathbf{v} + 5\mathbf{u} = \langle -2, 12\rangle + \langle 20, 0\rangle = \langle 18, 12\rangle$

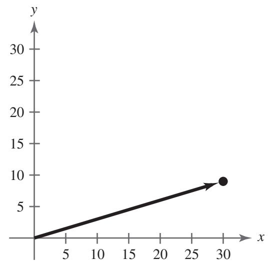
60. (a) $\mathbf{u} + \mathbf{v} = \langle -7, -3\rangle + \langle 4, -1\rangle = \langle -3, -4\rangle$

(b) $\mathbf{u} - \mathbf{v} = \langle -11, -2\rangle$

(c) $3\mathbf{u} = \langle -21, -9\rangle$

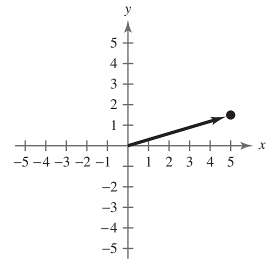
(d) $2\mathbf{v} + 5\mathbf{u} = \langle 8, -2\rangle + \langle -35, -15\rangle = \langle -27, -17\rangle$

61. $3\mathbf{v} = 3(10\mathbf{i} + 3\mathbf{j}) = 30\mathbf{i} + 9\mathbf{j} = \langle 30, 9\rangle$



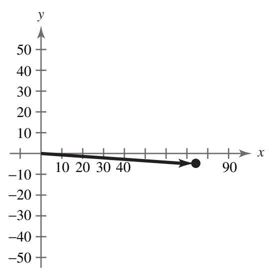
62. $\mathbf{v} = 10\mathbf{i} + 3\mathbf{j}$

$\frac{1}{2}\mathbf{v} = 5\mathbf{i} + \frac{3}{2}\mathbf{j}$

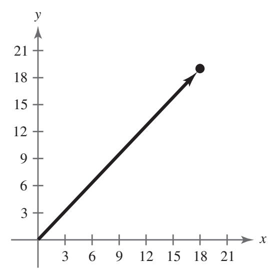


63. $\mathbf{w} = 4\mathbf{u} + 5\mathbf{v} = 4(6\mathbf{i} - 5\mathbf{j}) + 5(10\mathbf{i} + 3\mathbf{j})$

$= 74\mathbf{i} - 5\mathbf{j}$



64. $3\mathbf{v} - 2\mathbf{u} = 3(10\mathbf{i} + 3\mathbf{j}) - 2(6\mathbf{i} - 5\mathbf{j}) = 18\mathbf{i} + 19\mathbf{j}$



65. $\|\mathbf{u}\| = 6$

Unit vector: $\frac{1}{6}\langle 0, -6\rangle = \langle 0, -1\rangle$

66. $\|\mathbf{v}\| = \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$

Unit vector: $\left\langle -\frac{12}{13}, -\frac{5}{13}\right\rangle$

67. $\|\mathbf{v}\| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$

Unit vector: $\dfrac{1}{\sqrt{29}}\langle 5, -2\rangle = \left\langle \dfrac{5}{\sqrt{29}}, \dfrac{-2}{\sqrt{29}}\right\rangle$

68. $\|\mathbf{w}\| = 7$

Unit vector: $\dfrac{1}{7}(-7\mathbf{i}) = -\mathbf{i}$

69. $\mathbf{u} = \langle 1 - (-8), -5 - 3\rangle = \langle 9, -8\rangle = 9\mathbf{i} - 8\mathbf{j}$

70. $\mathbf{u} = \langle -6.4 - 2, 10.8 - (-3.2)\rangle = -8.4\mathbf{i} + 14\mathbf{j}$

71. $\mathbf{v} = -10\mathbf{i} + 10\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{(-10)^2 + (10)^2} = \sqrt{200} = 10\sqrt{2}$

$\tan\theta = \dfrac{10}{-10} = -1 \implies \theta = 135°$ since $\mathbf{v}$ is in Quadrant II.

$\mathbf{v} = 10\sqrt{2}(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

72. $\mathbf{v} = 4\mathbf{i} - \mathbf{j}$

$\|\mathbf{v}\| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$

$\tan\theta = \dfrac{-1}{4} = -\dfrac{1}{4} \implies \theta \approx 346°$, since θ is in Quadrant IV.

$\mathbf{v} = \sqrt{17}(\cos 346°\mathrm{i} + \sin 346°\,\mathbf{j})$

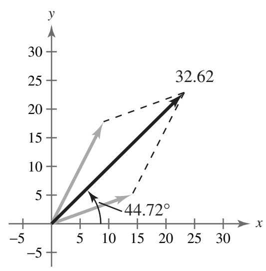
73. $\mathbf{u} = 15[(\cos 20°)\mathbf{i} + (\sin 20°)\mathbf{j}]$

$\mathbf{v} = 20[(\cos 63°)\mathbf{i} + (\sin 63°)\mathbf{j}]$

$\mathbf{u} + \mathbf{v} \approx 23.1752\mathbf{i} + 22.9504\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx 32.62$

$\tan\theta = \dfrac{22.9504}{23.1752} \implies \theta \approx 44.72°$



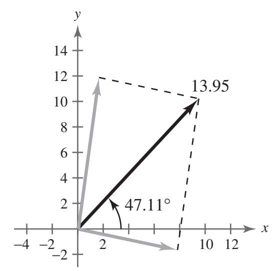
74. $\mathbf{u} = 12[(\cos 82°)\mathbf{i} + (\sin 82°)\mathbf{j}]$

$\mathbf{v} = 8[(\cos(-12°))\mathbf{i} + (\sin(-12°))\mathbf{j}]$

$\mathbf{u} + \mathbf{v} \approx 9.4953\mathbf{i} + 10.2199\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| \approx 13.95$

$\tan\theta = \dfrac{10.2199}{9.4953} \implies \theta \approx 47.11°$



75. $\mathbf{F}_1 = 250\langle \cos 60°, \sin 60° \rangle$

$\mathbf{F}_2 = 100\langle \cos 150°, \sin 150° \rangle$

$\mathbf{F}_3 = 200\langle \cos(-90°), \sin(-90°) \rangle$

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \approx \langle 38.39746, 66.50635 \rangle$

$\tan\theta \approx \dfrac{66.50635}{38.39746} \implies \theta \approx 60°$

$\|\mathbf{F}\| = \sqrt{38.39746^2 + 66.50635^2} \approx 76.8$ pounds

76. Force One: $\mathbf{u} = 85\mathbf{i}$

Force Two: $\mathbf{v} = 50\cos 15°\mathbf{i} + 50\sin 15°\mathbf{j}$

Resultant Force:

$\mathbf{u} + \mathbf{v} = (85 + 50\cos 15°)\mathbf{i} + (50\sin 15°)\mathbf{j}$

$\|\mathbf{u} + \mathbf{v}\| = \sqrt{(85 + 50\cos 15°)^2 + (50\sin 15°)^2}$

$= \sqrt{85^2 + 8500\cos 15° + 50^2}$

$= 133.92$ lb

$\tan\theta = \dfrac{50\sin 15°}{85 + 50\cos 15°} \implies \theta \approx 5.5°$ from the 85-pound force.

77. Rope One: $\mathbf{u} = \|\mathbf{u}\|(\cos 30°\mathbf{i} - \sin 30°\mathbf{j}) = \|\mathbf{u}\|\left(\dfrac{\sqrt{3}}{2}\mathbf{i} - \dfrac{1}{2}\mathbf{j}\right)$

Rope Two: $\mathbf{v} = \|\mathbf{u}\|(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j}) = \|\mathbf{u}\|\left(-\dfrac{\sqrt{3}}{2}\mathbf{i} - \dfrac{1}{2}\mathbf{j}\right)$

Resultant: $\mathbf{u} + \mathbf{v} = -\|\mathbf{u}\|\mathbf{j} = -180\mathbf{j}$

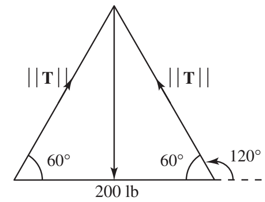
$\|\mathbf{u}\| = 180$

Therefore, the tension on each rope is $\|\mathbf{u}\| = 180$ pounds.

78. By symmetry, the magnitudes of the tensions are equal.

$\mathbf{T} = \|\mathbf{T}\|(\cos 120°\mathbf{i} + \sin 120°\mathbf{j})$

$\|\mathbf{T}\| \sin 120° = \frac{1}{2}(200) \implies \|\mathbf{T}\| = \frac{100}{\sqrt{3}/2} = \frac{200}{\sqrt{3}} \approx 115.5 \text{ lbs}$



79. Airplane velocity: $\mathbf{u} = 430\langle \cos 315°, \sin 315° \rangle$

Wind velocity: $\mathbf{w} = 35\langle \cos 60°, \sin 60° \rangle$

$\mathbf{u} + \mathbf{w} \approx \langle 321.5559, -273.7450 \rangle$

$\|\mathbf{u} + \mathbf{w}\| \approx 422.3 \text{ mph}$

$\tan(\mathbf{u} + \mathbf{w}) = \frac{-273.7450}{321.5559}$

$\approx -0.8513 \implies \theta \approx -40.4°$

The bearing from the north is $90° + 40.4° = 130.4°$.

80. Airspeed: $\mathbf{u} = 724(\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$

$= 362(\mathbf{i} + \sqrt{3}\mathbf{j})$

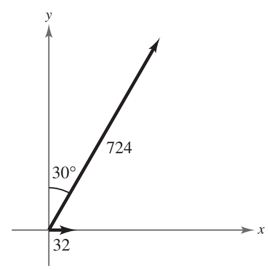
Wind: $\mathbf{w} = 32\mathbf{i}$

Groundspeed $= \mathbf{u} + \mathbf{w} = (394\mathbf{i} + 362\sqrt{3}\mathbf{j})$

$\|\mathbf{u} + \mathbf{w}\| = \sqrt{(394)^2 + (362\sqrt{3})^2} \approx 740.5 \text{ km/hr}$

$\tan \theta = \frac{362\sqrt{3}}{394} \implies \theta \approx 57.9°$

Bearing: N 32.1° E (or 32.1° in airplane navigation)



81. $\mathbf{u} \cdot \mathbf{v} = \langle 0, -2 \rangle \cdot \langle 1, 10 \rangle = 0 - 20 = -20$

82. $\mathbf{u} \cdot \mathbf{v} = \langle -4, 5 \rangle \cdot \langle 3, -1 \rangle = -12 - 5 = -17$

83. $\mathbf{u} \cdot \mathbf{v} = \langle 6, -1 \rangle \cdot \langle 2, 5 \rangle = 6(2) + (-1)(5) = 7$

84. $\mathbf{u} \cdot \mathbf{v} = (8\mathbf{i} - 7\mathbf{j}) \cdot (3\mathbf{i} - 4\mathbf{j}) = 24 + 28 = 52$

85. $\mathbf{u} \cdot \mathbf{u} = \langle -3, -4 \rangle \cdot \langle -3, -4 \rangle$

$= 9 + 16 = 25 = \|\mathbf{u}\|^2$

86. $\|\mathbf{v}\| - 3 = \sqrt{(2)^2 + (1)^2} - 3 = \sqrt{5} - 3$

87. $4\mathbf{u} \cdot \mathbf{v} = 4\langle -3, -4 \rangle \cdot \langle 2, 1 \rangle = 4(-6 - 4) = -40$

88. $(\mathbf{u} \cdot \mathbf{v})\mathbf{u} = (-10)\langle -3, -4 \rangle = \langle 30, 40 \rangle$

89. $\mathbf{u} = \langle 2\sqrt{2}, -4 \rangle, \mathbf{v} = \langle -\sqrt{2}, 1 \rangle$

$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{-8}{(\sqrt{24})(\sqrt{3})} \implies \theta \approx 160.5°$

90. $\mathbf{u} = \langle 3, 1 \rangle, \mathbf{v} = \langle 4, 5 \rangle$

$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{17}{(\sqrt{10})(\sqrt{41})} \implies \theta \approx 32.9°$

91. $\mathbf{u} = \cos \frac{7\pi}{4}\mathbf{i} + \sin \frac{7\pi}{4}\mathbf{j} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

$\mathbf{v} = \cos \frac{5\pi}{6}\mathbf{i} + \sin \frac{5\pi}{6}\mathbf{j} = \left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

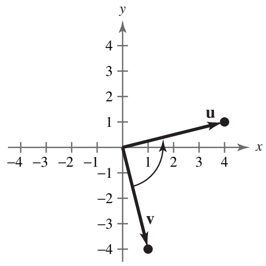
$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} = \frac{(-\sqrt{3}/(2\sqrt{2})) - (1/(2\sqrt{2}))}{(1)(1)}$

$\approx -0.966 \implies \theta \approx 165° \text{ or } \frac{11\pi}{12}$

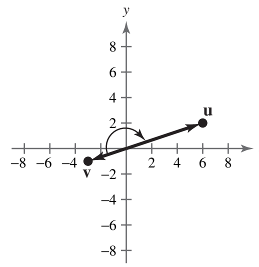
92. Angle $= 45° + 60° = 105°$

93. $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$

$= 0 \implies \theta = 90°$



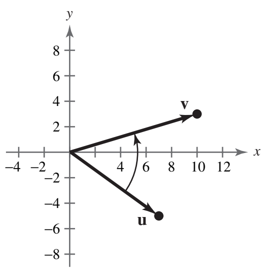
94. 180°



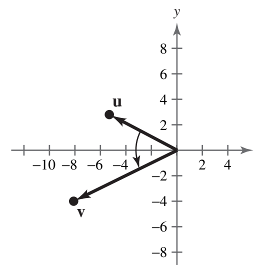
95. $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{70 - 15}{\sqrt{74}\sqrt{109}}$

≈ 0.612

$\implies \theta \approx 52.2°$



96. 54.1°



97. $\mathbf{u} = \langle 39, -12\rangle, \mathbf{v} = \langle -26, 8\rangle$

$\mathbf{u}\cdot\mathbf{v} = 39(-26) + (-12)(8)$

$= -1110 \neq 0 \implies$ **u** and **v** are not orthogonal.

$\mathbf{v} = -\frac{2}{3}\mathbf{u} \implies$ **u** and **v** are parallel.

98. $\mathbf{u}\cdot\mathbf{v} = \langle 8, -4\rangle \cdot \langle 5, 10\rangle = 40 - 40 = 0 \implies$ orthogonal

99. $\mathbf{u} = \langle 8, 5\rangle, \mathbf{v} = \langle -2, 4\rangle$

$\mathbf{u}\cdot\mathbf{v} = 8(-2) + (5)(4)$

$= 4 \neq 0 \implies$ **u** and **v** are not orthogonal.

$\mathbf{u} \neq k\mathbf{v} \implies$ **u** and **v** are not parallel.

Neither

100. $-\frac{3}{4}\mathbf{v} = -\frac{3}{4}\langle 20, -68\rangle = \langle -15, 51\rangle = \mathbf{u} \implies$ parallel

101. $\langle 1, -k\rangle \cdot \langle 1, 2\rangle = 1 - 2k = 0 \implies k = \frac{1}{2}$

102. $\langle 2, 1\rangle \cdot \langle -1, -k\rangle = -2 - k = 0 \implies k = -2$

103. $\langle k, -1\rangle \cdot \langle 2, -2\rangle = 2k + 2 = 0 \implies k = -1$

104. $\langle k, -2\rangle \cdot \langle 1, 4\rangle = k - 8 = 0 \implies k = 8$

105. $\mathbf{u} = \langle -4, 3\rangle,\ \mathbf{v} = \langle -8, -2\rangle$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \left(\frac{26}{68}\right)\langle -8, -2\rangle = -\frac{13}{17}\langle 4, 1\rangle$$

$$\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u} = \langle -4, 3\rangle - \left\langle \frac{-52}{17}, \frac{-13}{17}\right\rangle$$

$$= \left\langle -\frac{16}{17}, \frac{64}{17}\right\rangle$$

$$\mathbf{u} = \left\langle -\frac{52}{17}, -\frac{13}{17}\right\rangle + \left\langle -\frac{16}{17}, \frac{64}{17}\right\rangle$$

106. $\mathbf{u} = \langle 5, 6\rangle, \mathbf{v} = \langle 10, 0\rangle$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{50}{100}\langle 10, 0\rangle = \langle 5, 0\rangle$$

$$\mathbf{u} = \langle 5, 0\rangle + \langle 0, 6\rangle$$

107. $\mathbf{u} = \langle 2, 7\rangle, \mathbf{v} = \langle 1, -1\rangle$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{-5}{2}\langle 1, -1\rangle = \left\langle -\frac{5}{2}, \frac{5}{2}\right\rangle$$

$$\mathbf{u} - \text{proj}_{\mathbf{v}}\mathbf{u} = \langle 2, 7\rangle - \left\langle -\frac{5}{2}, \frac{5}{2}\right\rangle = \left\langle \frac{9}{2}, \frac{9}{2}\right\rangle$$

$$\mathbf{u} = \left\langle -\frac{5}{2}, \frac{5}{2}\right\rangle + \left\langle \frac{9}{2}, \frac{9}{2}\right\rangle$$

108. $\mathbf{u} = \langle -3, 5\rangle, \mathbf{v} = \langle -5, 2\rangle$

$$\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \frac{25}{29}\langle -5, 2\rangle$$

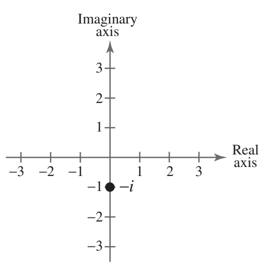
$$\mathbf{u} = \left\langle -\frac{125}{29}, \frac{50}{29}\right\rangle + \left\langle \frac{38}{29}, \frac{95}{29}\right\rangle$$

109. 48 inches = 4 feet

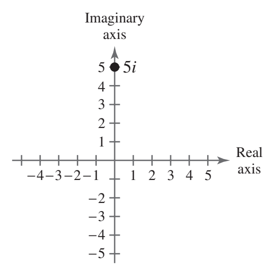
Work = 18,000(4) = 72,000 ft · lb

110. Force = 500 sin 12° ≈ 104 lbs

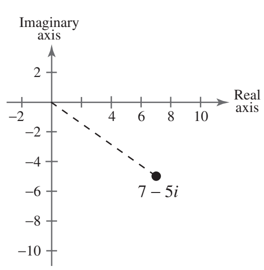
111. $|-i| = 1$



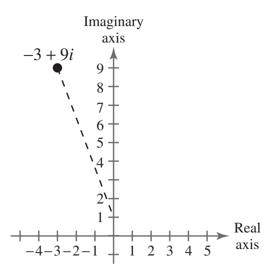
112. $|5i| = 5$



113. $|7 - 5i| = \sqrt{7^2 + (-5)^2} = \sqrt{74}$



114. $|-3 + 9i| = \sqrt{9 + 81} = \sqrt{90} = 3\sqrt{10}$



115. $z = 2 - 2i$

$$|z| = \sqrt{4 + 4} = 2\sqrt{2}$$

$$\theta = \frac{7\pi}{4}$$

$$z = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

116. $z = -2 + 2i$

$$|z| = \sqrt{4 + 4} = 2\sqrt{2}$$

$$\theta = \frac{3\pi}{4}$$

$$z = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

117. $z = -\sqrt{3} - i$

$|z| = \sqrt{3 + 1} = 2$

$\theta = \dfrac{7\pi}{6}$

$z = 2\left(\cos \dfrac{7\pi}{6} + i \sin \dfrac{7\pi}{6}\right)$

118. $z = -\sqrt{3} + i$

$|z| = \sqrt{3 + 1} = 2$

$\theta = \dfrac{5\pi}{6}$

$z = 2\left(\cos \dfrac{5\pi}{6} + i \sin \dfrac{5\pi}{6}\right)$

119. $z = -2i$

$|z| = 2$

$\theta = \dfrac{3\pi}{2}$

$z = 2\left(\cos \dfrac{3\pi}{2} + i \sin \dfrac{3\pi}{2}\right)$

120. $z = 4i$

$|z| = 4$

$\theta = \dfrac{\pi}{2}$

$z = 4\left(\cos \dfrac{\pi}{2} + i \sin \dfrac{\pi}{2}\right)$

121. $\left[\dfrac{5}{2}\left(\cos \dfrac{\pi}{2} + i \sin \dfrac{\pi}{2}\right)\right]\left[4\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)\right] = 10\left[\cos \dfrac{3\pi}{4} + i \sin \dfrac{3\pi}{4}\right]$

122. $6\left[\cos\left(\dfrac{2\pi}{3} + \dfrac{\pi}{6}\right) + i \sin\left(\dfrac{2\pi}{3} + \dfrac{\pi}{6}\right)\right] = 6\left(\cos \dfrac{5\pi}{6} + i \sin \dfrac{5\pi}{6}\right) = -3\sqrt{3} + 3i$

123. $\dfrac{20(\cos 320° + i \sin 320°)}{5(\cos 80° + i \sin 80°)} = 4[\cos 240° + i \sin 240°]$

124. $\dfrac{3}{9}[\cos(230° - 95°) + i \sin(230° - 95°)] = \dfrac{1}{3}(\cos 135° + i \sin 135°)$

$= -\dfrac{\sqrt{2}}{6} + \dfrac{\sqrt{2}}{6}i$

125. (a) $2 - 2i = 2\sqrt{2}\left(\cos \dfrac{7\pi}{4} + i \sin \dfrac{7\pi}{4}\right)$

$3 + 3i = 3\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)$

(b) $2\sqrt{2}\left(\cos \dfrac{7\pi}{4} + i \sin \dfrac{7\pi}{4}\right)3\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right) = 12(\cos 2\pi + i \sin 2\pi) = 12$

(c) $(2 - 2i)(3 + 3i) = 6 + 6 = 12$

126. (a) $4 + 4i = 4\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)$

$-1 - i = \sqrt{2}\left(\cos \dfrac{5\pi}{4} + i \sin \dfrac{5\pi}{4}\right)$

(b) $4\sqrt{2}\left(\cos \dfrac{\pi}{4} + i \sin \dfrac{\pi}{4}\right)\sqrt{2}\left(\cos \dfrac{5\pi}{4} + i \sin \dfrac{5\pi}{4}\right) = 8\left(\cos \dfrac{3\pi}{2} + i \sin \dfrac{3\pi}{2}\right) = -8i$

(c) $(4 + 4i)(-1 - i) = -4 - 4i - 4i + 4 = -8i$

127. (a) $-i = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$

$2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b) $\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$= 2\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= 2 - 2i$

(c) $-i(2 + 2i) = -2i + 2 = 2 - 2i$

128. (a) $4i = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$1 - i = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

(b) $4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right) = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$= 4\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= 4 + 4i$

(c) $4i(1 - i) = 4i + 4 = 4 + 4i$

129. (a) $3 - 3i = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$2 + 2i = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b) $\dfrac{3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}{2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)} = \frac{3}{2}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$= \frac{3}{2}(-i) = -\frac{3}{2}i$

(c) $\dfrac{3 - 3i}{2 + 2i}\cdot\dfrac{(2 - 2i)}{(2 - 2i)} = \dfrac{6 - 12i - 6}{8} = -\dfrac{12i}{8} = -\dfrac{3}{2}i$

130. (a) $-1 - i = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$-2 - 2i = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

(b) $\dfrac{-1 - i}{-2 - 2i} = \dfrac{\sqrt{2}}{2\sqrt{2}}(\cos 0 + i\sin 0) = \dfrac{1}{2}$

(c) $\dfrac{-1 - i}{-2 - 2i} = \dfrac{-1 - i}{2(-1 - i)} = \dfrac{1}{2}$

131. $\left[5\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]^4 = 5^4\left(\frac{4\pi}{12} + i\sin\frac{4\pi}{12}\right)$

$= 625\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 625\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= \frac{625}{2} + \frac{625\sqrt{3}}{2}i$

132. $\left[2\left(\cos\frac{4\pi}{15} + i\sin\frac{4\pi}{15}\right)\right]^5 = 2^5\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= 32\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= -16 - 16\sqrt{3}i$

133. $(2 + 3i)^6 \approx \left[\sqrt{13}(\cos 56.3° + i\sin 56.3°)\right]^6$

$= 13^3(\cos 337.9° + i\sin 337.9°)$

$\approx 13^3(0.9263 - 0.3769i)$

$\approx 2035 - 828i$

134. $(1 - i)^8 = \left[\sqrt{2}(\cos 315° + i\sin 315°)\right]^8$

$= 16(\cos 2520° + i\sin 2520°)$

$= 16(\cos 0° + i\sin 0°)$

$= 16$

135. $-\sqrt{3} + i = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

Square roots:

$\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) \approx 0.3660 + 1.3660i$

$\sqrt{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right) \approx -0.3660 - 1.3660i$

136. $\sqrt{3} - i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

Square roots:

$\sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) \approx -1.3660 + 0.3660i$

$\sqrt{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right) \approx 1.3660 - 0.3660i$

137. $-2i = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

Square roots:

$\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -1 + i$

$\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = 1 - i$

138. $-5i = 5\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

Square roots:

$\sqrt{5}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = \sqrt{5}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\frac{\sqrt{10}}{2} + \frac{\sqrt{10}}{2}i$

$\sqrt{5}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2}i$

139. $-2 - 2i = 2\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

Square roots:

$2^{3/4}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right) \approx -0.6436 + 1.5538i$

$2^{3/4}\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right) \approx 0.6436 - 1.5538i$

140. $-2 + 2i = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Square roots:

$2^{3/4}\left(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right) \approx 0.6436 + 1.5538i$

$2^{3/4}\left(\cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}\right) \approx -0.6436 - 1.5538i$

141. (a) Sixth roots of $-729i = 729\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$:

$\sqrt[6]{729}\left(\cos\frac{(3\pi/2) + 2k\pi}{6} + i\sin\frac{(3\pi/2) + 2k\pi}{6}\right),\ k = 0, 1, 2, 3, 4, 5$

$3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$3\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

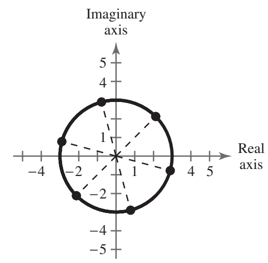
$3\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$3\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$3\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$

$3\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

(b)



(c) $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i, -0.7765 + 2.898i, -2.898 + 0.7765i, -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i, 0.7765 - 2.898i, 2.898 - 0.7765i$

142. (a) Fourth roots of $256i = 256\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$:

$\sqrt[4]{256}\left(\cos\frac{(\pi/2) + 2k\pi}{4} + i\sin\frac{(\pi/2) + 2k\pi}{4}\right), k = 0, 1, 2, 3$

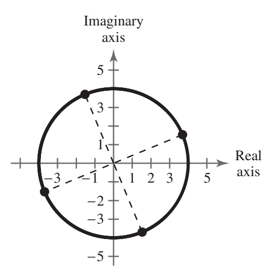
$4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

$4\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$

$4\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right)$

$4\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$

(b)



(c) $3.696 + 1.531i, -1.531 + 3.696i, -3.696 - 1.531i, 1.531 - 3.696i$

143. (a) Cube roots of $8 = 8(\cos 0 + i \sin 0)$:

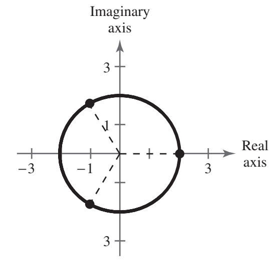
$$\sqrt[3]{8}\left(\cos\left(\frac{2k\pi}{3}\right) + i \sin\left(\frac{2k\pi}{3}\right)\right),\ k = 0, 1, 2$$

$$2(\cos 0 + i \sin 0)$$

$$2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$$

$$2\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$$

(b)



(c) $2, -1 + \sqrt{3}i, -1 - \sqrt{3}i$

144. (a) Fifth roots of $-1024 = 1024(\cos \pi + i \sin \pi)$:

$$\sqrt[5]{1024}\left(\cos\left(\frac{\pi + 2\pi k}{5}\right) + i \sin\left(\frac{\pi + 2\pi k}{5}\right)\right),\ k = 0, 1, 2, 3, 4$$

$$4\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$$

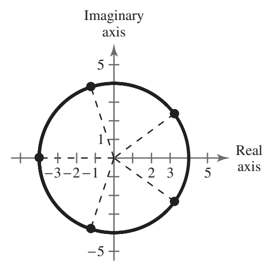
$$4\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$$

$$4\left(\cos \frac{5\pi}{5} + i \sin \frac{5\pi}{5}\right) = -4$$

$$4\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$$

$$4\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$$

(b)



(c) $3.236 \pm 2.351i, -1.236 \pm 3.804i, -4$

145. $x^4 + 256 = 0$

$$x^4 = -256 = 256(\cos \pi + i \sin \pi)$$

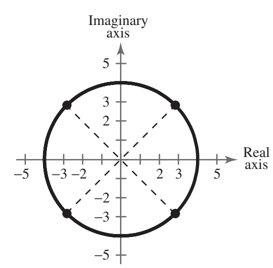
$$\sqrt[4]{-256} = 4\left[\cos\left(\frac{\pi + 2\pi k}{4}\right) + i \sin\left(\frac{\pi + 2\pi k}{4}\right)\right],\ k = 0, 1, 2, 3$$

$$4\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \frac{4\sqrt{2}}{2} + \frac{4\sqrt{2}}{2}i = 2\sqrt{2} + 2\sqrt{2}i$$

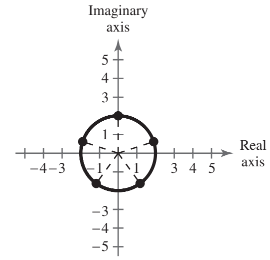
$$4\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right) = -\frac{4\sqrt{2}}{2} + \frac{4\sqrt{2}}{2}i = -2\sqrt{2} + 2\sqrt{2}i$$

$$4\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right) = -\frac{4\sqrt{2}}{2} - \frac{4\sqrt{2}}{2}i = -2\sqrt{2} - 2\sqrt{2}i$$

$$4\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right) = \frac{4\sqrt{2}}{2} - \frac{4\sqrt{2}}{2}i = 2\sqrt{2} - 2\sqrt{2}i$$



146. $x^5 = 32i = 32\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$



$\sqrt[5]{32}\left(\cos\left(\frac{\pi/2 + 2k\pi}{5}\right) + i\sin\left(\frac{\pi/2 + 2k\pi}{5}\right)\right), k = 0, 1, 2, 3, 4$

$2\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$

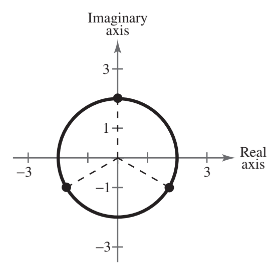
$2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$

$2\left(\cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10}\right)$

$2\left(\cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10}\right)$

$2\left(\cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}\right)$

147. $x^3 + 8i = 0$



$x^3 = -8i$

$-8i = 8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

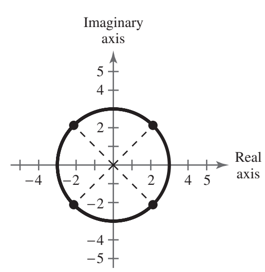
$\sqrt[3]{-8i} = \sqrt[3]{8}\left[\cos\frac{(3\pi/2) + 2\pi k}{3} + i\sin\frac{(3\pi/2) + 2\pi k}{3}\right],\ k = 0, 1, 2$

$2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$

$2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right) = -\sqrt{3} - i$

$2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = \sqrt{3} - i$

148. $x^4 = -81 = 81(\cos\pi + i\sin\pi)$



$\sqrt[4]{81}\left(\cos\frac{\pi + 2k\pi}{4} + i\sin\frac{\pi + 2k\pi}{4}\right), k = 0, 1, 2, 3$

$3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$3\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$3\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

149. True

150. False. There may be no solution, one solution, or two solutions.

151. Length and direction characterize vectors in plane.

152. **A** and **C** appear equivalent.

153.
$$\begin{aligned} z_1z_2 &= 2(\cos\theta + i\sin\theta)2(\cos(\pi - \theta) + i\sin(\pi - \theta)) \\ &= 4(\cos\theta + i\sin\theta)(-\cos\theta + i\sin\theta) \\ &= 4(-\cos^2\theta - \sin^2\theta) = -4 \end{aligned}$$

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{2(\cos\theta + i\sin\theta)}{2(\cos(\pi - \theta) + i\sin(\pi - \theta))} \\ &= \frac{\cos\theta + i\sin\theta}{-\cos\theta + i\sin\theta} \cdot \frac{-\cos\theta - i\sin\theta}{-\cos\theta - i\sin\theta} \\ &= -\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta i \\ &= -(\cos\theta + i\sin\theta)^2 \\ &= -\left(\frac{z_1}{2}\right)^2 \\ &= -\frac{z_1{}^2}{4} \end{aligned}$$

154. (a) Three roots are not shown.

(b) The modulus of each is 2, and the arguments are 120°, 210° and 300°.

Chapter 6 Practice Test

For Exercises 1 and 2, use the Law of Sines to find the remaining sides and angles of the triangle.

1. $A = 40°,\ B = 12°,\ b = 100$

2. $C = 150°,\ a = 5,\ c = 20$

3. Find the area of the triangle: $a = 3,\ b = 6,\ C = 130°$

4. Determine the number of solutions to the triangle: $a = 10,\ b = 35,\ A = 22.5°$

For Exercises 5 and 6, use the Law of Cosines to find the remaining sides and angles of the triangle.

5. $a = 49,\ b = 53,\ c = 38$

6. $C = 29°,\ a = 100,\ b = 300$

7. Use Heron's Formula to find the area of the triangle: $a = 4.1,\ b = 6.8,\ c = 5.5$.

8. A ship travels 40 miles due east, then adjusts its course 12° southward. After traveling 70 miles in that direction, how far is the ship from its point of departure?

9. $\mathbf{w} = 4\mathbf{u} - 7\mathbf{v}$ where $\mathbf{u} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$. Find $\mathbf{w}$.

10. Find a unit vector in the direction of $\mathbf{v} = 5\mathbf{i} - 3\mathbf{j}$.

11. Find the dot product and the angle between $\mathbf{u} = 6\mathbf{i} + 5\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$.

12. $\mathbf{v}$ is a vector of magnitude 4 making an angle of 30° with the positive x-axis. Find $\mathbf{v}$ in component form.

13. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$ given $\mathbf{u} = \langle 3, -1 \rangle$ and $\mathbf{v} = \langle -2, 4 \rangle$.

14. Give the trigonometric form of $z = 5 - 5i$.

15. Give the standard form of $z = 6(\cos 225° + i \sin 225°)$.

16. Multiply $[7(\cos 23° + i \sin 23°)][4(\cos 7° + i \sin 7°)]$.

17. Divide $\dfrac{9\left(\cos \dfrac{5\pi}{4} + i \sin \dfrac{5\pi}{4}\right)}{3(\cos \pi + i \sin \pi)}$.

18. Find $(2 + 2i)^8$.

19. Find the cube roots of $8\left(\cos \dfrac{\pi}{3} + i \sin \dfrac{\pi}{3}\right)$.

20. Find all the solutions to $x^4 + i = 0$.