# C H A P T E R 3
# Exponential and Logarithmic Functions

# CHAPTER 3
# Exponential and Logarithmic Functions

## Section 3.1 Exponential Functions and Their Graphs

- You should know that a function of the form $y = a^x$, where $a > 0$, $a \neq 1$, is called an exponential function with base $a$.
- You should be able to graph exponential functions.
- You should be familiar with the number $e$ and the natural exponential function $f(x) = e^x$.
- You should know formulas for compound interest.

  (a) For $n$ compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

  (b) For continuous compoundings: $A = Pe^{rt}$.

### Vocabulary Check

1. algebraic
2. transcendental
3. natural exponential, natural
4. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
5. $A = Pe^{rt}$

**1.** $(3.4)^{6.8} \approx 4112.033$

**2.** $1.2^{1/3} \approx 1.063$

**3.** $5^{-\pi} \approx 0.006$

**4.** $8.6^{-3(-\sqrt{2})} = 8.6^{3\sqrt{2}} \approx 9220.217$

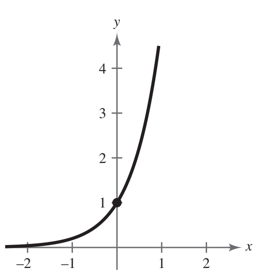
**5.** $g(x) = 5^x$

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $\frac{1}{25}$ | $\frac{1}{5}$ | 1 | 5 | 25 |

Asymptote: $y = 0$

Intercept: $(0, 1)$

Increasing



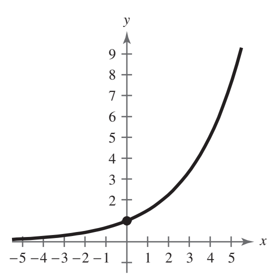
**6.** $f(x) = \left(\frac{3}{2}\right)^x$

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $\frac{4}{9}$ | $\frac{2}{3}$ | 1 | $\frac{3}{2}$ | $\frac{9}{4}$ |

Asymptote: $y = 0$

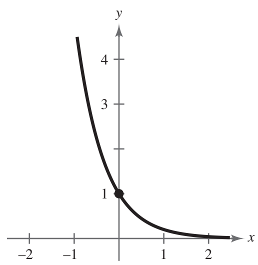
Intercept: $(0, 1)$

Increasing



**7.** $f(x) = \left(\frac{1}{5}\right)^x = 5^{-x}$

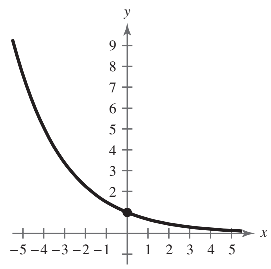
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $25$ | $5$ | $1$ | $\frac{1}{5}$ | $\frac{1}{25}$ |




Asymptote: $y = 0$
Intercept: $(0, 1)$
Decreasing

**8.** $h(x) = \left(\frac{3}{2}\right)^{-x}$

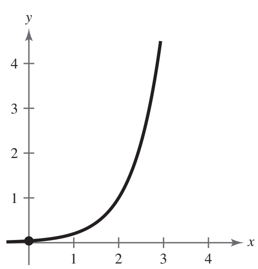
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $\frac{9}{4}$ | $\frac{3}{2}$ | $1$ | $\frac{2}{3}$ | $\frac{4}{9}$ |




Asymptote: $y = 0$
Intercept: $(0, 1)$
Decreasing

**9.** $h(x) = 5^{x-2}$

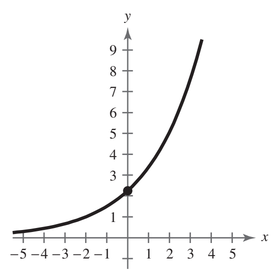
| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|
| $y$ | $\frac{1}{125}$ | $\frac{1}{25}$ | $\frac{1}{5}$ | $1$ | $5$ |




Asymptote: $y = 0$
Intercept: $\left(0, \frac{1}{25}\right)$
Increasing

**10.** $g(x) = \left(\frac{3}{2}\right)^{x+2}$

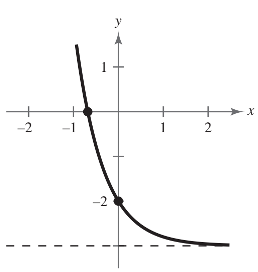
| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|---|---|
| $y$ | $\frac{4}{9}$ | $\frac{2}{3}$ | $1$ | $\frac{3}{2}$ | $\frac{9}{4}$ |




Asymptote: $y = 0$
Intercept: $\left(0, \frac{9}{4}\right)$
Increasing

**11.** $g(x) = 5^{-x} - 3$

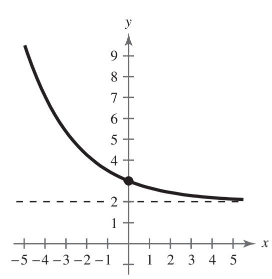
| $x$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|
| $y$ | $2$ | $-2$ | $-2\frac{4}{5}$ | $-2\frac{24}{25}$ |




Asymptote: $y = -3$
Intercepts:
$(0, -2)$, $(-0.683, 0)$
Decreasing

**12.** $f(x) = \left(\frac{3}{2}\right)^{-x} + 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $\frac{17}{4}$ | $\frac{7}{2}$ | $3$ | $\frac{8}{3}$ | $\frac{22}{9}$ |




Asymptote: $y = 2$
Intercept: $(0, 3)$
Decreasing

**13.** $f(x) = 2^{x-2}$ rises to the right.
Asymptote: $y = 0$
Intercept: $\left(0, \frac{1}{4}\right)$
Matches graph (d).

**14.** $f(x) = 2^{-x}$ is positive and decreasing.
Matches graph (a).

**15.** $f(x) = 2^x - 4$ rises to the right.

Asymptote: $y = -4$

Intercept: $(0, -3)$

Matches graph (c).

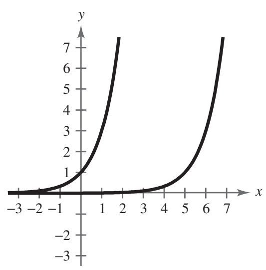
**16.** $f(x) = 2^x + 1$ is increasing and has $(0, 2)$ intercept.

Matches graph (b).

**17.** $f(x) = 3^x$

$g(x) = 3^{x-5} = f(x - 5)$

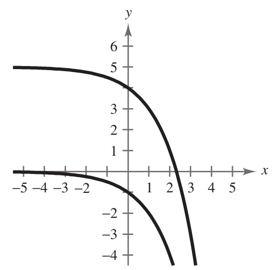
Horizontal shift five units to the right




**18.** $f(x) = -2^x$

$g(x) = 5 - 2^x = 5 + f(x)$

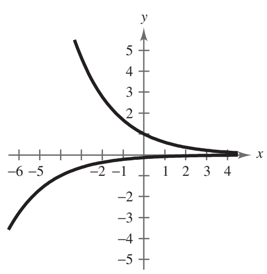
Vertical shift five units upward




**19.** $f(x) = \left(\frac{3}{5}\right)^x$

$g(x) = -\left(\frac{3}{5}\right)^{x+4} = -f(x + 4)$

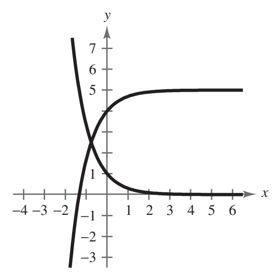
Horizontal shift four units to the left, followed by reflection in $x$-axis




**20.** $f(x) = 0.3^x$

$g(x) = -0.3^x + 5 = -f(x) + 5$

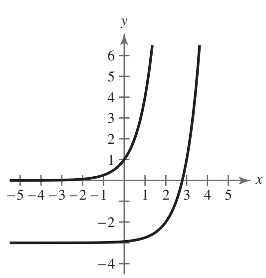
Reflection in $x$-axis followed by vertical shift five units upward




**21.** $f(x) = 4^x$

$g(x) = 4^{x-2} - 3 = f(x - 2) - 3$

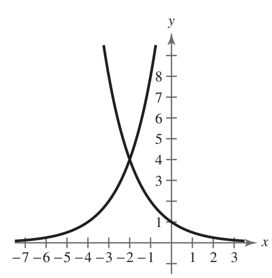
Horizontal shift two units to the right followed by vertical shift three units downward




**22.** $f(x) = \left(\frac{1}{2}\right)^x$

$g(x) = \left(\frac{1}{2}\right)^{-(x+4)}$

Reflection in the $y$-axis followed by left shift of four units






**23.** $e^{9.2} \approx 9897.129$

**24.** $e^{-(-3/4)} = e^{3/4}$

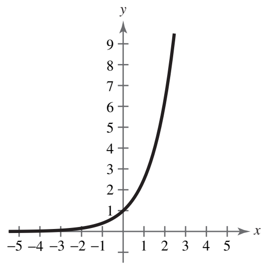
$\approx 2.117$

**25.** $50e^{4(0.02)} \approx 54.164$

**26.** $-5.5e^{-200} = 7.611 \times 10^{-87} \approx 0$

**27.** $f(x) = \left(\frac{5}{2}\right)^x$

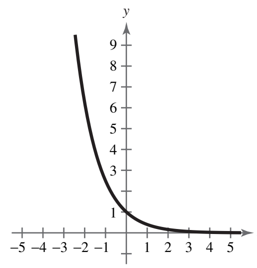
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 0.16 | 0.4 | 1 | 2.5 | 6.25 |




Asymptote: $y = 0$

**28.** $f(x) = \left(\frac{5}{2}\right)^{-x}$

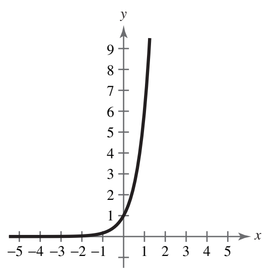
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 6.25 | 2.5 | 1 | 0.4 | 0.16 |




Asymptote: $y = 0$

**29.** $f(x) = 6^x$

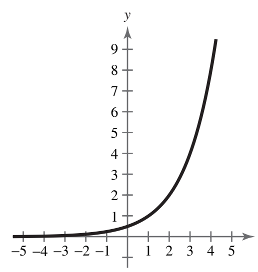
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 0.03 | 0.17 | 1 | 6 | 36 |




Asymptote: $y = 0$

**30.** $f(x) = 2^{x-1}$

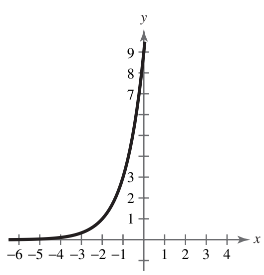
| $x$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 0.25 | 0.5 | 1 | 2 | 4 |




Asymptote: $y = 0$

**31.** $f(x) = 3^{x+2}$

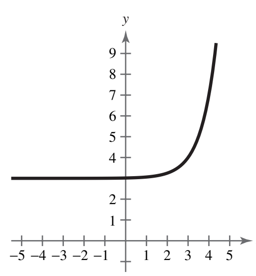
| $x$ | $-3$ | $-2$ | $-1$ | 0 | 1 |
|---|---|---|---|---|---|
| $f(x)$ | 0.33 | 1 | 3 | 9 | 27 |




Asymptote: $y = 0$

**32.** $f(x) = 4^{x-3} + 3$

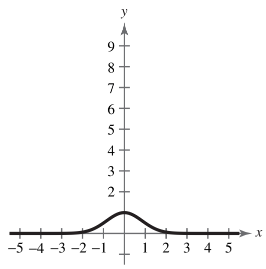
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f(x)$ | 3.016 | 3.063 | 3.25 | 4 | 7 | 19 |




Asymptote: $y = 3$

**33.** $y = 2^{-x^2}$

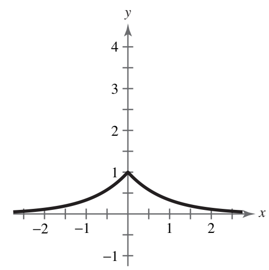
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0.06 | 0.5 | 1 | 0.5 | 0.06 |




Asymptote: $y = 0$

**34.** $y = 3^{-|x|}$

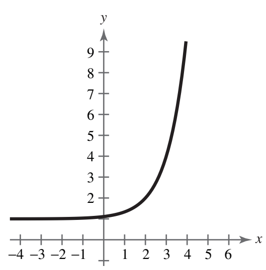
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0.11 | 0.33 | 1 | 0.33 | 0.11 |




Asymptote: $y = 0$

**35.** $y = 3^{x-2} + 1$

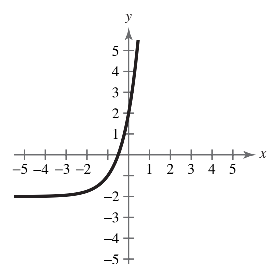
| $x$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $y$ | 1.04 | 1.11 | 1.33 | 2 | 4 | 10 |




Asymptote: $y = 1$

**36.** $y = 4^{x+1} - 2$

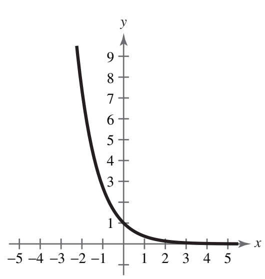
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | $-1.75$ | $-1$ | 2 | 14 | 62 |




Asymptote: $y = -2$

**37.** $f(x) = e^{-x}$

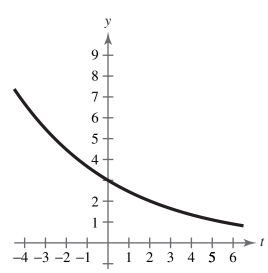
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 7.39 | 2.72 | 1 | 0.37 | 0.14 |




Asymptote: $y = 0$

**38.** $s(t) = 3e^{-0.2t}$

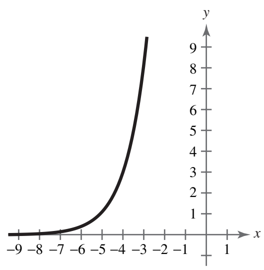
| $t$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $s(t)$ | 3.66 | 3 | 2.46 | 2.011 | 1.65 | 1.35 |




Asymptote: $y = 0$

**39.** $f(x) = 3e^{x+4}$

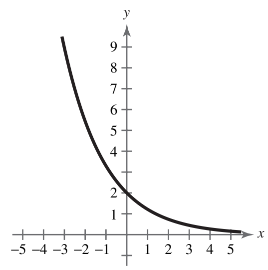
| $x$ | $-6$ | $-5$ | $-4$ | $-3$ | $-2$ |
|---|---|---|---|---|---|
| $f(x)$ | 0.41 | 1.10 | 3 | 8.15 | 22.17 |



Asymptote: $y = 0$

**40.** $f(x) = 2e^{-0.5x}$

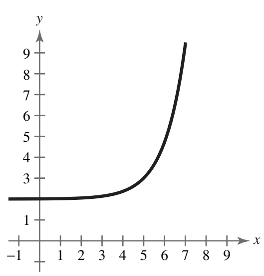
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $f(x)$ | 5.44 | 3.30 | 2 | 1.21 | 0.74 |



Asymptote: $y = 0$

**41.** $f(x) = 2 + e^{x-5}$

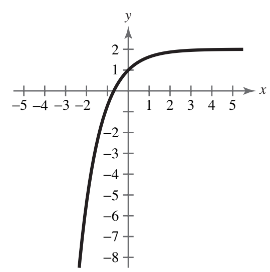
| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $f(x)$ | 2.14 | 2.37 | 3 | 4.72 | 9.39 |



Asymptote: $y = 2$

**42.** $g(x) = 2 - e^{-x}$

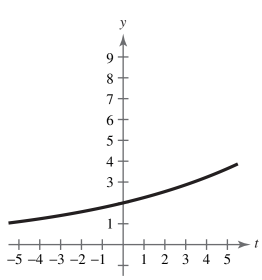
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $g(x)$ | $-5.39$ | $-0.72$ | 1 | 1.63 | 1.86 |



Asymptote: $y = 2$

**43.** $s(t) = 2e^{0.12t}$

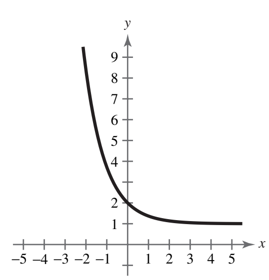
| $t$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $s(t)$ | 1.57 | 1.77 | 2 | 2.26 | 2.54 |



Asymptote: $y = 0$

**44.** $g(x) = 1 + e^{-x}$

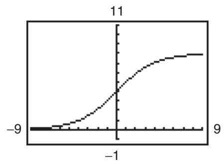
| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $g(x)$ | 8.39 | 3.72 | 2 | 1.37 | 1.14 |



Asymptote: $y = 1$

**45.** $f(x) = \dfrac{8}{1 + e^{-0.5x}}$

(a)

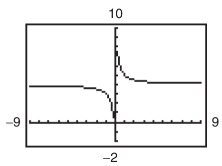



(b)

| $x$ | $-30$ | $-20$ | $-10$ | $0$ | $10$ | $20$ | $30$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $\approx 0$ | $\approx 0$ | $0.05$ | $4$ | $7.95$ | $\approx 8$ | $\approx 8$ |

Horizontal asymptotes: $y = 0, y = 8$

**46.** (a)




(b)

| $x$ | $-15$ | $-2$ | $-1$ | $-0.2$ | $-0.1$ |
|---|---|---|---|---|---|
| $f(x)$ | $3.93$ | $3.5$ | $3.0$ | $0.61$ | $0.05$ |

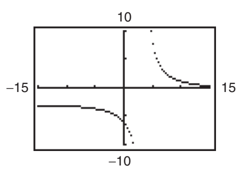
| $x$ | $0$ | $0.01$ | $0.2$ | $1$ | $5$ |
|---|---|---|---|---|---|
| $f(x)$ | undef. | $8$ | $7.4$ | $5.0$ | $4.2$ |

Horizontal asymptote: $y = 4$

Vertical asymptote: $x = 0$

**47.** $f(x) = \dfrac{-6}{2 - e^{0.2x}}$

(a)




(b)

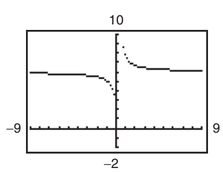
| $x$ | $-20$ | $-10$ | $0$ | $3$ | $3.4$ | $3.46$ |
|---|---|---|---|---|---|---|
| $f(x)$ | $-3.03$ | $-3.22$ | $-6$ | $-34$ | $-230$ | $-2617$ |

| $x$ | $3.47$ | $4$ | $5$ | $10$ | $20$ |
|---|---|---|---|---|---|
| $f(x)$ | $3516$ | $26.6$ | $8.4$ | $1.11$ | $0.11$ |

Horizontal asymptotes: $y = -3, y = 0$

Vertical asymptote: $x \approx 3.47$

**48.** (a)




(b)

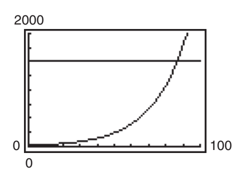
| $x$ | $-15$ | $-10$ | $-1$ | $-0.1$ | $-0.01$ |
|---|---|---|---|---|---|
| $f(x)$ | $5.9$ | $5.9$ | $5.1$ | $3.2$ | $3$ |

| $x$ | $\dfrac{0.2}{\ln 2}$ | $0.289$ | $1$ | $4$ | $10$ |
|---|---|---|---|---|---|
| $f(x)$ | undef. | $2715$ | $7.7$ | $6.3$ | $6.1$ |

Asymptotes: $y = 6$

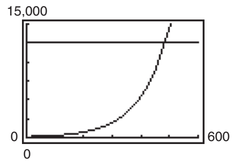
$$x = \frac{0.2}{\ln 2} \approx 0.2885$$

**49.**




Intersection: $(86.350, 1500)$

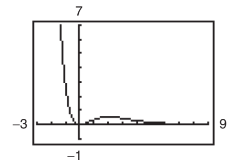
**50.**




Intersection: (482.831, 12,500)

**51.** $f(x) = x^2e^{-x}$

(a)




(b) Decreasing: $(-\infty, 0), (2, \infty)$

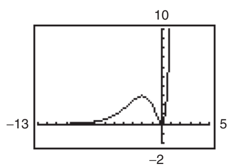
Increasing: $(0, 2)$

(c) Relative maximum: $(2, 4e^{-2}) \approx (2, 0.541)$

Relative minimum: $(0, 0)$

**52.** $f(x) = 2x^2e^{x+1}$

(a)




(b) Increasing on $(-\infty, -2)$ and $(0, \infty)$

Decreasing on $(-2, 0)$

(c) Relative maximum: $(-2, 2.943)$

Relative minimum: $(0, 0)$

**53.** $P = 2500, r = 2.5\% = 0.025, t = 10$

Compounded $n$ times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.025}{n}\right)^{10n}$

Compounded continuously: $A = Pe^{rt} = 2500e^{(0.025)(10)}$

| $n$ | 1 | 2 | 4 | 12 | 365 | Continuous |
|---|---|---|---|---|---|---|
| $A$ | 3200.21 | 3205.09 | 3207.57 | 3209.23 | 3210.04 | 3210.06 |

**54.** $P = 1000, r = 6\% = 0.06, t = 10$

| $n$ | 1 | 2 | 4 | 12 | 365 | Continuous |
|---|---|---|---|---|---|---|
| $A$ | 1790.85 | 1806.11 | 1814.02 | 1819.40 | 1822.03 | 1822.12 |

**55.** $P = 2500, r = 4\% = 0.04, t = 20$

Compounded $n$ times per year: $A = P\left(1 + \frac{r}{n}\right)^{nt} = 2500\left(1 + \frac{0.04}{n}\right)^{20n}$

Compounded continuously: $A = Pe^{rt} = 2500e^{(0.04)(20)}$

| $n$ | 1 | 2 | 4 | 12 | 365 | Continuous |
|---|---|---|---|---|---|---|
| $A$ | 5477.81 | 5520.10 | 5541.79 | 5556.46 | 5563.61 | 5563.85 |

**56.** $P = 1000, r = 3\% = 0.03, t = 40$

| $n$ | 1 | 2 | 4 | 12 | 365 | Continuous |
|---|---|---|---|---|---|---|
| $A$ | 3262.04 | 3290.66 | 3305.28 | 3315.15 | 3319.95 | 3320.12 |

**57.** $P = 12{,}000,\ r = 4\% = 0.04$

$A = Pe^{rt} = 12000e^{0.04t}$

| $t$ | 1 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $A$ | 12,489.73 | 17,901.90 | 26,706.49 | 39,841.40 | 59,436.39 | 88,668.67 |

**58.** $P = 12{,}000,\ r = 6\% = 0.06$, compounded continuously: $A = Pe^{rt} = 12{,}000e^{(0.06)t}$

| $t$ | 1 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $A$ | 12,742.04 | 21,865.43 | 39,841.40 | 75,595.77 | 132,278.12 | 241,026.44 |

**59.** $P = 12{,}000,\ r = 3.5\% = 0.035$

$A = Pe^{rt} = 12{,}000e^{0.035t}$

| $t$ | 1 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $A$ | 12,427.44 | 17,028.81 | 24,165.03 | 34,291.81 | 48,662.40 | 69,055.23 |

**60.** $P = 12{,}000,\ r = 2.5\% = 0.025$, compounded continuously: $A = Pe^{rt} = 12{,}000e^{0.025t}$

| $t$ | 1 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $A$ | 12,303.78 | 15,408.31 | 19,784.66 | 25,404.00 | 32,619.38 | 41,884.12 |

**61.** $A = 25\left[\dfrac{(1 + 0.12/12)^{48} - 1}{0.12/12}\right]$

$= 25\left[\dfrac{1.01^{48} - 1}{0.01}\right]$

$= \$1530.57$

**62.** $A = 100\left[\dfrac{(1 + 0.09/12)^{60} - 1}{0.09/12}\right]$

$= \$7542.41$

**63.** $A = 200\left[\dfrac{(1 + 0.06/12)^{72} - 1}{0.06/12}\right]$

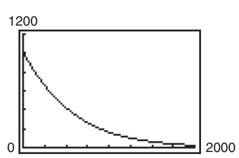
$= \$17{,}281.77$

**64.** $A = 75\left[\dfrac{(1 + 0.03/12)^{24} - 1}{0.03/12}\right]$

$= \$1852.71$

**65.** $p = 5000\left(1 - \dfrac{4}{4 + e^{-0.002x}}\right)$

(a)




(b) If $x = 500$, $p \approx \$421.12$.

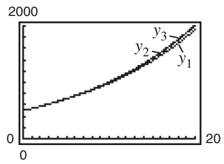
(c) For $x = 600$, $p \approx \$350.13$.

| $x$ | 100 | 200 | 300 | 400 | 500 | 600 | 700 |
|---|---|---|---|---|---|---|---|
| $p$ | 849.53 | 717.64 | 603.25 | 504.94 | 421.12 | 350.13 | 290.35 |

**66.** (a) $y_1 = 500(1 + 0.07)^x$

$y_2 = 500\left(1 + \frac{0.07}{4}\right)^{4x}$

$y_3 = 500e^{0.07x}$




(b) $y_3$ has the highest return.

After 20 years,

$y_2 - y_1 = 2003.20 - 1934.84 = \$68.36$

$y_3 - y_2 = 2027.60 - 2003.20 = \$24.40$

$y_3 - y_1 = 2027.60 - 1934.84 = \$92.76$

**67.** $Q = 25\left(\frac{1}{2}\right)^{t/1599}$

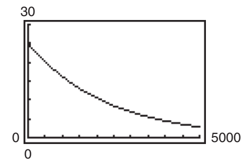
(a) When $t = 0$,

$Q = 25\left(\frac{1}{2}\right)^{0/1599} = 25(1) = 25$ grams.

(b) When $t = 1000$,

$Q = 25\left(\frac{1}{2}\right)^{1000/1599} \approx 16.21$ grams.

(c)




(d) Never. The graph has a horizontal asymptote $Q = 0$.

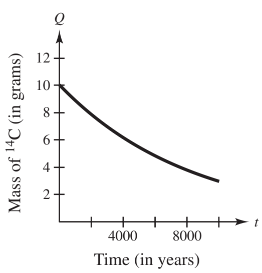
**68.** $Q = 10\left(\frac{1}{2}\right)^{t/5715}$

(a) When $t = 0$, $Q = 10$.

(b) When $t = 2000$,

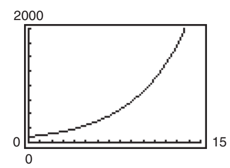
$Q = 10\left(\frac{1}{2}\right)^{2000/5715} \approx 7.85$ grams.

(c)




**69.** $P(t) = 100e^{0.2197t}$

(a)




(b) $P(0) = 100$

$P(5) \approx 300$

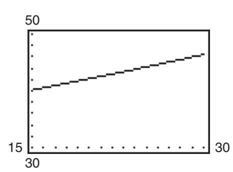
$P(10) \approx 900$

(c) $P(0) = 100e^{0.2197(0)} = 100$

$P(5) = 100e^{0.2197(5)} = 299.966 \approx 300$

$P(10) = 100e^{0.2197(10)} = 899.798 \approx 900$

**70.** (a)




(b)

| Year | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 |
|---|---|---|---|---|---|---|---|---|
| $P$ | 40.1 | 40.5 | 40.9 | 41.3 | 41.7 | 42.1 | 42.5 | 43.0 |

| Year | 2023 | 2024 | 2025 | 2026 | 2027 | 2028 | 2029 | 2030 |
|---|---|---|---|---|---|---|---|---|
| $P$ | 43.4 | 43.8 | 44.2 | 44.7 | 45.1 | 45.5 | 46.0 | 46.4 |

(c) $P = 34.706e^{0.0097t} = 50$

$e^{0.0097t} = 1.441$

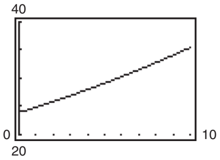
$0.0097t = \ln(1.441)$

$t \approx 38$, or 2038

(Answers will vary.)

**71.** $C(t) = P(1.04)^t$

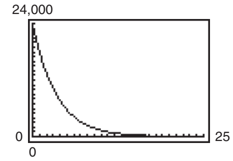
(a)




(b) $C(10) \approx 35.45$

(c) $C(10) = 23.95(1.04)^{10} \approx 35.45$

**72.** (a)




(b)

| $t$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $V$ | 17,978 | 13,483 | 10,112 | 7584 | 5688 |

| $t$ | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| $V$ | 4266 | 3200 | 2400 | 1800 | 1350 |

(c) According to the model, $V(t) \to 0$ as $t$ increases. However, $V \neq 0$.

**73.** True. $f(x) = 1^x$ is not an exponential function.

**74.** False. $e$ is an irrational number.

**75.** The graph decreases for all $x$ and has positive $y$-intercept. Matches (d).

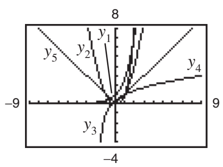
**76.** $y_1 = e^x$

$y_2 = x^2$

$y_3 = x^3$

$y_4 = \sqrt{x}$

$y_5 = |x|$



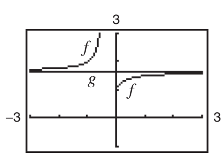

(a) $y_1 = e^x$ increases at the fastest rate.

(b) For any positive integer $n$, $e^x > x^n$ for $x$ sufficiently large. That is, $e^x$ grows faster than $x^n$.

(c) A quantity is growing exponentially if its growth rate is of the form $y = ce^{rx}$. This is a faster rate than any polynomial growth rate.

**77.** $f(x) = \left(1 + \dfrac{0.5}{x}\right)^x$ and $g(x) = e^{0.5} \approx 1.6487$

(Horizontal line)




As $x \to \infty, f(x) \to g(x)$.

**78.** $y = 3^x$ (c) and $y = 2^{-x}$ (d) are exponential functions because the exponents are variable.

**79.** $e^{\pi} \approx 23.14, \pi^e \approx 22.46$

$e^{\pi} > \pi^e$

**80.** $2^{10} = 1024, 10^2 = 100$

$2^{10} > 10^2$

**81.** $5^{-3} = 0.008, 3^{-5} \approx 0.0041$

$5^{-3} > 3^{-5}$

**82.** $4^{1/2} = 2, \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$4^{1/2} > \left(\frac{1}{2}\right)^4$

**83.** $f$ has an inverse because $f$ is one-to-one.

$$y = 5x - 7$$
$$x = 5y - 7$$
$$x + 7 = 5y$$
$$f^{-1}(x) = \frac{1}{5}(x + 7)$$

**84.** $f$ is one-to-one, so it has an inverse.

$$f(x) = -\frac{2}{3}x + \frac{5}{2}$$
$$y = -\frac{2}{3}x + \frac{5}{2}$$
$$x = -\frac{2}{3}y + \frac{5}{2}$$
$$x - \frac{5}{2} = -\frac{2}{3}y$$
$$-\frac{3}{2}\left(x - \frac{5}{2}\right) = y$$
$$f^{-1}(x) = -\frac{3}{2}x + \frac{15}{4}$$

**85.** $f$ has an inverse because $f$ is one-to-one.

$$y = \sqrt[3]{x + 8}$$
$$x = \sqrt[3]{y + 8}$$
$$x^3 = y + 8$$
$$x^3 - 8 = y$$
$$f^{-1}(x) = x^3 - 8$$

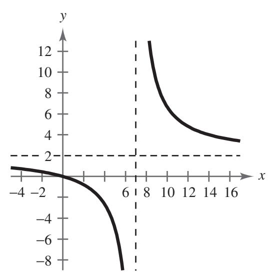
**86.** $f$ is not one-to-one, so it does not have an inverse.

**87.** $f(x) = \dfrac{2x}{x - 7}$

Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 2$

Intercept: $(0, 0)$

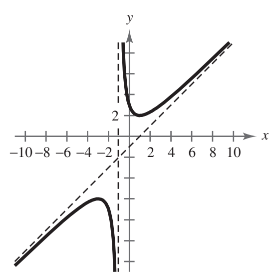



**88.** $f(x) = \dfrac{x^2 + 3}{x + 1} = x - 1 + \dfrac{4}{x + 1}$

Slant asymptote: $y = x - 1$

Vertical asymptote: $x = -1$

Intercept: $(0, 3)$




**89.** Answers will vary.

## Section 3.2 Logarithmic Functions and Their Graphs

- You should know that a function of the form $y = \log_a x$, where $a > 0$, $a \neq 1$, and $x > 0$, is called a logarithm of $x$ to base $a$.
- You should be able to convert from logarithmic form to exponential form and vice versa.

  $y = \log_a x \iff a^y = x$
- You should know the following properties of logarithms.

  (a) $\log_a 1 = 0$ since $a^0 = 1$. (c) $\log_a a^x = x$ since $a^x = a^x$.

  (b) $\log_a a = 1$ since $a^1 = a$. (d) If $\log_a x = \log_a y$, then $x = y$.
- You should know the definition of the natural logarithmic function.

  $\log_e x = \ln x, x > 0$
- You should know the properties of the natural logarithmic function.

  (a) $\ln 1 = 0$ since $e^0 = 1$. (c) $\ln e^x = x$ since $e^x = e^x$.

  (b) $\ln e = 1$ since $e^1 = e$. (d) If $\ln x = \ln y$, then $x = y$.
- You should be able to graph logarithmic functions.

### Vocabulary Check

**1.** logarithmic function **2.** 10 **3.** natural logarithmic

**4.** $a^{\log_a x} = x$ **5.** $x = y$

**1.** $\log_4 64 = 3 \implies 4^3 = 64$

**2.** $\log_3 81 = 4 \implies 3^4 = 81$

**3.** $\log_7 \frac{1}{49} = -2 \implies 7^{-2} = \frac{1}{49}$

**4.** $\log_{10} \frac{1}{1000} = -3 \implies 10^{-3} = \frac{1}{1000}$

**5.** $\log_{32} 4 = \frac{2}{5} \implies 32^{2/5} = 4$

**6.** $\log_{16} 8 = \frac{3}{4} \implies 16^{3/4} = 8$

**7.** $\ln 1 = 0 \implies e^0 = 1$

**8.** $\ln 4 = 1.3862\ldots \implies e^{1.3862\ldots} = 4$

**9.** $\ln e = 1 \implies e^1 = e$

**10.** $\ln e^3 = 3 \implies e^3 = e^3$

**11.** $\ln \sqrt{e} = \dfrac{1}{2} \implies e^{1/2} = \sqrt{e}$

**12.** $\ln \dfrac{1}{e^2} = -2 \implies e^{-2} = \dfrac{1}{e^2}$

**13.** $5^3 = 125 \implies \log_5 125 = 3$

**14.** $8^2 = 64 \implies \log_8 64 = 2$

**15.** $81^{1/4} = 3 \implies \log_{81} 3 = \frac{1}{4}$

**16.** $9^{3/2} = 27 \implies \log_9 27 = \frac{3}{2}$

**17.** $6^{-2} = \frac{1}{36} \implies \log_6 \frac{1}{36} = -2$

**18.** $10^{-3} = 0.001 \implies \log_{10} 0.001 = -3$

**19.** $e^3 = 20.0855\ldots \implies \ln 20.0855\ldots = 3$

**20.** $e^4 \approx 54.5981\ldots \implies \ln 54.5981\ldots = 4$

**21.** $e^{1.3} = 3.6692\ldots \implies \ln 3.6692\ldots = 1.3$

**22.** $e^{2.5} = 12.1824\ldots \implies \ln 12.1824\ldots = 2.5$

**23.** $\sqrt[3]{e} = 1.3956\ldots \implies \ln(1.3956\ldots) = \frac{1}{3}$

**24.** $\frac{1}{e^4} = e^{-4} = 0.0183\ldots \Rightarrow \ln 0.0183\ldots = -4$

**25.** $\log_2 16 = \log_2 2^4 = 4$

**26.** $\log_{16}\left(\frac{1}{4}\right) = -\frac{1}{2}$

because $16^{-1/2} = \frac{1}{16^{1/2}} = \frac{1}{4}$.

**27.** $g\left(\frac{1}{1000}\right) = \log_{10}\left(\frac{1}{1000}\right)$
$= \log_{10}(10^{-3})$
$= -3$

**28.** $g(10{,}000) = \log_{10}(10{,}000)$
$= \log_{10}(10^4)$
$= 4$

**29.** $\log_{10} 345 \approx 2.538$

**30.** $\log_{10}\left(\frac{4}{5}\right) \approx -0.097$

**31.** $6 \log_{10} 14.8 \approx 7.022$

**32.** $1.9 \log_{10}(4.3) \approx 1.204$

**33.** $\log_7 x = \log_7 9$
$x = 9$

**34.** $x = \log_5 5 = 1$

**35.** $\log_6 6^2 = x$
$2 \log_6 6 = x$
$2 = x$

**36.** $\log_2 2^{-1} = x$
$-1 = x$

**37.** $\log_8 x = \log_8 10^{-1}$
$x = 10^{-1} = \frac{1}{10}$

**38.** $x = \log_4(4^3)$
$= 3$

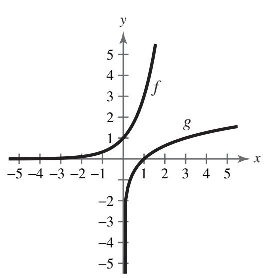
**39.** $\log_4 4^{3x} = (3x) \log_4 4 = 3x$

**40.** $6^{\log_6 36} = 36$

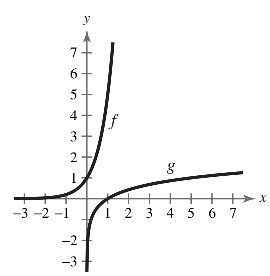
**41.** $3 \log_2\left(\frac{1}{2}\right) = 3 \log_2(2^{-1})$
$= 3(-1) = -3$

**42.** $\frac{1}{4} \log_4 16 = \frac{1}{4} \log_4 4^2 = \frac{1}{4}(2) = \frac{1}{2}$

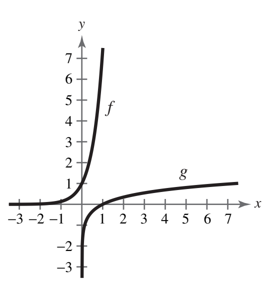
**43.** $f(x) = 3^x$ and $g(x) = \log_3 x$ are inverses of each other.



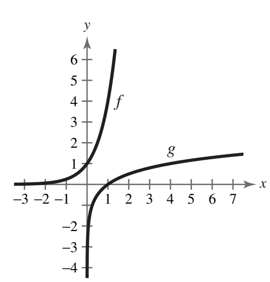

**44.** $f(x) = 5^x$ and $g(x) = \log_5 x$ are inverses of each other.




**45.** $f(x) = e^{2x}$ and $g(x) = \frac{1}{2} \ln x$ are inverses of each other.




**46.** $f(x) = 4^x$ and $g(x) = \log_4 x$ are inverses of each other.



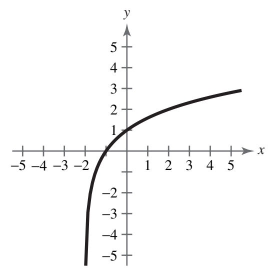
**47.** $y = \log_2(x + 2)$

Domain: $x + 2 > 0 \implies x > -2$

Vertical asymptote: $x = -2$

$$\log_2(x + 2) = 0$$
$$x + 2 = 1$$
$$x = -1$$

$x$-intercept: $(-1, 0)$



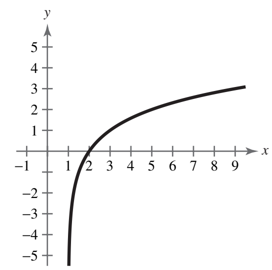
**48.** $y = \log_2(x - 1)$

Domain: $x - 1 > 0 \implies x > 1$

Vertical asymptote: $x = 1$

$$\log_2(x - 1) = 0$$
$$x - 1 = 1$$
$$x = 2$$

$x$-intercept: $(2, 0)$



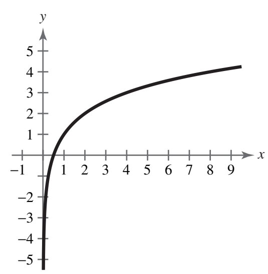
**49.** $y = 1 + \log_2 x$

Domain: $x > 0$

Vertical asymptote: $x = 0$

$$1 + \log_2 x = 0$$
$$\log_2 x = -1$$
$$x = 2^{-1} = \tfrac{1}{2}$$

$x$-intercept: $\left(\tfrac{1}{2}, 0\right)$



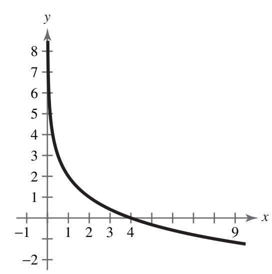
**50.** $y = 2 - \log_2 x$

Domain: $x > 0$

Vertical asymptote: $x = 0$

$$2 - \log_2 x = 0$$
$$\log_2 x = 2$$
$$x = 2^2 = 4$$

$x$-intercept: $(4, 0)$



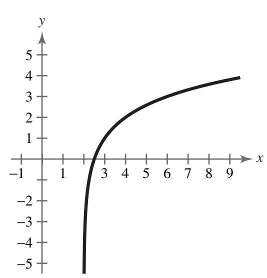
**51.** $y = 1 + \log_2(x - 2)$

Domain: $x - 2 > 0 \implies x > 2$

Vertical asymptote: $x = 2$

$$1 + \log_2(x - 2) = 0$$
$$\log_2(x - 2) = -1$$
$$x - 2 = 2^{-1} = \tfrac{1}{2}$$
$$x = \tfrac{5}{2}$$

$x$-intercept: $\left(\tfrac{5}{2}, 0\right)$



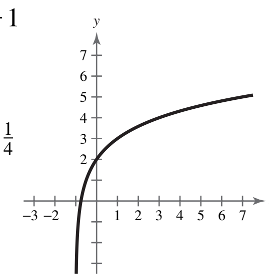
**52.** $y = 2 + \log_2(x + 1)$

Domain: $x + 1 > 0 \implies x > -1$

Vertical asymptote: $x = -1$

$$2 + \log_2(x + 1) = 0$$
$$\log_2(x + 1) = -2$$
$$x + 1 = 2^{-2} = \tfrac{1}{4}$$
$$x = -\tfrac{3}{4}$$

$x$-intercept: $\left(-\tfrac{3}{4}, 0\right)$



**53.** $f(x) = \log_3 x + 2$

Asymptote: $x = 0$

Point on graph: $(1, 2)$

Matches graph (b).

**54.** $f(x) = -\log_3 x$

Asymptote: $x = 0$

Point on graph: $(1, 0)$

Matches graph (c).

**55.** $f(x) = -\log_3(x + 2)$

Asymptote: $x = -2$

Point on graph: $(-1, 0)$

Matches graph (d).

**56.** $f(x) = \log_3(1 - x)$

Asymptote: $x = 1$

Domain: $1 - x > 0 \Rightarrow x < 1$

Point on graph: $(0, 0)$

Matches graph (a).

**57.** $f(x) = \log_{10} x$

$g(x) = -\log_{10} x$ is a reflection in the $x$-axis of the graph of $f$.

**58.** The graph of $g(x) = \log_{10}(x + 7)$ is a horizontal shift 7 units to the left of the graph of $f(x) = \log_{10} x$.

**59.** $f(x) = \log_2 x$

$g(x) = 4 - \log_2 x$ is obtained from $f$ by a reflection in the $x$-axis followed by a vertical shift four units upward.

**60.** The graph of $g(x) = \log_2 x + 3$ is a vertical shift three units upward of the graph of $f(x) = \log_2 x$.

**61.** Horizontal shift three units to the left and a vertical shift two units downward

**62.** Horizontal shift one unit to the right and a vertical shift four units upward

**63.** $\ln\sqrt{42} \approx 1.869$

**64.** $\ln 18.31 \approx 2.907$

**65.** $-\ln\left(\frac{1}{2}\right) \approx 0.693$

**66.** $3 \ln(0.75) \approx -0.863$

**67.** $\ln e^2 = 2$

(Inverse Property)

**68.** $-\ln e = -1$

**69.** $e^{\ln 1.8} = 1.8$

(Inverse Property)

**70.** $7 \ln e^0 = 7 \ln 1$

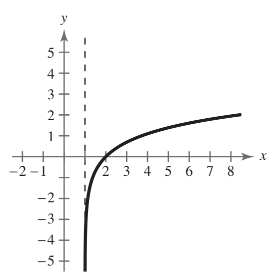
$= 7(0) = 0$

**71.** $f(x) = \ln(x - 1)$

Domain: $x > 1$

Vertical asymptote: $x = 1$

$x$-intercept: $(2, 0)$




**72.** $h(x) = \ln(x + 1)$

Domain: $x + 1 > 0 \Rightarrow x > -1$

The domain is $(-1, \infty)$.

Vertical asymptote: $x + 1 = 0 \Rightarrow x = -1$

$x$-intercept: $\ln(x + 1) = 0$

$e^0 = x + 1$

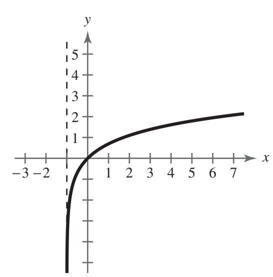
$1 = x + 1$

$0 = x$

The $x$-intercept is $(0, 0)$.

$y = \ln(x + 1) \Rightarrow e^y - 1 = x$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | $-0.39$ | 0 | 1.72 | 6.39 | 19.09 |
| $y$ | $-\frac{1}{2}$ | 0 | 1 | 2 | 3 |



**73.** $g(x) = \ln(-x)$

Domain: $-x > 0 \implies x < 0$

The domain is $(-\infty, 0)$.

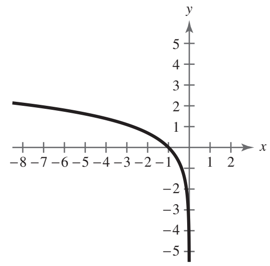
Vertical asymptote: $-x = 0 \implies x = 0$

$x$-intercept: $0 = \ln(-x)$

$$e^0 = -x$$

$$-1 = x$$

The $x$-intercept is $(-1, 0)$.




**74.** $f(x) = \ln(3 - x)$

Domain: $3 - x > 0 \implies x < 3$

The domain is $(-\infty, 3)$.

Vertical asymptote: $3 - x = 0 \implies x = 3$

$x$-intercept: $\ln(3 - x) = 0$

$$e^0 = 3 - x$$

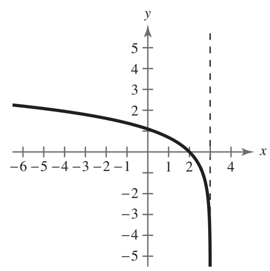
$$1 = 3 - x$$

$$2 = x$$

The $x$-intercept is $(2, 0)$.

$y = \ln(3 - x) \implies x = 3 - e^y$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 2.95 | 2.86 | 2.63 | 2 | 0.28 |
| $y$ | −3 | −2 | −1 | 0 | 1 |




**75.** $g(x) = \ln(x + 3)$ is a horizontal shift three units to the left.

**76.** $g(x) = \ln(x - 4)$ is a horizontal shift four units to the right.

**77.** $g(x) = \ln x - 5$ is a vertical shift five units downward.

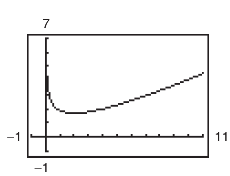
**78.** $g(x) = \ln x + 4$ is a vertical shift four units upward.

**79.** $g(x) = \ln(x - 1) + 2$ is a horizontal shift one unit to the right and a vertical shift two units upward.

**80.** $g(x) = \ln(x + 2) - 5$ is a horizontal shift two units to the left and a vertical shift five units downward.

**81.** $f(x) = \dfrac{x}{2} - \ln \dfrac{x}{4}$

(a)




(b) Domain: $(0, \infty)$

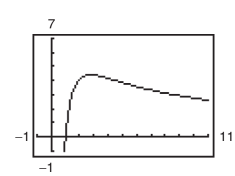
(c) Increasing on $(2, \infty)$

Decreasing on $(0, 2)$

(d) Relative minimum: $(2, 1.693)$

**82.** $g(x) = \dfrac{12 \ln x}{x}$

(a)




(b) Domain: $(0, \infty)$

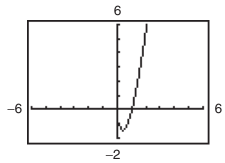
(c) Increasing on $(0, 2.72)$

Decreasing on $(2.72, \infty)$

(d) Relative maximum: $(2.72, 4.41)$

**83.** $h(x) = 4x \ln x$

(a)




(b) Domain: $(0, \infty)$

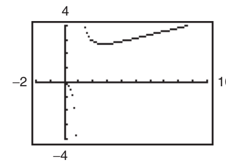
(c) Increasing on $(0.368, \infty)$

Decreasing on $(0, 0.368)$

(d) Relative minimum: $(0.368, -1.472)$

**84.** $f(x) = \dfrac{x}{\ln x}$

(a)




(b) Domain: $(0, 1)\ (1, \infty)$

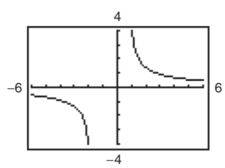
(c) Increasing on $(2.72, \infty)$

Decreasing on $(0, 1)\ (1, 2.72)$

(d) Relative minimum: $(2.72, 2.72)$

**85.** $f(x) = \ln\left(\dfrac{x + 2}{x - 1}\right)$

(a)




(b) $\dfrac{x + 2}{x - 1} > 0$; Critical numbers: $1, -2$

Test intervals: $(-\infty, -2), (-2, 1), (1, \infty)$

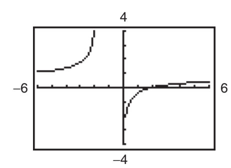
Testing these three intervals, we see that the domain is $(-\infty, -2) \cup (1, \infty)$.

(c) The graph is decreasing on $(-\infty, -2)$ and decreasing on $(1, \infty)$.

(d) There are no relative maximum or minimum values.

**86.** $f(x) = \ln\left(\dfrac{2x}{x + 2}\right)$

(a)




(b) $\dfrac{2x}{x + 2} > 0$; Critical numbers: $0, -2$

Test intervals: $(-\infty, -2), (-2, 0), (0, \infty)$

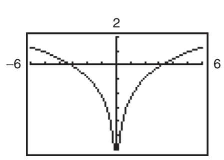
Testing these three intervals, we see that the domain is $(-\infty, -2) \cup (0, \infty)$.

(c) The graph is increasing on $(-\infty, -2)$ and increasing on $(0, \infty)$.

(d) There are no relative maximum or minimum values.

**87.** $f(x) = \ln\left(\dfrac{x^2}{10}\right)$

(a)



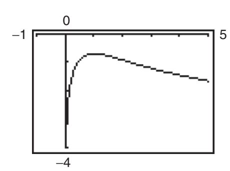

(b) $\dfrac{x^2}{10} > 0 \implies x \neq 0$; Domain: all $x \neq 0$

(c) The graph is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(d) There are no relative maximum or relative minimum values.

**88.** $f(x) = \ln\left(\dfrac{x}{x^2 + 1}\right)$

(a)

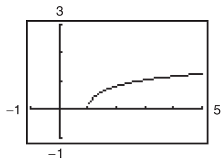



(b) Domain: $x > 0$

(c) The graph is increasing on $(0, 1)$ and decreasing on $(1, \infty)$.

(d) Relative maximum: $(1, -0.693)$

**89.** $f(x) = \sqrt{\ln x}$

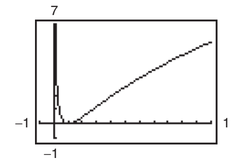
(a) 

(b) $\ln x \geq 0 \implies x \geq 1$; Domain: $x \geq 1$

(c) The graph is increasing on $(1, \infty)$.

(d) There are no relative maximum or relative minimum values.

**90.** $f(x) = (\ln x)^2$

(a) 

(b) Domain: $x > 0$

(c) The graph is decreasing on $(0, 1)$ and increasing on $(1, \infty)$.

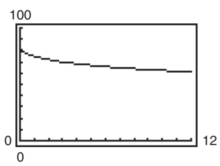
(d) Relative minimum: $(1, 0)$

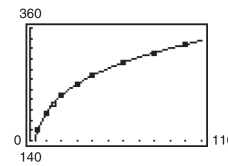
**91.** $f(t) = 80 - 17 \log_{10}(t + 1), \quad 0 \leq t \leq 12$

(a) $f(0) = 80 - 17 \log_{10}(0 + 1) = 80$

(b) $f(4) = 80 - 17 \log_{10}(4 + 1) \approx 68.1$

(c) $f(10) = 80 - 17 \log_{10}(10 + 1) \approx 62.3$

(d) 

**92.** (a) 

The model is a good fit.

(b) $T > 300°\text{F}$ when $p > 67.3$ pounds per square inch

[The graph of $T$ and $y = 300$ intersect at $p = 67.3$.]

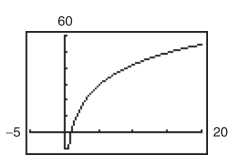
(c) $T(74) = 306.48°\text{F}$

**93.** $t = \dfrac{\ln K}{0.055}$

(a)

| $K$ | 1 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $t$ | 0 | 12.6 | 25.2 | 32.6 | 37.8 | 41.9 | 45.2 |

As the amount increases, the time increases, but at a lesser rate.

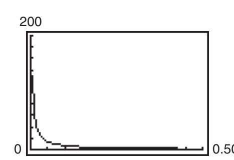
(b) 

**94.** (a)

| $r$ | 0.005 | 0.010 | 0.015 |
|---|---|---|---|
| $t$ | 138.6 yr | 69.3 yr | 46.2 yr |

| $r$ | 0.020 | 0.025 | 0.030 |
|---|---|---|---|
| $t$ | 34.7 yr | 27.7 yr | 23.1 yr |

The doubling time decreases as $r$ increases.

(b) 

**95.** $\beta = 10 \log_{10}\left(\dfrac{I}{10^{-12}}\right)$

(a) $I = 1$: $\beta = 10 \log_{10}\left(\dfrac{1}{10^{-12}}\right) = 10 \cdot \log_{10}(10^{12}) = 10(12) = 120$ decibels

(b) $I = 10^{-2}$: $\beta = 10 \log_{10}\left(\dfrac{10^{-2}}{10^{-12}}\right) = 10 \log_{10}(10^{10}) = 10(10) = 100$ decibels

(c) No, this is a logarithmic scale.

**96.** $t = 16.625 \ln\left(\frac{x}{x - 750}\right), x > 750$

(a) $16.625 \ln\left(\frac{897.72}{897.72 - 750}\right) \approx 30$ years

$16.625 \ln\left(\frac{1659.24}{1659.24 - 750}\right) \approx 10$ years

(b) $(897.72)(30)(12) = 323{,}179.20$

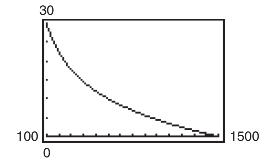
$(1659.24)(10)(12) = 199{,}108.80$

Interest for 30-year loan is
$323{,}179.20 - 150{,}000 = 173{,}179.20.$

Interest for 10-year loan is
$199{,}108.80 - 150{,}000 = 49{,}108.80.$

**97.** $y = 80.4 - 11 \ln x$

$y(300) = 80.4 - 11 \ln 300 \approx 17.66 \text{ ft}^3/\text{min}$




**98.** $y = 80.4 - 11 \ln x, \; 100 \le x \le 1500$

(a) $\frac{\text{450 cubic ft per minute}}{\text{30 children}} = 15$ cubic feet per minute per child

(b) From the graph, for $y = 15$ you get $x \approx 382$ cubic feet.

(c) If ceiling height is 30, then 382 square feet of floor space is needed.

**99.** False. You would reflect $y = 6^x$ in the line $y = x$.

**100.** True. $\log_3(27) = \log_3 3^3 = 3$

**101.** $5 = \log_b 32$

$b^5 = 32 = 2^5$

$b = 2$

**102.** $4 = \log_b 81$

$b^4 = 81 = 3^4$

$b = 3$

**103.** $2 = \log_b\left(\frac{1}{16}\right)$

$b^2 = \frac{1}{16} = \left(\frac{1}{4}\right)^2$

$b = \frac{1}{4}$

**104.** $3 = \log_b\left(\frac{1}{27}\right)$

$b^3 = \frac{1}{27} = \left(\frac{1}{3}\right)^3$

$b = \frac{1}{3}$

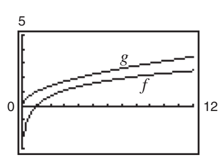
**105.** The vertical asymptote is to the right of the $y$-axis, and the graph increases. Matches (b).

**106.** The vertical asymptote is to the left of the $y$-axis.

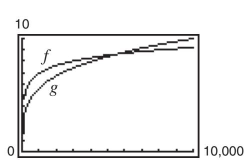
Matches (b).

**107.** $f(x) = \log_a x$ is the inverse of $g(x) = a^x$, where $a > 0, a \neq 1$.

**108.** (a) $f(x) = \ln x, g(x) = \sqrt{x}$




(b) $f(x) = \ln x, g(x) = \sqrt[4]{x}$




The rate of growth of the natural logarithmic function is slower than $g(x) = x^{1/n}$ for any $n$.

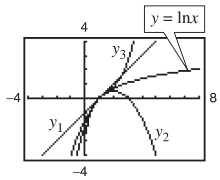
**109.** (a) False, $y$ is not an exponential function of $x$. ($y$ can never be 0.)

(b) True, $y$ could be $\log_2 x$.

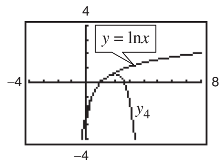
(c) True, $x$ could be $2^y$.

(d) False, $y$ is not linear. (The points are not collinear.)

**110.** (a)




(b) Pattern is $(x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4 + \cdots$.




As you use more terms, the graph better approximates the graph of ln $x$ on the interval $(0, 2)$.

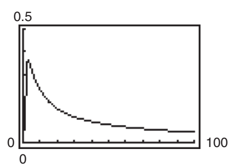
**111.** $f(x) = \dfrac{\ln x}{x}$

(a)

| $x$ | 1 | 5 | 10 | $10^2$ | $10^4$ | $10^6$ |
|---|---|---|---|---|---|---|
| $f(x)$ | 0 | 0.322 | 0.230 | 0.046 | 0.00092 | 0.0000138 |

(b) As $x$ increases without bound, $f(x)$ approaches 0.

(c)




**112.**

$$f(t) = 75 - 6\ln(t + 1)$$
$$60 = 75 - 6\ln(t + 1)$$
$$\ln(t + 1) = \tfrac{15}{6} = \tfrac{5}{2}$$
$$t = e^{5/2} - 1 \approx 11.18$$

Or, you could graph $f(t)$ and $y = 60$ together in the same viewing window, and determine their point of intersection.

**113.** $x^2 + 2x - 3 = (x + 3)(x - 1)$

**114.** $2x^2 + 3x - 5 = (2x + 5)(x - 1)$

**115.** $12x^2 + 5x - 3 = (4x + 3)(3x - 1)$

**116.** $16x^2 + 16x + 7$

$$x = \frac{-16 \pm \sqrt{256 - 448}}{32}$$
$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{4}i$$
$$\left(x + \frac{1}{2} - \frac{\sqrt{3}}{4}i\right)\left(x + \frac{1}{2} + \frac{\sqrt{3}}{4}i\right)$$

**117.** $16x^2 - 25 = (4x + 5)(4x - 5)$

**118.** $36x^2 - 49 = (6x - 7)(6x + 7)$

**119.** $2x^3 + x^2 - 45x = x(2x^2 + x - 45)$
$= x(2x - 9)(x + 5)$

**120.** $3x^3 - 5x^2 - 12x = x(3x^2 - 5x - 12)$
$= x(3x + 4)(x - 3)$

**121.** $(f + g)(2) = f(2) + g(2) = [3(2) + 2] + [2^3 - 1] = 8 + 7 = 15$

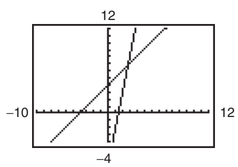
**122.** $(f - g)(-1) = (-1) - (-2) = 1$

**123.** $(fg)(6) = f(6)g(6) = [3(6) + 2][6^3 - 1] = [20][215] = 4300$

**124.** $\left(\frac{f}{g}\right)(0) = \frac{2}{-1} = -2$

**125.** $5x - 7 = x + 4$

The graphs of $y = 5x - 7$ and $y_2 = x + 4$ intersect when $x = 2.75$ or $\frac{11}{4}$.




**126.** $y = -2x + 3$ and $y = 8x$ intersect at $x = 0.3$

**127.** $\sqrt{3x - 2} = 9$

The graphs of $y_1 = \sqrt{3x - 2}$ and $y_2 = 9$ intersect when $x \approx 27.667$ or $\frac{83}{3}$.

**128.** $y = \sqrt{x - 11}$ and $y = x + 2$ do not intersect.

No solution

## Section 3.3 Properties of Logarithms

- You should know the following properties of logarithms.

  (a) $\log_a x = \dfrac{\log_b x}{\log_b a}$

  (b) $\log_a(uv) = \log_a u + \log_a v$ $\qquad \ln(uv) = \ln u + \ln v$

  (c) $\log_a(u/v) = \log_a u - \log_a v$ $\qquad \ln(u/v) = \ln u - \ln v$

  (d) $\log_a u^n = n \log_a u$ $\qquad \ln u^n = n \ln u$

- You should be able to rewrite logarithmic expressions using these properties.

### Vocabulary Check

**1.** change-of-base **2.** $\dfrac{\ln x}{\ln a}$ **3.** $\log_a u^n$ **4.** $\ln u + \ln v$

**1.** (a) $\log_5 x = \dfrac{\log_{10} x}{\log_{10} 5}$

(b) $\log_5 x = \dfrac{\ln x}{\ln 5}$

**2.** (a) $\log_3 x = \dfrac{\log_{10} x}{\log_{10} 3}$

(b) $\log_3 x = \dfrac{\ln x}{\ln 3}$

**3.** (a) $\log_{1/5} x = \dfrac{\log_{10} x}{\log_{10} 1/5} = \dfrac{\log_{10} x}{-\log_{10} 5}$

(b) $\log_{1/5} x = \dfrac{\ln x}{\ln 1/5} = \dfrac{\ln x}{-\ln 5}$

**4.** (a) $\log_{1/3} x = \dfrac{\log_{10} x}{\log_{10}(1/3)} = \dfrac{-\log_{10} x}{\log_{10} 3}$

(b) $\log_{1/3} x = \dfrac{\ln x}{\ln(1/3)} = \dfrac{-\ln x}{\ln 3}$

**5.** (a) $\log_a\left(\dfrac{3}{10}\right) = \dfrac{\log_{10}(3/10)}{\log_{10} a}$

(b) $\log_a\left(\dfrac{3}{10}\right) = \dfrac{\ln(3/10)}{\ln a}$

**6.** (a) $\log_a\left(\dfrac{3}{4}\right) = \dfrac{\log_{10}(3/4)}{\log_{10} a}$

(b) $\log_a\left(\dfrac{3}{4}\right) = \dfrac{\ln(3/4)}{\ln a}$

**7.** (a) $\log_{2.6} x = \dfrac{\log_{10} x}{\log_{10} 2.6}$

(b) $\log_{2.6} x = \dfrac{\ln x}{\ln 2.6}$

**8.** (a) $\log_{7.1} x = \dfrac{\log_{10} x}{\log_{10} 7.1}$

(b) $\log_{7.1} x = \dfrac{\ln x}{\ln 7.1}$

**9.** $\log_3 7 = \dfrac{\ln 7}{\ln 3} \approx 1.771$

**10.** $\log_7 4 = \dfrac{\ln 4}{\ln 7} \approx 0.712$

**11.** $\log_{1/2} 4 = \dfrac{\ln 4}{\ln(1/2)} = -2$

**12.** $\log_{1/8} 64 = \dfrac{\ln 64}{\ln(1/8)}$

$= \dfrac{\ln 8^2}{-\ln 8} = -2$

**13.** $\log_9(0.8) = \dfrac{\ln(0.8)}{\ln 9} \approx -0.102$

**14.** $\log_3(0.015) = \dfrac{\ln(0.015)}{\ln 3}$

$\approx -3.823$

**15.** $\log_{15} 1460 = \dfrac{\ln 1460}{\ln 15} \approx 2.691$

**16.** $\log_{20} 135 = \dfrac{\ln 135}{\ln 20} \approx 1.637$

**17.** $\ln 20 = \ln(4 \cdot 5)$

$= \ln 4 + \ln 5$

**18.** $\ln 500 = \ln(5^3 \cdot 4)$

$= \ln 5^3 + \ln 4$

$= 3 \ln 5 + \ln 4$

**19.** $\ln \frac{5}{64} = \ln 5 - \ln 64$

$= \ln 5 - \ln 4^3$

$= \ln 5 - 3 \ln 4$

**20.** $\ln \frac{2}{5} = \ln 2 - \ln 5$

$= \ln 4^{1/2} - \ln 5$

$= \frac{1}{2} \ln 4 - \ln 5$

**21.** $\log_b 25 = \log_b 5^2$

$= 2 \log_b 5$

$\approx 2(0.8271) \approx 1.6542$

**22.** $\log_b 30 = \log_b(2 \cdot 3 \cdot 5)$

$= \log_b 2 + \log_b 3 + \log_b 5$

$\approx 0.3562 + 0.5646 + 0.8271 \approx 1.7479$

**23.** $\log_b \sqrt{3} = \frac{1}{2} \log_b 3$

$\approx \frac{1}{2}(0.5646)$

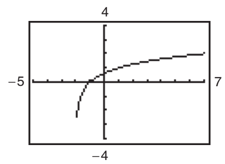
$\approx 0.2823$

**24.** $\log_b\left(\frac{25}{9}\right) = \log_b 5^2 - \log_b 3^2$

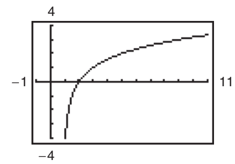
$= 2 \log_b 5 - 2 \log_b 3$

$\approx 2(0.8271) - 2(0.5646) \approx 0.5250$

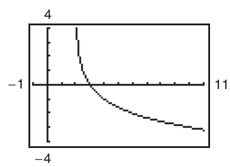
**25.** $f(x) = \log_3(x + 2) = \dfrac{\ln(x + 2)}{\ln 3}$



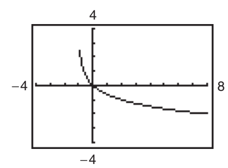

**26.** $f(x) = \log_2(x - 1) = \dfrac{\ln(x - 1)}{\ln 2}$



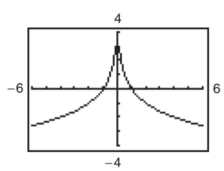

**27.** $f(x) = \log_{1/2}(x - 2) = \dfrac{\ln(x - 2)}{\ln(1/2)} = \dfrac{\ln(x - 2)}{-\ln 2}$



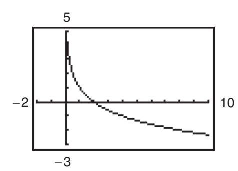

**28.** $f(x) = \log_{1/3}(x + 1) = \dfrac{\ln(x + 1)}{\ln(1/3)} = \dfrac{\ln(x + 1)}{-\ln 3}$




**29.** $f(x) = \log_{1/4}(x^2) = \dfrac{\ln x^2}{\ln(1/4)} = \dfrac{\ln x^2}{-\ln 4}$




**30.** $f(x) = \log_{1/2}\left(\dfrac{x}{2}\right) = \dfrac{\ln(x/2)}{\ln(1/2)} = \dfrac{\ln(x/2)}{-\ln 2}$




**31.** $\log_4 8 = \log_4 2^3 = 3 \log_4 2$

$= 3 \log_4 4^{1/2} = 3\left(\frac{1}{2}\right) \log_4 4$

$= \frac{3}{2}$

**32.** $\log_2(4^2 \cdot 3^4) = \log_2 4^2 + \log_2 3^4$

$= 2 \log_2 4 + 4 \log_2 3$

$= 2 \log_2 2^2 + 4 \log_2 3$

$= 4 \log_2 2 + 4 \log_2 3$

$= 4 + 4 \log_2 3$

**33.** $\ln(5e^6) = \ln 5 + \ln e^6 = \ln 5 + 6 = 6 + \ln 5$

**34.** $\ln \dfrac{6}{e^2} = \ln 6 - \ln e^2$

$= \ln 6 - 2 \ln e = \ln 6 - 2$

**35.** $\log_5 \frac{1}{250} = \log_5 1 - \log_5 250 = 0 - \log_5(125 \cdot 2)$

$= -\log_5(5^3 \cdot 2) = -[\log_5 5^3 + \log_5 2]$

$= -[3 \log_5 5 + \log_5 2] = -3 - \log_5 2$

**36.** $-\ln 24 = -\ln(2^3 \cdot 3)$

$= -\ln 2^3 - \ln 3$

$= -3 \ln 2 - \ln 3$

$= -(3 \ln 2 + \ln 3)$

**37.** $\log_{10} 5x = \log_{10} 5 + \log_{10} x$

**38.** $\log_{10} 10z = \log_{10} 10 + \log_{10} z = 1 + \log_{10} z$

**39.** $\log_{10} \dfrac{5}{x} = \log_{10} 5 - \log_{10} x$

**40.** $\log_{10} \dfrac{y}{2} = \log_{10} y - \log_{10} 2$

**41.** $\log_8 x^4 = 4 \log_8 x$

**42.** $\log_6 z^{-3} = -3 \log_6 z$

**43.** $\ln\sqrt{z} = \ln z^{1/2} = \frac{1}{2}\ln z$

**44.** $\ln\sqrt[3]{t} = \ln t^{1/3} = \frac{1}{3}\ln t$

**45.** $\ln xyz = \ln x + \ln y + \ln z$

**46.** $\ln\dfrac{xy}{z} = \ln x + \ln y - \ln z$

**47.** $\log_3(a^2bc^3) = \log_3 a^2 + \log_3 b + \log_3 c^3$
$= 2\log_3 a + \log_3 b + 3\log_3 c$

**48.** $\log_5(x^3y^3z) = \log_5 x^3 + \log_5 y^3 + \log_5 z$
$= 3\log_5 x + 3\log_5 y + \log_5 z$

**49.** $\ln\left(a^2\sqrt{a-1}\right) = \ln a^2 + \ln(a-1)^{1/2}$
$= 2\ln a + \frac{1}{2}\ln(a-1),\ a > 1$

**50.** $\ln[z(z-1)^2] = \ln z + \ln(z-1)^2$
$= \ln z + 2\ln(z-1)$

**51.** $\ln\sqrt[3]{\dfrac{x}{y}} = \dfrac{1}{3}\ln\dfrac{x}{y}$
$= \dfrac{1}{3}[\ln x - \ln y]$
$= \dfrac{1}{3}\ln x - \dfrac{1}{3}\ln y$

**52.** $\ln\sqrt{\dfrac{x^2}{y^3}} = \ln\left(\dfrac{x^2}{y^3}\right)^{1/2} = \dfrac{1}{2}\ln\left(\dfrac{x^2}{y^3}\right)$
$= \dfrac{1}{2}(\ln x^2 - \ln y^3)$
$= \dfrac{1}{2}(2\ln x - 3\ln y)$
$= \ln x - \dfrac{3}{2}\ln y$

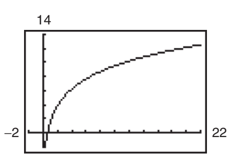
**53.** $\ln\left(\dfrac{x^2-1}{x^3}\right) = \ln(x^2-1) - \ln x^3$
$= \ln[(x-1)(x+1)] - 3\ln x$
$= \ln(x-1) + \ln(x+1) - 3\ln x,\ x > 1$

**54.** $\ln\left(\dfrac{x}{\sqrt{x^2+1}}\right) = \ln x - \ln\sqrt{x^2+1}$
$= \ln x - \ln(x^2+1)^{1/2}$
$= \ln x - \dfrac{1}{2}\ln(x^2+1)$

**55.** $\ln\left(\dfrac{x^4\sqrt{y}}{z^5}\right) = \ln x^4\sqrt{y} - \ln z^5$
$= \ln x^4 + \ln\sqrt{y} - \ln z^5$
$= 4\ln x + \dfrac{1}{2}\ln y - 5\ln z$

**56.** $\log_b \dfrac{\sqrt{x}y^4}{z^4} = \log_b \sqrt{x}y^4 - \log_b z^4$
$= \log_b x^{1/2} + \log_b y^4 - \log_b z^4$
$= \dfrac{1}{2}\log_b x + 4\log_b y - 4\log_b z$

**57.** $y_1 = \ln[x^3(x+4)]$
$y_2 = 3\ln x + \ln(x+4)$

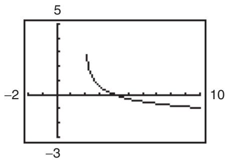
(a)



(b)

| $x$ | 0.5 | 1 | 1.5 | 2 | 3 | 10 |
|---|---|---|---|---|---|---|
| $y_1$ | −0.5754 | 1.6094 | 2.9211 | 3.8712 | 5.2417 | 9.5468 |
| $y_2$ | −0.5754 | 1.6094 | 2.9211 | 3.8712 | 5.2417 | 9.5468 |

(c) The graphs and table suggest that $y_1 = y_2$ for $x > 0$. In fact,
$y_1 = \ln[x^3(x+4)] = \ln x^3 + \ln(x+4)$
$= 3\ln x + \ln(x+4) = y_2.$

**58.** $y_1 = \ln\left(\frac{\sqrt{x}}{x-2}\right), y_2 = \frac{1}{2}\ln x - \ln(x-2)$

(a) 


(b)

| $x$ | 3 | 4 | 5 | 6 | 10 | 20 |
|---|---|---|---|---|---|---|
| $y_1$ | 0.5493 | 0 | −0.2939 | −0.4904 | −0.9281 | −1.393 |
| $y_2$ | 0.5493 | 0 | −0.2939 | −0.4904 | −0.9281 | −1.393 |

(c) The graphs and table suggest that $y_1 = y_2$.

In fact,

$$y_1 = \ln\left(\frac{\sqrt{x}}{x-2}\right) = \ln x^{1/2} - \ln(x-2) = \frac{1}{2}\ln x - \ln(x-2) = y_2.$$

**59.** $\ln x + \ln 4 = \ln 4x$

**60.** $\ln y + \ln z = \ln yz$

**61.** $\log_4 z - \log_4 y = \log_4 \frac{z}{y}$

**62.** $\log_5 8 - \log_5 t = \log_5 \frac{8}{t}$

**63.** $2\log_2(x+3) = \log_2(x+3)^2$

**64.** $\frac{5}{2}\log_7(z-4) = \log_7(z-4)^{5/2}$

**65.** $\frac{1}{2}\ln(x^2+4) = \ln(x^2+4)^{1/2}$
$= \ln\sqrt{x^2+4}$

**66.** $2\ln x + \ln(x+1) = \ln x^2 + \ln(x+1)$
$= \ln(x^2(x+1))$
$= \ln(x^3 + x^2)$

**67.** $\ln x - 3\ln(x+1) = \ln x - \ln(x+1)^3$
$= \ln\frac{x}{(x+1)^3}$

**68.** $\ln x - 2\ln(x+2) = \ln x - \ln(x+2)^2$
$= \ln\frac{x}{(x+2)^2}$

**69.** $\ln(x-2) - \ln(x+2) = \ln\left(\frac{x-2}{x+2}\right)$

**70.** $3\ln x + 2\ln y - 4\ln z = \ln x^3 + \ln y^2 - \ln z^4$
$= \ln x^3y^2 - \ln z^4$
$= \ln\frac{x^3y^2}{z^4}$

**71.** $\ln x - 2[\ln(x+2) + \ln(x-2)] = \ln x - 2\ln[(x+2)(x-2)]$
$= \ln x - 2\ln(x^2-4)$
$= \ln x - \ln(x^2-4)^2$
$= \ln\frac{x}{(x^2-4)^2}$

**72.** $4[\ln z + \ln(z+5)] - 2\ln(z-5) = 4[\ln z(x+5)] - \ln(z-5)^2$
$= \ln[z(z+5)]^4 - \ln(z-5)^2$
$= \ln\frac{z^4(z+5)^4}{(z-5)^2}$

**73.** $\frac{1}{3}[2\ln(x+3) + \ln x - \ln(x^2-1)] = \frac{1}{3}[\ln(x+3)^2 + \ln x - \ln(x^2-1)]$

$$= \frac{1}{3}[\ln[x(x+3)^2] - \ln(x^2-1)]$$

$$= \frac{1}{3}\ln\frac{x(x+3)^2}{x^2-1}$$

$$= \ln\sqrt[3]{\frac{x(x+3)^2}{x^2-1}}$$

**74.** $2[\ln x - \ln(x+1) - \ln(x-1)] = 2\left[\ln\frac{x}{x+1} - \ln(x-1)\right]$

$$= 2\left[\ln\frac{x}{(x+1)(x-1)}\right]$$

$$= 2\left[\ln\frac{x}{x^2-1}\right]$$

$$= \ln\left(\frac{x}{x^2-1}\right)^2$$

**75.** $\frac{1}{3}[\ln y + 2\ln(y+4)] - \ln(y-1) = \frac{1}{3}[\ln y + \ln(y+4)^2] - \ln(y-1)$

$$= \frac{1}{3}\ln[y(y+4)^2] - \ln(y-1)$$

$$= \ln\sqrt[3]{y(y+4)^2} - \ln(y-1)$$

$$= \ln\frac{\sqrt[3]{y(y+4)^2}}{y-1}$$

**76.** $\frac{1}{2}[\ln(x+1) + 2\ln(x-1)] + 3\ln x = \frac{1}{2}[\ln(x+1) + \ln(x-1)^2] + \ln x^3$

$$= \frac{1}{2}[\ln(x+1)(x-1)^2] + \ln x^3$$

$$= \ln[(x+1)(x-1)^2]^{1/2} + \ln x^3$$

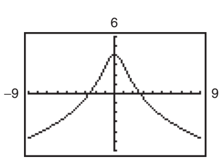
$$= \ln[(x+1)^{1/2}(x-1)] + \ln x^3$$

$$= \ln\left[x^3(x-1)\sqrt{x+1}\right]$$

**77.** $y_1 = 2[\ln 8 - \ln(x^2+1)]$

$y_2 = \ln\left[\frac{64}{(x^2+1)^2}\right]$

(a)




(c) The graphs and table suggest that $y_1 = y_2$. In fact,

$$y_1 = 2[\ln 8 - \ln(x^2+1)]$$

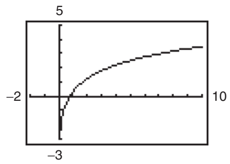
$$= 2\ln\frac{8}{x^2+1} = \ln\frac{64}{(x^2+1)^2} = y_2.$$

(b)

| $x$ | $-8$ | $-4$ | $-2$ | 0 | 2 | 4 | 8 |
|---|---|---|---|---|---|---|---|
| $y_1$ | $-4.1899$ | $-1.5075$ | 0.9400 | 4.1589 | 0.9400 | $-1.5075$ | $-4.1899$ |
| $y_2$ | $-4.1899$ | $-1.5075$ | 0.9400 | 4.1589 | 0.9400 | $-1.5075$ | $-4.1899$ |

**78.** $y_1 = \ln x + \frac{1}{2}\ln(x + 1),\ y_2 = \ln\left(x\sqrt{x + 1}\right),\ x > 0$

(a)




(b)

| $x$ | 0 | 1 | 2 | 5 | 10 |
|---|---|---|---|---|---|
| $y_1$ | ERROR | 0.34657 | 1.2425 | 2.5053 | 3.5015 |
| $y_2$ | ERROR | 0.34657 | 1.2425 | 2.5053 | 3.5015 |

(c) The graphs and table suggest that $y_1 = y_2$.

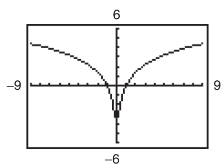
In fact,

$$y_1 = \ln x + \tfrac{1}{2}\ln(x + 1) = \ln x + \ln(x + 1)^{1/2} = \ln\left[x\sqrt{x + 1}\right] = y_2.$$

**79.** $y_1 = \ln x^2$

$y_2 = 2 \ln x$

(a)




(The domain of $y_2$ is $x > 0$.)

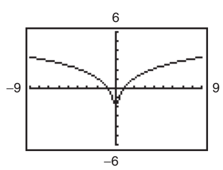
(b)

| $x$ | $-8$ | $-4$ | 1 | 2 | 4 |
|---|---|---|---|---|---|
| $y_1$ | 4.1589 | 2.7726 | 0 | 1.3863 | 2.7726 |
| $y_2$ | undefined | undefined | 0 | 1.3863 | 2.7726 |

(c) The graphs and table suggest that $y_1 = y_2$ for $x > 0$. The functions are not equivalent because the domains are different.

**80.** $y_1 = \frac{1}{4}\ln[x^4(x^2 + 1)],\ y_2 = \ln x + \frac{1}{4}\ln(x^2 + 1)$

(a)




(b)

| $x$ | $-10$ | $-1$ | 0 | 1 | 5 | 10 |
|---|---|---|---|---|---|---|
| $y_1$ | 3.4564 | 0.17329 | ERROR | 0.17329 | 2.4240 | 3.4564 |
| $y_2$ | ERROR | ERROR | ERROR | 0.17329 | 2.4240 | 3.4564 |

(c) No, the expressions are not equivalent. The domain of $y_1$ is all $x \neq 0$, whereas the domain of $y_2$ is $x > 0$.

**81.** $\log_3 9 = 2 \log_3 3 = 2$

**82.** $\log_6 \sqrt[3]{6} = \log_6 6^{1/3} = \frac{1}{3}\log_6 6 = \frac{1}{3}(1) = \frac{1}{3}$

**83.** $\log_4 16^{3.4} = 3.4 \log_4(4^2) = 6.8 \log_4 4 = 6.8$

**84.** $\log_5 \frac{1}{125} = \log_5 5^{-3} = -3 \log_5 5 = -3(1) = -3$

**85.** $\log_2(-4)$ is undefined. $-4$ is not in the domain of $f(x) = \log_2 x$.

**86.** $\log_4(-16)$ is undefined because $-16$ is not in the domain of $\log_4 x$.

**87.** $\log_5 75 - \log_5 3 = \log_5 \frac{75}{3} = \log_5 25 = \log_5 5^2 = 2$

**88.**
$$\begin{aligned}\log_4 2 + \log_4 32 &= \log_4 4^{1/2} + \log_4 4^{5/2}\\ &= \tfrac{1}{2}\log_4 4 + \tfrac{5}{2}\log_4 4\\ &= \tfrac{1}{2}(1) + \tfrac{5}{2}(1)\\ &= 3\end{aligned}$$

**89.** $\ln e^3 - \ln e^7 = 3 - 7 = -4$

**90.** $\ln e^6 - 2 \ln e^5 = 6 \ln e - 10 \ln e = 6 - 10 = -4$

**91.** $2 \ln e^4 = 2(4) \ln e = 8$

**92.** $\ln e^{4.5} = 4.5 \ln e = 4.5$

**93.** $\ln\left(\frac{1}{\sqrt{e}}\right) = \ln(1) - \ln e^{1/2} = 0 - \frac{1}{2}\ln e = -\frac{1}{2}$

**94.** $\ln \sqrt[5]{e^3} = \ln e^{3/5} = \frac{3}{5}\ln e = \frac{3}{5}$

**95.** (a) $\beta = 10 \cdot \log_{10}\left(\frac{I}{10^{-12}}\right) = 10[\log_{10} I - \log_{10} 10^{-12}]$

$= 10[\log_{10} I - (-12)\log_{10} 10]$

$= 10[\log_{10} I + 12] = 120 + 10 \cdot \log_{10} I$

(b)

| $I$ | $10^{-4}$ | $10^{-6}$ | $10^{-8}$ | $10^{-10}$ | $10^{-12}$ | $10^{-14}$ |
|---|---|---|---|---|---|---|
| $\beta$ | 80 | 60 | 40 | 20 | 0 | $-20$ |

(c) $\beta(10^{-4}) = 120 + 10 \cdot \log_{10} 10^{-4} = 120 - 40 = 80$

$\beta(10^{-6}) = 120 + 10 \cdot \log_{10} 10^{-6} = 120 - 60 = 60$

$\beta(10^{-8}) = 120 + 10 \cdot \log_{10} 10^{-8} = 120 - 80 = 40$

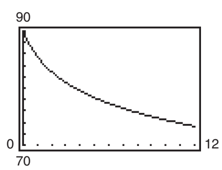
$\beta(10^{-10}) = 120 + 10 \cdot \log_{10} 10^{-10} = 120 - 100 = 20$

$\beta(10^{-12}) = 120 + 10 \cdot \log_{10} 10^{-12} = 120 - 120 = 0$

$\beta(10^{-14}) = 120 + 10 \cdot \log_{10} 10^{-14} = 120 - 140 = -20$

**96.** $f(t) = 90 - 15\log_{10}(t + 1), 0 \le t \le 12$

(a)



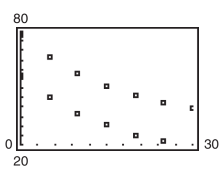

(b) When $t = 0, f(0) = 90$.

(c) $f(6) \approx 77$

(d) $f(12) \approx 73$

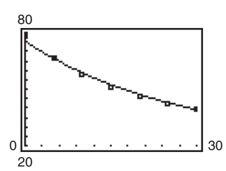
(e) $f(t) = 75$ when $t = 9$ months.

**97.** (a)




(b) $T - 21 = 54.4(0.964)^t$

$T = 21 + 54.4(0.964)^t$




The data $(t, T - 21)$ fits the model

$T - 21 = 54.4(0.964)^t$.

The model

$T = 21 + 54.4(0.964)^t$

fits the original data.

**—CONTINUED—**

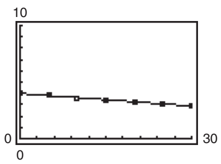
**97. —CONTINUED—**

(c) $\ln(T - 21) = -0.0372t + 3.9971$, linear model

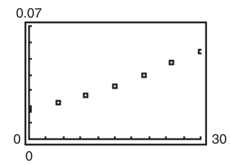
$$T - 21 = e^{-0.0372t + 3.9971}$$

$$T = 21 + 54.4e^{-0.0372t}$$

$$= 21 + 54.4(0.964)^t$$



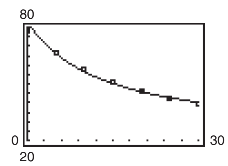

(d)




$$\frac{1}{T - 21} = 0.00121t + 0.01615, \quad \text{linear model}$$

$$T - 21 = \frac{1}{0.00121t + 0.01615}$$

$$T = 21 + \frac{1}{0.00121t + 0.01615}$$




**98.** If $y = ab^x$, then $\ln y = \ln(ab^x) = \ln a + x \ln b$, which is linear. If $y = \dfrac{1}{cx + d}$, then $\dfrac{1}{y} = cx + d$.

**99.** True

**100.** False. For example, let $x = 2$ and $a = 1$.
Then $f(x - a) = \ln(2 - 1) = 0$, but
$f(x) - f(a) = \ln(2) - \ln 1 = \ln 2$.

**101.** False. For example, let $x = 1$ and $a = 2$.
Then $f\left(\dfrac{x}{a}\right) = \ln\left(\dfrac{1}{2}\right)$. But $\dfrac{f(x)}{f(a)} = \dfrac{\ln 1}{\ln 2} = 0$.

**102.** False. For example, let $x = 1$ and $a = 1$.
Then $f(x + a) = \ln(1 + 1) = \ln 2$, but
$f(x)f(a) = (\ln 1)(\ln 1) = 0$.

**103.** False. $\sqrt{\ln x} \neq \frac{1}{2} \ln x$
In fact, $\ln x^{1/2} = \frac{1}{2} \ln x$.

**104.** False. For example, let $n = 2$ and $x = e$.
Then $[f(x)]^n = [\ln e]^2 = 1$, but
$nf(x) = 2 \ln e = 2$.

**105.** True. In fact, if $\ln x < 0$, then $0 < x < 1$.

**106.** False. For example, let $x = \sqrt{e}$.
Then $f(x) = \ln\sqrt{e} = \frac{1}{2}\ln e = \frac{1}{2} > 0$, but $\sqrt{e} < e$.

**107.** Let $y = \log_a x$ and $z = \log_{a/b} x$, then $a^y = x = \left(\dfrac{a}{b}\right)^z$ and

$$\left(\frac{1}{b}\right)^z = a^{y-z}$$

$$\frac{1}{b} = a^{(y-z)/z}$$

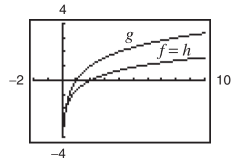
$$\log_a\left(\frac{1}{b}\right) = \frac{y - z}{z} = \frac{y}{z} - 1 \implies 1 + \log_a\left(\frac{1}{b}\right) = \frac{\log_a x}{\log_{a/b} x}.$$

**108.** $f(x) = \ln \frac{x}{2}$

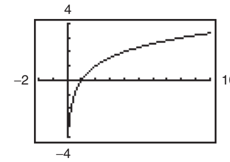
$g(x) = \frac{\ln x}{\ln 2}$

$h(x) = \ln x - \ln 2$

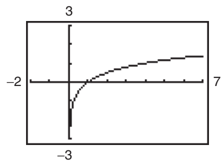
$f(x) = h(x)$ by Property 2.



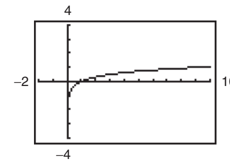

**109.** $f(x) = \log_2 x = \frac{\ln x}{\ln 2}$



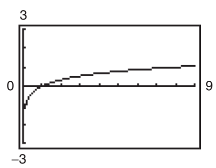

**110.** $f(x) = \log_4 x = \frac{\ln x}{\ln 4}$



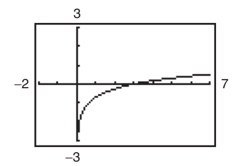

**111.** $f(x) = \log_3 \sqrt{x} = \frac{1}{2}\frac{\ln x}{\ln 3}$



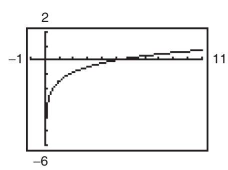

**112.** $f(x) = \log_2 \sqrt[3]{x} = \frac{1}{3}\frac{\ln x}{\ln 2}$




**113.** $f(x) = \log_5\left(\frac{x}{3}\right) = \frac{\ln(x/3)}{\ln 5}$




**114.** $f(x) = \log_3 \frac{x}{5} = \frac{\ln x - \ln 5}{\ln 3}$




**115.** $\ln 1 = 0, \ln 2 \approx 0.6931, \ln 3 \approx 1.0986, \ln 5 \approx 1.6094$

$\ln 2 \approx 0.6931$

$\ln 3 \approx 1.0986$

$\ln 4 = \ln 2 + \ln 2 \approx 0.6931 + 0.6931 = 1.3862$

$\ln 5 \approx 1.6094$

$\ln 6 = \ln 2 + \ln 3 \approx 0.6931 + 1.0986 = 1.7917$

$\ln 8 = \ln 2^3 = 3 \ln 2 \approx 3(0.6931) = 2.0793$

$\ln 9 = \ln 3^2 = 2 \ln 3 \approx 2(1.0986) = 2.1972$

$\ln 10 = \ln 5 + \ln 2 \approx 1.6094 + 0.6931 = 2.3025$

$\ln 12 = \ln 2^2 + \ln 3 = 2 \ln 2 + \ln 3 \approx 2(0.6931) + 1.0986 = 2.4848$

$\ln 15 = \ln 5 + \ln 3 \approx 1.6094 + 1.0986 = 2.7080$

$\ln 16 = \ln 2^4 = 4 \ln 2 \approx 4(0.6931) = 2.7724$

$\ln 18 = \ln 3^2 + \ln 2 = 2 \ln 3 + \ln 2 \approx 2(1.0986) + 0.6931 = 2.8903$

$\ln 20 = \ln 5 + \ln 2^2 = \ln 5 + 2 \ln 2 \approx 1.6094 + 2(0.6931) = 2.9956$

**116.** $\dfrac{24xy^{-2}}{16x^{-3}y} = \dfrac{24xx^3}{16yy^2} = \dfrac{3x^4}{2y^3}$

**117.** $\left(\dfrac{2x^2}{3y}\right)^{-3} = \left(\dfrac{3y}{2x^2}\right)^3$

$= \dfrac{(3y)^3}{(2x^2)^3}$

$= \dfrac{27y^3}{8x^6}$

**118.** $(18x^3y^4)^{-3}(18x^3y^4)^3 = \dfrac{(18x^3y^4)^3}{(18x^3y^4)^3} = 1$ if $x \neq 0, y \neq 0$

**119.** $xy(x^{-1} + y^{-1})^{-1} = \dfrac{xy}{x^{-1} + y^{-1}}$

$= \dfrac{xy}{\dfrac{1}{x} + \dfrac{1}{y}}$

$= \dfrac{xy}{\dfrac{y + x}{xy}}$

$= \dfrac{(xy)^2}{x + y}, x \neq 0, y \neq 0$

**120.** $x^2 - 6x + 2 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 4(2)}}{2} = 3 \pm \sqrt{7}$

**121.** $2x^3 + 20x^2 + 50x = 0$

$2x(x^2 + 10x + 25) = 0$

$2x(x + 5)^2 = 0$

$x = 0, -5, -5$

**122.** $x^4 - 19x^2 + 48 = 0$

$(x^2 - 16)(x^2 - 3) = 0$

$(x - 4)(x + 4)(x - \sqrt{3})(x + \sqrt{3}) = 0$

$x = \pm 4, \pm\sqrt{3}$

**123.** $9x^4 - 37x^2 + 4 = 0$

$(x^2 - 4)(9x^2 - 1) = 0$

$(x - 2)(x + 2)(3x - 1)(3x + 1) = 0$

$x = \pm 2, \pm\frac{1}{3}$

**124.** $x^3 - 6x^2 - 4x + 24 = 0$

$x^2(x - 6) - 4(x - 6) = 0$

$(x^2 - 4)(x - 6) = 0$

$(x - 2)(x + 2)(x - 6) = 0$

$x = 2, -2, 6$

**125.** $9x^4 - 226x^2 + 25 = 0$

$(x^2 - 25)(9x^2 - 1) = 0$

$(x - 5)(x + 5)(3x + 1)(3x - 1) = 0$

$x = \pm 5, \pm\frac{1}{3}$

# Section 3.4 Solving Exponential and Logarithmic Equations

- To solve an exponential equation, isolate the exponential expression, then take the logarithm of both sides. Then solve for the variable.
  1. $\log_a a^x = x$
  2. $\ln e^x = x$
- To solve a logarithmic equation, rewrite it in exponential form. Then solve for the variable.
  1. $a^{\log_a x} = x$
  2. $e^{\ln x} = x$
- If $a > 0$ and $a \neq 1$ we have the following:
  1. $\log_a x = \log_a y \implies x = y$
  2. $a^x = a^y \implies x = y$
- Use your graphing utility to approximate solutions.

## Vocabulary Check

**1.** solve

**2.** (a) $x = y$ (b) $x = y$ (c) $x$ (d) $x$

**3.** extraneous

**1.** $4^{2x-7} = 64$

(a) $x = 5$

$4^{2(5)-7} = 4^3 = 64$

Yes, $x = 5$ is a solution.

(b) $x = 2$

$4^{2(2)-7} = 4^{-3} = \frac{1}{64} \neq 64$

No, $x = 2$ is not a solution.

**2.** $2^{3x+1} = 32$

(a) $x = -1$

$2^{3(-1)+1} = 2^{-2} = \frac{1}{4}$

No, $x = -1$ is not a solution.

(b) $x = 2$

$2^{3(2)+1} = 2^7 = 128$

No, $x = 2$ is not a solution.

**3.** $3e^{x+2} = 75$

(a) $x = -2 + e^{25}$

$3e^{(-2+e^{25})+2} = 3e^{e^{25}} \neq 75$

No, $x = -2 + e^{25}$ is not a solution.

(b) $x = -2 + \ln 25$

$3e^{(-2+\ln 25)+2} = 3e^{\ln 25} = 3(25) = 75$

Yes, $x = -2 + \ln 25$ is a solution.

(c) $x \approx 1.2189$

$3e^{1.2189+2} = 3e^{3.2189} \approx 75$

Yes, $x \approx 1.2189$ is a solution.

**4.** $4e^{x-1} = 60$

(a) $x = 1 + \ln 15$

$4e^{(1+\ln 15)-1} = 4e^{\ln 15} = 4(15) = 60$

Yes

(b) $x \approx 3.7081$

$4e^{3.7081-1} = 4e^{2.7081} \approx 60$

Yes

(c) $x = \ln 16$

$4e^{\ln 16 - 1} \approx 23.5 \neq 60$

No

**5.** $\log_4(3x) = 3$

$4^3 = 3x$

$x = \frac{64}{3} \approx 21.333$

(a) $x = 21.3560$ is an approximate solution.

(b) No, $x = -4$ is not a solution.

(c) Yes, $x = \frac{64}{3}$ is a solution.

**6.** $\log_6\left(\frac{5}{3}x\right) = 2 \Leftrightarrow \frac{\ln\left(\frac{5}{3}x\right)}{\ln 6} = 2$

(a) $x \approx 20.2882;\ \frac{\ln\left(\frac{5}{3} \cdot 20.2882\right)}{\ln 6} = 1.965 \neq 2;$ No

(b) $x = \frac{108}{5};\ \log_6\left(\frac{5}{3} \cdot \frac{108}{5}\right) = \log_6(36) = 2;$ Yes

(c) $x = 7.2;\ \frac{\ln\left(\frac{5}{3}(7.2)\right)}{\ln 6} \approx 1.3869 \neq 2;$ No

**7.** $\ln(x - 1) = 3.8$

(a) $x = 1 + e^{3.8}$

$\ln(1 + e^{3.8} - 1) = \ln e^{3.8} = 3.8$

Yes, $x = 1 + e^{3.8}$ is a solution.

(b) $x \approx 45.7012$

$\ln(45.7012 - 1) = \ln(44.7012) \approx 3.8$

Yes, $x \approx 45.7012$ is a solution.

(c) $x = 1 + \ln 3.8$

$\ln(1 + \ln 3.8 - 1) = \ln(\ln 3.8) \approx 0.289$

No, $x = 1 + \ln 3.8$ is not a solution.

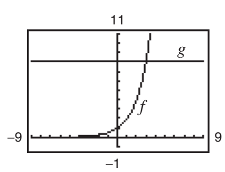
**8.** $\ln(2 + x) = 2.5$

(a) $x = e^{2.5} - 2;\ \ln(2 + e^{2.5} - 2) = \ln e^{2.5} = 2.5;$ Yes

(b) $x \approx \frac{4073}{400};\ \ln\left(2 + \frac{4073}{400}\right) \approx 2.5;$ Yes

(c) $x = \frac{1}{2};\ \ln\left(2 + \frac{1}{2}\right) \approx 0.9163 \neq 2.5;$ No

**9.**




Point of intersection: (3, 8)

Algebraically: $2^x = 8$

$2^x = 2^3$

$x = 3 \implies y = 8 \implies (3, 8)$

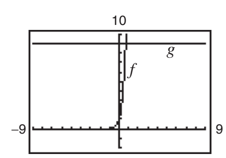
**10.** $f(x) = g(x)$

$27^x = 9$

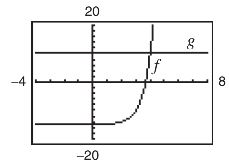
$27^x = 27^{2/3}$

$x = \frac{2}{3}$

Point of intersection: $\left(\frac{2}{3}, 9\right)$



**11.** Point of intersection: $(4, 10)$




Algebraically: $5^{x-2} - 15 = 10$

$$5^{x-2} = 25 = 5^2$$

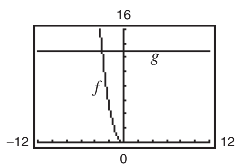
$$x - 2 = 2$$

$$x = 4$$

$(4, 10)$

**12.** $f(x) = 2^{-x+1} - 3$

$g(x) = 13$




Point of intersection: $(-3, 13)$

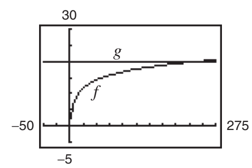
$2^{-x+1} - 3 = 13$

$2^{-x+1} = 16 = 2^4$

$-x + 1 = 4$

$x = -3 \implies (-3, 13)$

**13.**




Point of intersection: $(243, 20)$

Algebraically: $4 \log_3 x = 20$

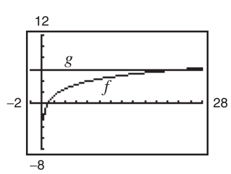
$$\log_3 x = 5$$

$$x = 3^5 = 243$$

$(243, 20)$

**14.** $f(x) = 3 \log_5 x = 3 \cdot \dfrac{\ln x}{\ln 5}$

$g(x) = 6$



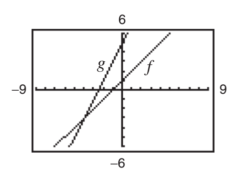

Point of intersection: $(25, 6)$

$3 \log_5 x = 6$

$\log_5 x = 2$

$x = 5^2 = 25 \implies (25, 6)$

**15.**




Point of intersection: $(-4, -3)$

Algebraically: $\ln e^{x+1} = 2x + 5$

$$x + 1 = 2x + 5$$

$$-4 = x$$

$(-4, -3)$

**16.** $f(x) = \ln e^{x-2} = x - 2$

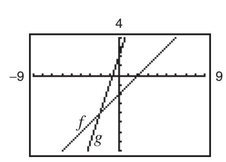
$g(x) = 3x + 2$

Point of intersection: $(-2, -4)$

$x - 2 = 3x + 2$

$-4 = 2x$

$x = -2 \implies (-2, -4)$



**17.** $4^x = 16$

$4^x = 4^2$

$x = 2$

**18.** $3^x = 243$

$3^x = 3^5$

$x = 5$

**19.** $5^x = \dfrac{1}{625}$

$5^x = \dfrac{1}{5^4} = 5^{-4}$

$x = -4$

**20.** $7^x = \dfrac{1}{49}$

$7^x = 7^{-2}$

$x = -2$

**21.** $\left(\frac{1}{8}\right)^x = 64$

$8^{-x} = 8^2$

$-x = 2$

$x = -2$

**22.** $\left(\frac{1}{2}\right)^x = 32$

$\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{-5}$

$x = -5$

**23.** $\left(\frac{2}{3}\right)^x = \frac{81}{16}$

$\left(\frac{3}{2}\right)^{-x} = \left(\frac{3}{2}\right)^4$

$-x = 4$

$x = -4$

**24.** $\left(\frac{3}{4}\right)^x = \frac{27}{64}$

$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^3$

$x = 3$

**25.** $6(10^x) = 216$

$10^x = 36$

$\log_{10} 10^x = \log_{10} 36$

$x = \log_{10} 36 \approx 1.5563$

**26.** $5(8^x) = 325$

$8^x = 65$

$x = \log_8 65$

$= \dfrac{\ln 65}{\ln 8} \approx 2.0075$

**27.** $2^{x+3} = 256$

$2^x \cdot 2^3 = 256$

$2^x = 32$

$x = 5$

*Alternate solution:*

$2^{x+3} = 2^8$

$x + 3 = 8$

$x = 5$

**28.** $3^{x-1} = \frac{1}{81}$

$3^{x-1} = 3^{-4}$

$x - 1 = -4$

$x = -3$

**29.** $\ln x - \ln 5 = 0$

$\ln x = \ln 5$

$x = 5$

**30.** $\ln x - \ln 2 = 0$

$\ln x = \ln 2$

$x = 2$

**31.** $\ln x = -7$

$x = e^{-7}$

**32.** $\ln x = -1$

$e^{-1} = x$

$x = \dfrac{1}{e} \approx 0.368$

**33.** $\log_x 625 = 4$

$x^4 = 625$

$x^4 = 5^4$

$x = 5$

**34.** $\log_x 25 = 2$

$x^2 = 25$

$x = 5$

**35.** $\log_{10} x = -1$

$x = 10^{-1}$

$x = \frac{1}{10}$

**36.** $\log_{10} x = -\dfrac{1}{2}$

$x = 10^{-1/2} = \dfrac{1}{\sqrt{10}}$

$\approx 0.316$

**37.** $\ln(2x - 1) = 5$

$2x - 1 = e^5$

$x = \dfrac{1 + e^5}{2} \approx 74.707$

**38.** $\ln(3x + 5) = 8$

$e^8 = 3x + 5$

$x = \dfrac{1}{3}(e^8 - 5)$

$\approx 991.986$

**39.** $\ln e^{x^2} = x^2 \ln e^x = x^2$

**40.** $\ln e^{2x-1} = 2x - 1$

**41.** $e^{\ln(5x+2)} = 5x + 2$

**42.** $e^{\ln x^2} = x^2$

**43.** $-1 + \ln e^{2x} = -1 + 2x = 2x - 1$

**44.** $-8 + e^{\ln x^3} = -8 + x^3 = x^3 - 8$

**45.**
$$\begin{aligned} 8^{3x} &= 360 \\ \ln 8^{3x} &= \ln 360 \\ 3x \ln 8 &= \ln 360 \\ 3x &= \frac{\ln 360}{\ln 8} \\ x &= \frac{1}{3}\frac{\ln 360}{\ln 8} \\ x &\approx 0.944 \end{aligned}$$

**46.**
$$\begin{aligned} 6^{5x} &= 3000 \\ \ln 6^{5x} &= \ln 3000 \\ (5x)\ln 6 &= \ln 3000 \\ 5x &= \frac{\ln 3000}{\ln 6} \\ x &= \frac{\ln 3000}{5\ln 6} \approx 0.894 \end{aligned}$$

**47.**
$$\begin{aligned} 5^{-t/2} &= 0.20 = \frac{1}{5} \\ -\frac{t}{2}\ln 5 &= \ln\left(\frac{1}{5}\right) \\ -\frac{t}{2}\ln 5 &= -\ln 5 \\ \frac{t}{2} &= 1 \\ t &= 2 \end{aligned}$$

**48.**
$$\begin{aligned} 4^{-3t} &= 0.10 \\ \ln 4^{-3t} &= \ln 0.10 \\ (-3t)\ln 4 &= \ln 0.10 \\ -3t &= \frac{\ln 0.10}{\ln 4} \\ t &= -\frac{\ln 0.10}{3\ln 4} \approx 0.554 \end{aligned}$$

**49.**
$$\begin{aligned} 5(2^{3-x}) - 13 &= 100 \\ 5(2^{3-x}) &= 113 \\ 2^{3-x} &= \frac{113}{5} \\ \ln 2^{3-x} &= \ln\left(\frac{113}{5}\right) \\ 3 - x &= \frac{\ln(113/5)}{\ln 2} \\ x &= 3 - \frac{\ln(113/5)}{\ln 2} \\ x &\approx -1.498 \end{aligned}$$

**50.**
$$\begin{aligned} 6(8^{-2-x}) + 15 &= 2601 \\ 6(8^{-2-x}) &= 2586 \\ 8^{-2-x} &= 431 \\ (-2 - x)\ln 8 &= \ln 431 \\ -2 - x &= \frac{\ln 431}{\ln 8} \\ x &= -2 - \frac{\ln 431}{\ln 8} \\ x &\approx -4.917 \end{aligned}$$

**51.**
$$\begin{aligned} \left(1 + \frac{0.10}{12}\right)^{12t} &= 2 \\ \left(\frac{12.1}{12}\right)^{12t} &= 2 \\ (12t)\ln\left(\frac{12.1}{12}\right) &= \ln 2 \\ t &= \frac{1}{12}\frac{\ln 2}{\ln(12.1/12)} \\ t &\approx 6.960 \end{aligned}$$

**52.**
$$\begin{aligned} \left(16 + \frac{0.878}{26}\right)^{3t} &= 30 \\ 3t\ln\left(16 + \frac{0.878}{26}\right) &= \ln 30 \\ t &= \frac{\ln 30}{3\ln\left(16 + \frac{0.878}{26}\right)} \\ &\approx \frac{3.4012}{8.3241} \approx 0.409 \end{aligned}$$

**53.**
$$\begin{aligned} 5000\left[\frac{(1 + 0.005)^x}{0.005}\right] &= 250{,}000 \\ 5000(1.005)^x &= 1250 \\ 1.005^x &= 0.25 \\ x\ln(1.005) &= \ln 0.25 \\ x &= \frac{\ln 0.25}{\ln(1.005)} \\ x &\approx -277.951 \end{aligned}$$

**54.**
$$\begin{aligned} 250\left[\frac{(1 + 0.01)^x}{0.01}\right] &= 150{,}000 \\ 250(1.01)^x &= 1500 \\ 1.01^x &= 6 \\ x\ln 1.01 &= \ln 6 \\ x &= \frac{\ln 6}{\ln 1.01} \\ x &\approx 180.070 \end{aligned}$$

**55.** $2e^{5x} = 18$
$e^{5x} = 9$
$5x = \ln 9$
$x = \frac{1}{5}\ln 9$
$x \approx 0.439$

**56.** $4e^{2x} = 40$
$e^{2x} = 10$
$2x = \ln 10$
$x = \frac{1}{2}\ln 10 \approx 1.151$

**57.** $500e^{-x} = 300$
$e^{-x} = \frac{3}{5}$
$-x = \ln\frac{3}{5}$
$x = -\ln\frac{3}{5} = \ln\frac{5}{3} \approx 0.511$

**58.** $1000e^{-4x} = 75$
$e^{-4x} = \frac{3}{40}$
$\ln e^{-4x} = \ln\frac{3}{40}$
$-4x = \ln\frac{3}{40}$
$x = -\frac{1}{4}\ln\frac{3}{40} \approx 0.648$

**59.** $7 - 2e^x = 5$
$-2e^x = -2$
$e^x = 1$
$x = \ln 1 = 0$

**60.** $-14 + 3e^x = 11$
$3e^x = 25$
$e^x = \frac{25}{3}$
$\ln e^x = \ln\frac{25}{3}$
$x = \ln\frac{25}{3} \approx 2.120$

**61.** $e^{2x} - 4e^x - 5 = 0$
$(e^x - 5)(e^x + 1) = 0$
$e^x = 5$ or $e^x = -1$
$x = \ln 5 \approx 1.609$
($e^x = -1$ is impossible.)

**62.** $e^{2x} - 5e^x + 6 = 0$
$(e^x - 2)(e^x - 3) = 0$
$e^x = 2$ or $e^x = 3$
$x = \ln 2 \approx 0.693$ or
$x = \ln 3 \approx 1.099$

**63.** $250e^{0.02x} = 10{,}000$
$e^{0.02x} = 40$
$0.02x = \ln 40$
$x = \dfrac{\ln 40}{0.02}$
$x \approx 184.444$

**64.** $100e^{0.005x} = 125{,}000$
$e^{0.005x} = 1250$
$0.005x = \ln 1250$
$x = \dfrac{\ln 1250}{0.005}$
$x \approx 1426.180$

**65.** $e^x = e^{x^2-2}$
$x = x^2 - 2$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2, -1$

**66.** $e^{2x} = e^{x^2-8}$
$2x = x^2 - 8$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x = 4, -2$

**67.** $e^{x^2-3x} = e^{x-2}$
$x^2 - 3x = x - 2$
$x^2 - 4x + 2 = 0$
$x = \dfrac{4 \pm \sqrt{16-8}}{2}$
$x = 2 \pm \sqrt{2}$
$x \approx 3.414, 0.586$

**68.** $e^{-x^2} = e^{x^2-2x}$
$-x^2 = x^2 - 2x$
$2x^2 - 2x = 0$
$2x(x - 1) = 0$
$x = 0, 1$

**69.** $\dfrac{400}{1 + e^{-x}} = 350$
$1 + e^{-x} = \dfrac{400}{350} = \dfrac{8}{7}$
$e^{-x} = \dfrac{1}{7}$
$-x = \ln\left(\dfrac{1}{7}\right) = -\ln 7$
$x = \ln 7 \approx 1.946$

**70.** $\dfrac{525}{1 + e^{-x}} = 275$

$1 + e^{-x} = \dfrac{525}{275}$

$e^{-x} = \dfrac{525}{275} - 1 = \dfrac{250}{275} = \dfrac{10}{11}$

$-x = \ln \dfrac{10}{11}$

$x = -\ln \dfrac{10}{11} = \ln \dfrac{11}{10} \approx 0.095$

**71.** $\dfrac{40}{1 - 5e^{-0.01x}} = 200$

$1 - 5e^{-0.01x} = \dfrac{40}{200} = \dfrac{1}{5}$

$5e^{-0.01x} = \dfrac{4}{5}$

$e^{-0.01x} = \dfrac{4}{25}$

$-0.01x = \ln\left(\dfrac{4}{25}\right)$

$x = \dfrac{\ln(4/25)}{-0.01}$

$x \approx 183.258$

**72.** $\dfrac{50}{1 - 2e^{-0.001x}} = 1000$

$1 - 2e^{-0.001x} = \dfrac{50}{1000} = 0.05$

$2e^{-0.001x} = 0.95$

$e^{-0.001x} = 0.475$

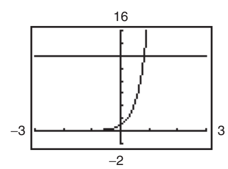
$-0.001x = \ln 0.475$

$x = \dfrac{\ln 0.475}{-0.001}$

$x \approx 744.440$

**73.** $e^{3x} = 12$

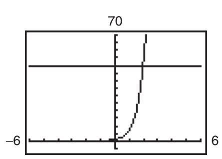
| $x$ | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|
| $f(x)$ | 6.05 | 8.17 | 11.02 | 14.88 | 20.09 |




$x \approx 0.828$

**74.** $e^{2x} = 50$

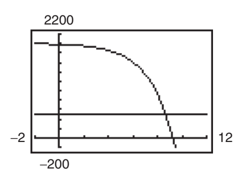
| $x$ | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|---|
| $e^{2x}$ | 24.53 | 29.96 | 36.60 | 44.70 | 54.60 |




$x \approx 1.956$

**75.** $20(100 - e^{x/2}) = 500$

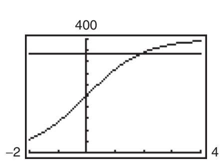
| $x$ | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $f(x)$ | 1756 | 1598 | 1338 | 908 | 200 |




$x \approx 8.635$

**76.** $\dfrac{400}{1 + e^{-x}} = 350$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $\dfrac{400}{1 + e^{-x}}$ | 200 | 292 | 352 | 381 | 393 |




$x \approx 1.946$

**77.** $\left(1 + \frac{0.065}{365}\right)^{365t} = 4 \implies t = 21.330$

**78.** $\left(4 - \frac{2.471}{40}\right)^{9t} = 21$

$3.938225^{9t} = 21$

The zero of $y = 3.938225^{9t} - 21$ is $t \approx 0.247$.

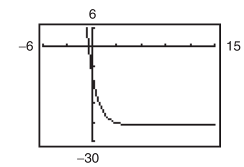
**79.** $\frac{3000}{2 + e^{2x}} = 2$

The zero of $y = \frac{3000}{2 + e^{2x}} - 2$ is $x \approx 3.656$.

**80.** $\frac{119}{e^{6x} - 14} = 7$

The zero of $y = \frac{119}{e^{6x} - 14} - 7$ is $x \approx 0.572$.

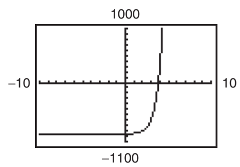
**81.** $g(x) = 6e^{1-x} - 25$



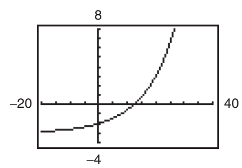

Zero at $x = -0.427$

**82.** $f(x) = 3e^{3x/2} - 962$

The zero is $x \approx 3.847$.



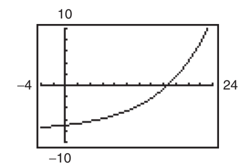

**83.** $g(t) = e^{0.09t} - 3$




Zero at $t = 12.207$

**84.** $h(t) = e^{0.125t} - 8$

The zero is $t \approx 16.636$.




**85.** $\ln x = -3$

$x = e^{-3} \approx 0.050$

**86.** $\ln x = -2$

$x = e^{-2} = \frac{1}{e^2} \approx 0.135$

**87.** $\ln 4x = 2.1$

$4x = e^{2.1}$

$x = \frac{1}{4}e^{2.1} \approx 2.042$

**88.** $\ln 2x = 1.5$

$e^{1.5} = 2x$

$x = \frac{1}{2}e^{1.5} \approx 2.241$

**89.** $-2 + 2\ln 3x = 17$

$2\ln 3x = 19$

$\ln 3x = \frac{19}{2}$

$3x = e^{19/2}$

$x = \frac{1}{3}e^{19/2}$

$x \approx 4453.242$

**90.** $3 + 2\ln x = 10$

$2\ln x = 7$

$\ln x = \frac{7}{2} = 3.5$

$x = e^{3.5} \approx 33.115$

**91.** $\log_5(3x + 2) = \log_5(6 - x)$

$3x + 2 = 6 - x$

$4x = 4$

$x = 1$

**92.** $\log_9(4 + x) = \log_9(2x - 1)$

$4 + x = 2x - 1$

$x = 5$

**93.** $\log_{10}(z - 3) = 2$

$z - 3 = 10^2$

$z = 10^2 + 3$

$= 103$

**94.** $\log_{10} x^2 = 6$

$x^2 = 10^6$

$x = \pm\sqrt{10^6} = \pm 1000$

**95.** $7 \log_4(0.6x) = 12$

$\log_4(0.6x) = \frac{12}{7}$

$4^{12/7} = 0.6x = \frac{3}{5}x$

$x = \frac{5}{3} 4^{12/7}$

$\approx 17.945$

**96.** $4 \log_{10}(x - 6) = 11$

$\log_{10}(x - 6) = \frac{11}{4}$

$10^{11/4} = x - 6$

$x = 6 + 10^{11/4}$

$\approx 568.341$

**97.** $\ln\sqrt{x + 2} = 1$

$\sqrt{x + 2} = e^1$

$x + 2 = e^2$

$x = e^2 - 2 \approx 5.389$

**98.** $\ln\sqrt{x - 8} = 5$

$\frac{1}{2}\ln(x - 8) = 5$

$\ln(x - 8) = 10$

$e^{10} = x - 8$

$x = 8 + e^{10}$

$\approx 22{,}034.466$

**99.** $\ln(x + 1)^2 = 2$

$e^{\ln(x+1)^2} = e^2$

$(x + 1)^2 = e^2$

$x + 1 = e$ or $x + 1 = -e$

$x = e - 1 \approx 1.718$

or

$x = -e - 1 \approx -3.718$

**100.** $\ln(x^2 + 1) = 8$

$e^8 = x^2 + 1$

$x = \pm\sqrt{e^8 - 1} = \pm 54.589$

**101.** $\log_4 x - \log_4(x - 1) = \frac{1}{2}$

$\log_4\left(\frac{x}{x - 1}\right) = \frac{1}{2}$

$4^{\log_4(x/x-1)} = 4^{1/2}$

$\frac{x}{x - 1} = 2$

$x = 2(x - 1)$

$x = 2x - 2$

$2 = x$

**102.** $\log_3 x + \log_3(x - 8) = 2$

$\log_3[x(x - 8)] = 2$

$3^2 = x(x - 8) = x^2 - 8x$

$x^2 - 8x - 9 = 0$

$(x - 9)(x + 1) = 0$

$x = 9$

($x = -1$ is extraneous.)

**103.** $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

$\ln(x + 5) = \ln\left(\frac{x - 1}{x + 1}\right)$

$x + 5 = \frac{x - 1}{x + 1}$

$(x + 5)(x + 1) = x - 1$

$x^2 + 6x + 5 = x - 1$

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

$x = -2$ or $x = -3$

Both of these solutions are extraneous, so the equation has no solution.

**104.** $\ln(x + 1) - \ln(x - 2) = \ln x, \ (x > 2)$

$$\ln\left(\frac{x+1}{x-2}\right) = \ln x$$

$$\frac{x+1}{x-2} = x$$

$$x + 1 = x(x - 2) = x^2 - 2x$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}$$

Taking the positive solution, $x = \dfrac{3 + \sqrt{13}}{2} \approx 3.303$.

**105.** $\log_{10} 8x - \log_{10}(1 + \sqrt{x}) = 2$

$$\log_{10} \frac{8x}{1 + \sqrt{x}} = 2$$

$$\frac{8x}{1 + \sqrt{x}} = 10^2$$

$$8x = 100 + 100\sqrt{x}$$

$$8x - 100\sqrt{x} - 100 = 0$$

$$2x - 25\sqrt{x} - 25 = 0$$

$$\sqrt{x} = \frac{25 \pm \sqrt{25^2 - 4(2)(-25)}}{4}$$

$$= \frac{25 \pm 5\sqrt{33}}{4}$$

Choosing the positive value, we have $\sqrt{x} \approx 13.431$ and $x \approx 180.384$.

**106.** $\log_{10} 4x - \log_{10}(12 + \sqrt{x}) = 2, (x > 0)$

$$\log_{10} \frac{4x}{12 + \sqrt{x}} = 2$$

$$\frac{4x}{12 + \sqrt{x}} = 10^2 = 100$$

$$4x = 1200 + 100\sqrt{x}$$

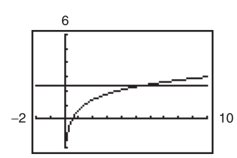
$$x - 25\sqrt{x} - 300 = 0, \text{ Quadratic in } \sqrt{x}$$

$$\sqrt{x} = \frac{25 \pm \sqrt{(-25)^2 - 4(-300)}}{2} = \frac{25 \pm \sqrt{1825}}{2}$$

Taking the positive root and squaring, $x \approx 1146.5$.

**107.** $\ln 2x = 2.4$

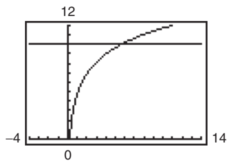
| $x$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $f(x)$ | 1.39 | 1.79 | 2.08 | 2.30 | 2.48 |




$x \approx 5.512$

**108.**

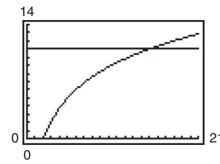
| $x$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $f(x)$ | 8.99 | 9.66 | 10.20 | 10.67 | 11.07 |




$x \approx 5.606$

**109.** $6 \log_3(0.5x) = 11$

| $x$ | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| $f(x)$ | 9.79 | 10.22 | 10.63 | 11.00 | 11.36 |

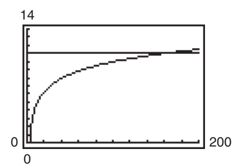



$x \approx 14.988$

**110.**

| $x$ | 150 | 155 | 160 | 165 | 170 |
|---|---|---|---|---|---|
| $f(x)$ | 10.85 | 10.92 | 10.99 | 11.06 | 11.13 |

$x \approx 160.489$




**111.** $\log_{10} x = x^3 - 3$

Graphing $y = \log_{10} x - x^3 + 3$, you obtain two zeros, $x \approx 1.469$ and $x \approx 0.001$.

**112.** Solving $y = \log_{10} x^2 - 4 = 0$, $x = \pm 100$.

**113.** $\ln x + \ln(x - 2) = 1$

Graphing $y = \ln x + \ln(x - 2) - 1$, you obtain one zero, $x \approx 2.928$.

**114.** $\ln x + \ln(x + 1) = 2$

Graphing $y = \ln x + \ln(x + 1) - 2$, you obtain one zero, $x \approx 2.264$.

**115.** $\ln(x - 3) + \ln(x + 3) = 1$

Graphing $y = \ln(x - 3) + \ln(x + 3) - 1$, you obtain $x \approx 3.423$.

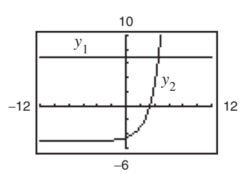
**116.** $\ln x + \ln(x^2 + 4) = 10$

Solving $y = \ln x + \ln(x^2 + 4) - 10 = 0$, $x = 27.984$.

**117.** $y_1 = 7$

$y_2 = 2^{x-1} - 5$

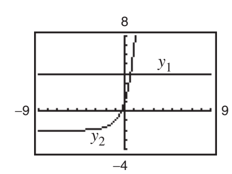
Intersection: $(4.585, 7)$




**118.** $y_1 = 4$

$y_2 = 3^{x+1} - 2$

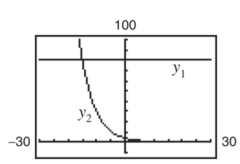
The graphs intersect at $(x, y) \approx (0.631, 4)$.




**119.** $y_1 = 80$

$y_2 = 4e^{-0.2x}$

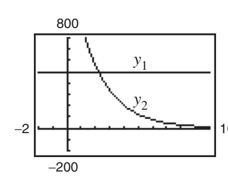
Intersection: $(-14.979, 80)$




**120.** $y_1 = 500$

$y_2 = 1500e^{-x/2}$

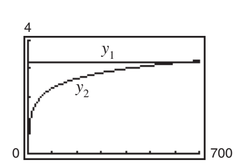
From the graph, we have $(x, y) \approx (2.197, 500)$.




**121.** $y_1 = 3.25$

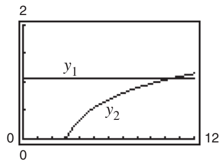
$y_2 = \frac{1}{2} \ln(x + 2)$

Intersection: $(663.142, 3.25)$



**122.** $y_1 = 1.05$

$y_2 = \ln\sqrt{x-2} = \frac{1}{2}\ln(x-2)$




The graphs intersect at (10.166, 1.05).

**123.**
$$2x^2e^{2x} + 2xe^{2x} = 0$$
$$(2x^2 + 2x)e^{2x} = 0$$
$$2x^2 + 2x = 0 \quad (\text{since } e^{2x} \neq 0)$$
$$2x(x+1) = 0$$
$$x = 0, -1$$

**124.**
$$-x^2e^{-x} + 2xe^{-x} = 0$$
$$(-x^2 + 2x)e^{-x} = 0$$
$$-x^2 + 2x = 0 \quad (\text{since } e^{-x} \neq 0)$$
$$-x(x-2) = 0$$
$$x = 0, 2$$

**125.**
$$-xe^{-x} + e^{-x} = 0$$
$$(-x+1)e^{-x} = 0$$
$$-x + 1 = 0 \quad (\text{since } e^{-x} \neq 0)$$
$$x = 1$$

**126.**
$$e^{-2x} - 2xe^{-2x} = 0$$
$$(1 - 2x)e^{-2x} = 0$$
$$1 - 2x = 0 \quad (\text{since } e^{-2x} \neq 0)$$
$$x = \frac{1}{2}$$

**127.**
$$2x\ln x + x = 0$$
$$x(2\ln x + 1) = 0$$
$$2\ln x + 1 = 0 \quad (\text{since } x > 0)$$
$$\ln x = -\frac{1}{2}$$
$$x = e^{-1/2} \approx 0.607$$

**128.**
$$\frac{1 - \ln x}{x^2} = 0$$
$$1 - \ln x = 0 \quad (x > 0)$$
$$\ln x = 1$$
$$x = e \approx 2.718$$

**129.**
$$\frac{1 + \ln x}{2} = 0$$
$$1 + \ln x = 0$$
$$\ln x = -1$$
$$x = e^{-1} = \frac{1}{e} \approx 0.368$$

**130.**
$$2x\ln\left(\frac{1}{x}\right) - x = 0$$
$$x\left(2\ln\left(\frac{1}{x}\right) - 1\right) = 0$$
$$2\ln\left(\frac{1}{x}\right) - 1 = 0 \quad (\text{since } x > 0)$$
$$\ln\left(\frac{1}{x}\right) = \frac{1}{2}$$
$$\frac{1}{x} = e^{1/2}$$
$$x = e^{-1/2} \approx 0.607$$

**131.** (a)
$$2000 = 1000e^{0.075t}$$
$$2 = e^{0.075t}$$
$$\ln 2 = 0.075t$$
$$t = \frac{\ln 2}{0.075} \approx 9.24 \text{ years}$$

(b)
$$3000 = 1000e^{0.075t}$$
$$3 = e^{0.075t}$$
$$\ln 3 = 0.075t$$
$$t = \frac{\ln 3}{0.075} \approx 14.65 \text{ years}$$

**132.** (a)
$$\begin{aligned} 2000 &= 1000e^{0.06t} \\ 2 &= e^{0.06t} \\ \ln 2 &= 0.06t \\ t &= \frac{\ln 2}{0.06} \approx 11.55 \text{ years} \end{aligned}$$

(b)
$$\begin{aligned} 3000 &= 1000e^{0.06t} \\ 3 &= e^{0.06t} \\ \ln 3 &= 0.06t \\ t &= \frac{\ln 3}{0.06} \approx 18.31 \text{ years} \end{aligned}$$

**133.** (a)
$$\begin{aligned} 2000 &= 1000e^{0.025t} \\ 2 &= e^{0.025t} \\ \ln 2 &= 0.025t \\ t &= \frac{\ln 2}{0.025} \approx 27.73 \text{ years} \end{aligned}$$

(b)
$$\begin{aligned} 3000 &= 1000e^{0.025t} \\ 3 &= e^{0.025t} \\ \ln 3 &= 0.025t \\ t &= \frac{\ln 3}{0.025} \approx 43.94 \text{ years} \end{aligned}$$

**134.** (a)
$$\begin{aligned} 2000 &= 1000e^{0.0375t} \\ 2 &= e^{0.0375t} \\ \ln 2 &= 0.0375t \\ t &= \frac{\ln 2}{0.0375} \approx 18.48 \text{ years} \end{aligned}$$

(b)
$$\begin{aligned} 3000 &= 1000e^{0.0375t} \\ 3 &= e^{0.0375t} \\ \ln 3 &= 0.0375t \\ t &= \frac{\ln 3}{0.0375} \approx 29.30 \text{ years} \end{aligned}$$

**135.** $p = 500 - 0.5(e^{0.004x})$

(a)
$$\begin{aligned} p &= 350 \\ 350 &= 500 - 0.5(e^{0.004x}) \\ 300 &= e^{0.004x} \\ 0.004x &= \ln 300 \\ x &\approx 1426 \text{ units} \end{aligned}$$

(b)
$$\begin{aligned} p &= 300 \\ 300 &= 500 - 0.5(e^{0.004x}) \\ 400 &= e^{0.004x} \\ 0.004x &= \ln 400 \\ x &\approx 1498 \text{ units} \end{aligned}$$

**136.** $p = 5000\left(1 - \dfrac{4}{4 + e^{-0.002x}}\right)$

(a) When $p = \$600$:
$$\begin{aligned} 600 &= 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right) \\ 0.12 &= 1 - \frac{4}{4 + e^{-0.002x}} \\ \frac{4}{4 + e^{-0.002x}} &= 0.88 \\ 4 &= 3.52 + 0.88e^{-0.002x} \\ 0.48 &= 0.88e^{-0.002x} \\ \frac{6}{11} &= e^{-0.002x} \\ \ln \frac{6}{11} &= \ln e^{-0.002x} \\ \ln \frac{6}{11} &= -0.002x \\ x &= \frac{\ln(6/11)}{-0.002} \approx 303 \text{ units} \end{aligned}$$

(b) When $p = \$400$:
$$\begin{aligned} 400 &= 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right) \\ 0.08 &= 1 - \frac{4}{4 + e^{-0.002x}} \\ \frac{4}{4 + e^{-0.002x}} &= 0.92 \\ 4 &= 3.68 + 0.92e^{-0.002x} \\ 0.32 &= 0.92e^{-0.002x} \\ \frac{8}{23} &= e^{-0.002x} \\ \ln \frac{8}{23} &= \ln e^{-0.002x} \\ x &= \frac{\ln(8/23)}{-0.002} \approx 528 \text{ units} \end{aligned}$$

**137.** $7247 - 596.5 \ln t = 5800$

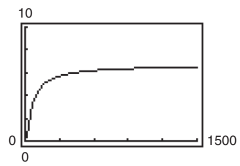
$$-596.5 \ln t = -1447$$

$$\ln t \approx 2.4258$$

$$t \approx 11.3, \text{ or } 2001$$

**138.** $V = 6.7e^{-48.1/t}, \; t > 0$

(a)




(b) As $t \to \infty$, $V \to 6.7$.

Horizontal asymptote: $y = 6.7$

The yield will approach 6.7 million cubic feet per acre.

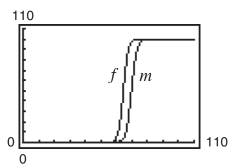
(c) $1.3 = 6.7e^{-48.1/t}$

$$\frac{1.3}{6.7} = e^{-48.1/t}$$

$$\ln \frac{13}{67} = \frac{-48.1}{t}$$

$$t = \frac{-48.1}{\ln(13/67)} \approx 29.3 \text{ years}$$

**139.** (a)




(b) From the graph we see horizontal asymptotes at $y = 0$ and $y = 100$. These represent the lower and upper percent bounds.

(c) Males: $50 = \dfrac{100}{1 + e^{-0.6114(x-69.71)}}$

$$1 + e^{-0.6114(x-69.71)} = 2$$

$$e^{-0.6114(x-69.71)} = 1$$

$$-0.6114(x - 69.71) = \ln 1$$

$$-0.6114(x - 69.71) = 0$$

$$x = 69.71 \text{ inches}$$

Females: $50 = \dfrac{100}{1 + e^{-0.66607(x-64.51)}}$

$$1 + e^{-0.66607(x-64.51)} = 2$$

$$e^{-0.66607(x-64.51)} = 1$$

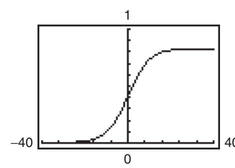
$$-0.66607(x - 64.51) = \ln 1$$

$$-0.66607(x - 64.51) = 0$$

$$x = 64.51 \text{ inches}$$

**140.** $P = \dfrac{0.83}{1 + e^{-0.2n}}$

(a)




(b) Horizontal asymptotes: $y = 0, y = 0.83$
The upper asymptote, $y = 0.83$, indicates that the proportion of correct responses will approach 0.83 as the number of trials increases.

(c) When $P = 60\%$ or $P = 0.60$:

$$0.60 = \frac{0.83}{1 + e^{-0.2n}}$$

$$1 + e^{-0.2n} = \frac{0.83}{0.60}$$

$$e^{-0.2n} = \frac{0.83}{0.60} - 1$$

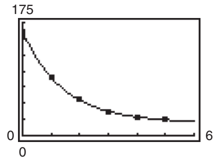
$$\ln e^{-0.2n} = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$-0.2n = \ln\left(\frac{0.83}{0.60} - 1\right)$$

$$n = -\frac{\ln\left(\frac{0.83}{0.60} - 1\right)}{0.2} \approx 5 \text{ trials}$$

**141.** $T = 20[1 + 7(2^{-h})]$

(a)

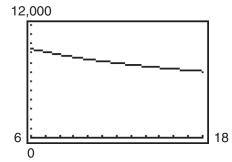



(b) We see a horizontal asymptote at $y = 20$. This represents the room temperature.

(c)
$$100 = 20[1 + 7(2^{-h})]$$
$$5 = 1 + 7(2^{-h})$$
$$4 = 7(2^{-h})$$
$$\frac{4}{7} = 2^{-h}$$
$$\ln\left(\frac{4}{7}\right) = \ln 2^{-h}$$
$$\ln\left(\frac{4}{7}\right) = -h \ln 2$$
$$\frac{\ln(4/7)}{-\ln 2} = h$$
$$h \approx 0.81 \text{ hour}$$

**142.** (a)
$$13{,}387 - 2190.5 \ln t = 7250$$
$$2190.5 \ln t = 6137$$
$$\ln t = 2.8016$$
$$t \approx 16.5, \text{ or } 2006$$

(b)




(c) Let $y_1 = 13{,}387 - 2190.5 \ln t$ and $y_2 = 7250$. The graphs of $y_1$ and $y_2$ intersect at $t \approx 16.5$.

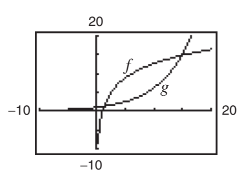
**143.** False. The equation $e^x = 0$ has no solutions.

**144.** False. A logarithmic equation can have any number of extraneous solutions. For example $\ln(2x - 1) + \ln(x + 2) = \ln(x^2 - x - 5)$ has two extraneous solutions, $x = -1$ and $x = -3$.

**145.** Answers will vary.

**146.** $f(x) = \log_a x, g(x) = a^x, a > 1.$

(a) $a = 1.2$




The curves intersect twice: $(1.258, 1.258)$ and $(14.767, 14.767)$

(b) If $f(x) = \log_a x = a^x = g(x)$ intersect exactly once, then

$x = \log_a x = a^x \implies a = x^{1/x}$.

The graphs of $y = x^{1/x}$ and $y = a$ intersect once for $a = e^{1/e} \approx 1.445$. Then

$\log_a x = x \implies (e^{1/e})^x = x \implies e^{x/e} = x \implies x = e.$

For $a = e^{1/e}$, the curves intersect once at $(e, e)$.

(c) For $1 < a < e^{1/e}$ the curves intersect twice. For $a > e^{1/e}$, the curves do not intersect.

**147.** Yes. The doubling time is given by

$$2P = Pe^{rt}$$
$$2 = e^{rt}$$
$$\ln 2 = rt$$
$$t = \frac{\ln 2}{r}.$$

The time to quadruple is given by

$$4P = Pe^{rt}$$
$$4 = e^{rt}$$
$$\ln 4 = rt$$
$$t = \frac{\ln 4}{r} = \frac{\ln 2^2}{r} = \frac{2 \ln 2}{r} = 2\left[\frac{\ln 2}{r}\right]$$

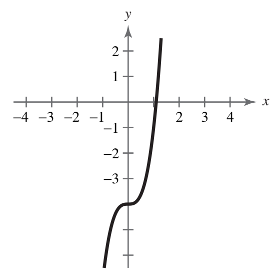
which is twice as long.

**148.** To find the length of time it takes for an investment $P$ to double to $2P$, solve

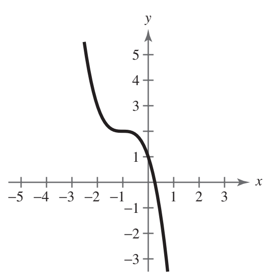
$$2P = Pe^{rt}$$
$$2 = e^{rt}$$
$$\ln 2 = rt$$
$$\frac{\ln 2}{r} = t.$$

Thus, you can see that the time is not dependent on the size of the investment, but rather the interest rate.

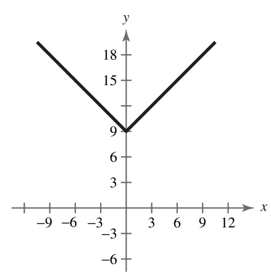
**149.** $f(x) = 3x^3 - 4$



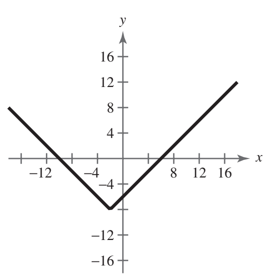
**150.** $f(x) = -(x + 1)^3 + 2$



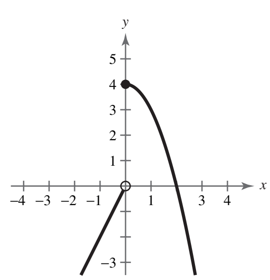
**151.** $f(x) = |x| + 9$



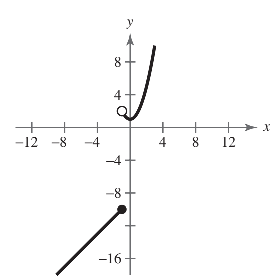
**152.** $f(x) = |x + 2| - 8$



**153.** $f(x) = \begin{cases} 2x, & x < 0 \\ -x^2 + 4, & x \geq 0 \end{cases}$



**154.** $f(x) = \begin{cases} x - 9, & x \leq -1 \\ x^2 + 1, & x > -1 \end{cases}$



## Section 3.5 Exponential and Logarithmic Models

- You should be able to solve compound interest problems.
  1. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
  2. $A = Pe^{rt}$
- You should be able to solve growth and decay problems.
  (a) Exponential growth if $b > 0$ and $y = ae^{bx}$.
  (b) Exponential decay if $b > 0$ and $y = ae^{-bx}$.
- You should be able to use the Gaussian model
  $y = ae^{-(x-b)^2/c}$.
- You should be able to use the logistics growth model
  $y = \dfrac{a}{1 + be^{-(x-c)/d}}$.
- You should be able to use the logarithmic models
  $y = \ln(ax + b)$ and $y = \log_{10}(ax + b)$.

### Vocabulary Check

**1.** (a) iv (b) i (c) vi (d) iii (e) vii (f) ii (g) v

**2.** Normally **3.** Sigmoidal **4.** Bell-shaped, mean

**1.** $y = 2e^{x/4}$

This is an exponential growth model.

Matches graph (c).

**2.** $y = 6e^{-x/4}$

This is an exponential decay model.

Matches graph (e).

**3.** $y = 6 + \log_{10}(x + 2)$

This is a logarithmic model, and contains $(-1, 6)$.

Matches graph (b).

**4.** $y = 3e^{-(x-2)^2/5}$

Gaussian model

Matches (a).

**5.** $y = \ln(x + 1)$

This is a logarithmic model.

Matches graph (d).

**6.** $y = \dfrac{4}{1 + e^{-2x}}$

Logistics model

Matches (f ).

**7.** Since $A = 10{,}000e^{0.035t}$, the time to double is given by

$$20{,}000 = 10{,}000e^{0.035t}$$
$$2 = e^{0.035t}$$
$$\ln 2 = 0.035t$$
$$t = \frac{\ln 2}{0.035} \approx 19.8 \text{ years.}$$

Amount after 10 years:

$$A = 10{,}000e^{0.035(10)} \approx \$14{,}190.68$$

**8.** Since $A = 2000e^{0.015t}$, the time to double is given by

$$4000 = 2000e^{0.015t}$$
$$2 = e^{0.015t}$$
$$\ln 2 = 0.015t$$
$$t = \frac{\ln 2}{0.015} \approx 46.2 \text{ years.}$$

Amount after 10 years:

$$A = 2000e^{0.015(10)} \approx \$2323.67$$

**9.** Since $A = 7500e^{rt}$ and $A = 15{,}000$ when $t = 21$, we have the following.

$$15{,}000 = 7500e^{21r}$$
$$2 = e^{21r}$$
$$\ln 2 = 21r$$
$$r = \frac{\ln 2}{21} \approx 0.033 = 3.3\%$$

Amount after 10 years:

$$A = 7500e^{0.033(10)} \approx \$10{,}432.26$$

**10.** Since $A = 1000e^{rt}$ and $A = 2000$ when $t = 12$, we have the following.

$$2000 = 1000e^{12r}$$
$$2 = e^{12r}$$
$$\ln 2 = 12r$$
$$r = \frac{\ln 2}{12} \approx 0.058 = 5.8\%$$

Amount after 10 years:

$$A = 1000e^{0.058(10)} \approx \$1786.04$$

**11.** Since $A = 5000e^{rt}$ and $A = 5665.74$ when $t = 10$, we have the following.

$$5665.74 = 5000e^{10r}$$
$$\frac{5665.74}{5000} = e^{10r}$$
$$\ln\left(\frac{5665.74}{5000}\right) = 10r$$
$$r = \frac{1}{10}\ln\left(\frac{5665.74}{5000}\right)$$
$$\approx 0.0125 = 1.25\%$$

The time to double is given by

$$10{,}000 = 5000e^{0.0125t}$$
$$2 = e^{0.0125t}$$
$$\ln 2 = 0.0125t$$
$$t = \frac{\ln 2}{0.0125} \approx 55.5 \text{ years.}$$

**12.** Since $A = 300e^{rt}$ and $A = 385.21$ when $t = 10$, we have the following.

$$385.21 = 300e^{10r}$$
$$\frac{385.21}{300} = e^{10r}$$
$$\ln\left(\frac{385.21}{300}\right) = 10r$$
$$r = \frac{1}{10}\ln\left(\frac{385.21}{300}\right)$$
$$\approx 0.025 = 2.5\%$$

The time to double is given by

$$600 = 300e^{0.025t}$$
$$2 = e^{0.025t}$$
$$\ln 2 = 0.025t$$
$$t = \frac{\ln 2}{0.025} \approx 27.7 \text{ years.}$$

**13.** Since $A = Pe^{0.045t}$ and $A = 100{,}000$ when $t = 10$, we have the following.

$$100{,}000 = Pe^{0.045(10)}$$
$$\frac{100{,}000}{e^{0.45}} = P \approx 63{,}762.82$$

The time to double is given by

$$127{,}525.64 = 63{,}762.82e^{0.045t}$$
$$2 = e^{0.045t}$$
$$\ln 2 = 0.045t$$
$$t = \frac{\ln 2}{0.045} \approx 15.4 \text{ years.}$$

**14.** Since $A = Pe^{0.02t}$ and $A = 2500$ when $t = 10$, we have the following.

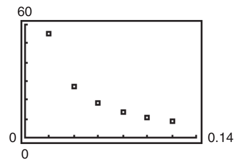
$$2500 = Pe^{0.02(10)}$$
$$\frac{2500}{e^{0.02}} = P \approx \$2046.83$$

The time to double is given by

$$4093.66 = 2046.83e^{0.02t}$$
$$2 = e^{0.02t}$$
$$\ln 2 = 0.02t$$
$$t = \frac{\ln 2}{0.02} \approx 34.7 \text{ years.}$$

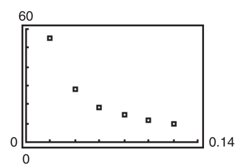
**15.**
$$3P = Pe^{rt}$$
$$3 = e^{rt}$$
$$\ln 3 = rt$$
$$\frac{\ln 3}{r} = t$$

| $r$ | 2% | 4% | 6% | 8% | 10% | 12% |
|---|---|---|---|---|---|---|
| $t = \dfrac{\ln 3}{r}$ | 54.93 | 27.47 | 18.31 | 13.73 | 10.99 | 9.16 |

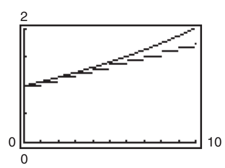



**16.**
$$3P = P(1 + r)^t$$
$$3 = (1 + r)^t$$
$$\ln 3 = \ln(1 + r)^t$$
$$\ln 3 = t \ln(1 + r)$$
$$\frac{\ln 3}{\ln(1 + r)} = t$$

| $r$ | 2% | 4% | 6% | 8% | 10% | 12% |
|---|---|---|---|---|---|---|
| $t = \dfrac{\ln 3}{\ln(1 + r)}$ | 55.48 | 28.01 | 18.85 | 14.27 | 11.53 | 9.69 |




**17.**

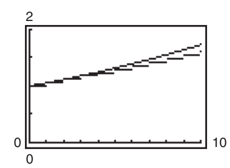



Continuous compounding results in faster growth.

$A = 1 + 0.075[\![t]\!]$

and $A = e^{0.07t}$

**18.**




$A = 1 + 0.06[\![t]\!]$

$$A = \left(1 + \frac{0.055}{365}\right)^{[\![365t]\!]}$$

From the graph, $5\frac{1}{2}\%$ compounded daily grows faster than 6% simple interest.

**19.**
$$\frac{1}{2}C = Ce^{k(1599)}$$
$$\frac{1}{2} = e^{1599k}$$
$$k = \frac{\ln(1/2)}{1599}$$
$$y = Ce^{kt}$$
$$= 10e^{[\ln(1/2)/1599]1000}$$
$$\approx 6.48 \text{ g}$$

**20.**
$$\frac{1}{2}C = Ce^{k(1599)}$$
$$\frac{1}{2} = e^{1599k}$$
$$k = \frac{\ln(1/2)}{1599}$$
$$y = Ce^{kt}$$
$$1.5 = Ce^{[\ln(1/2)/1599]1000}$$
$$1.5 \approx C(0.64824)$$
$$C \approx 2.31 \text{ g}$$

**21.**
$$\frac{1}{2}C = Ce^{k(5715)}$$
$$\frac{1}{2} = e^{5715k}$$
$$k = \frac{\ln(1/2)}{5715}$$
$$y = Ce^{kt}$$
$$= 3e^{[\ln(1/2)/5715]1000}$$
$$\approx 2.66 \text{ g}$$

**22.** $\frac{1}{2}C = Ce^{k(24,100)}$

$\frac{1}{2} = e^{24,100k}$

$k = \frac{\ln(1/2)}{24,100}$

$0.4 = Ce^{[\ln(1/2)/24,100]1000}$

$\approx C(0.97165)$

$C \approx 0.41$ g

**23.** $y = ae^{bx}$

$1 = ae^{b(0)} \Rightarrow 1 = a$

$10 = e^{b(3)}$

$\ln 10 = 3b$

$\frac{\ln 10}{3} = b \Rightarrow b \approx 0.7675$

Thus, $y = e^{0.7675x}$.

**24.** $y = ae^{bx}$

$\frac{1}{2} = ae^{b(0)} \Rightarrow a = \frac{1}{2}$

$5 = \frac{1}{2}e^{b(4)}$

$10 = e^{4b}$

$\ln 10 = 4b$

$b = \frac{\ln 10}{4} \approx 0.5756$

Thus, $y = \frac{1}{2}e^{0.5756x}$.

**25.** $(0, 4) \Rightarrow a = 4$

$(5, 1) \Rightarrow 1 = 4e^{b(5)} \Rightarrow b = \frac{1}{5}\ln\left(\frac{1}{4}\right)$

$= -\frac{1}{5}\ln 4 \approx -0.2773$

$y = 4e^{-0.2773x}$

**26.** $y = ae^{bx}$

$1 = ae^{b(0)} \Rightarrow 1 = a$

$\frac{1}{4} = e^{b(3)}$

$\ln\left(\frac{1}{4}\right) = 3b$

$\frac{\ln(1/4)}{3} = b \Rightarrow b \approx -0.4621$

Thus, $y = e^{-0.4621x}$.

**27.** (a) Australia: (0, 19.2), (10, 20.9)

$a = 19.2$ and $20.9 = 19.2e^{b(10)} \Rightarrow b = 0.008484$

$y = 19.2e^{0.008484t}$

For 2030, $y \approx 24.8$ million.

Canada: (0, 31.3), (10, 34.3)

$a = 31.3$ and $34.3 = 31.3e^{b(10)} \Rightarrow b = 0.009153$

$y = 31.3e^{0.009153t}$

For 2030, $y \approx 41.2$ million.

Philippines: (0, 79.7), (10, 95.9)

$a = 79.7$ and $95.9 = 79.7e^{b(10)} \Rightarrow b = 0.0185$

$y = 79.7e^{0.0185t}$

For 2030, $y \approx 138.8$ million.

South Africa: (0, 44.1), (10, 43.3)

$a = 44.1$ and $43.3 = 44.1e^{b(10)} \Rightarrow b = -0.00183$

$y = 44.1e^{-0.00183t}$

For 2030, $y \approx 41.7$ million.

**—CONTINUED—**

**27.** (a) **—CONTINUED—**

Turkey: $(0, 65.7)$, $(10, 73.3)$

$a = 65.7$ and $73.3 = 65.7e^{b(10)} \implies b = 0.01095$

$y = 65.7e^{0.01095t}$

For 2030, $y \approx 91.2$ million.

(b) The constant $b$ gives the growth rates.

(c) The constant $b$ is negative for South Africa.

**28.** $P = 372.55e^{-0.01052t}$

(a) Decreasing because the exponent is negative.

(b) For 1990, $t = 0$ and $P \approx 372{,}550$ people.

For 2000, $t = 10$ and $P \approx 335{,}349$ people.

For 2004, $t = 14$ and $P \approx 321{,}530$ people.

(c) $300 = 372.55e^{-0.01052t} \implies t \approx 20.6$, or 2010

**29.** (a) $180 = 134.0e^{k(10)}$

$$10k = \ln \frac{180}{134.0}$$

$$k \approx 0.0295$$

(b) For 2010, $t = 20$ and

$P = 134.0e^{0.0295(20)} \approx 241{,}734$ people.

**30.** $P = 258.0e^{kt}$

(a) $478 = 258.0e^{k(10)}$

$$10k = \ln\left(\frac{478}{258}\right)$$

$$k \approx 0.0617$$

(b) For 2010, $t = 20$ and

$P = 258e^{0.0617(20)} \approx 886{,}215$ people.

**31.**

$$y = Ce^{kt}$$

$$\frac{1}{2}C = Ce^{(1599)k}$$

$$\ln\left(\frac{1}{2}\right) = 1599k$$

$$k = \frac{\ln(1/2)}{1599}$$

When $t = 100$, we have

$y = Ce^{[(\ln(1/2)\cdot 100)/1599]} \approx 0.958C$, or 95.8%.

**32.**

$$y = Ce^{kt}$$

$$\frac{1}{2}C = Ce^{5715k}$$

$$\ln\left(\frac{1}{2}\right) = 5715k$$

$$k = \frac{\ln(1/2)}{5715}$$

The ancient charcoal has only 15% as much radioactive carbon.

$$0.15C = Ce^{[(\ln(1/2)/5715)t]}$$

$$\ln 0.15 = \frac{\ln(1/2)}{5715}t$$

$$t \approx 15{,}642 \text{ years}$$

**33.** (a) $V = mt + b$, $V(0) = 30{,}788 \implies b = 30{,}788$

$V(2) = 24{,}000 \implies 24{,}000 = 2m + 30{,}788$

$\implies m = -3394$

$V(t) = -3394t + 30{,}788$

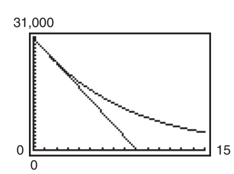
(b) $V = ae^{kt}$, $V(0) = 30{,}788 \implies b = 30{,}788$

$V(2) = 24{,}000 \implies 24{,}000 = 30{,}788e^{2k}$

$$\implies k = \frac{1}{2}\ln\left(\frac{24{,}000}{30{,}788}\right) \approx -0.1245$$

$V = 30{,}788e^{-0.1245t}$

(c)




(d) The exponential model depreciates faster in the first year.

(e) Answers will vary.

**34.** Let $t = 0$ correspond to 2005.

(a) $V = mt + b$, $V(0) = 1150 \Rightarrow b = 1150$

$V(2) = 550 \Rightarrow 550 = m(2) + 1150 \Rightarrow m = -300$

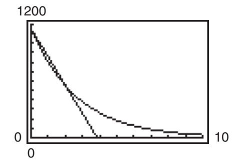
$V(t) = -300t + 1150$

(b) $V = ae^{kt}$, $V(0) = 1150 \Rightarrow a = 1150$

$V(2) = 550 \Rightarrow 550 = 1150e^{2k}$

$\Rightarrow k = \frac{1}{2}\ln\left(\frac{55}{115}\right) \approx -0.3688$

$V(t) = 1150e^{-0.3688t}$

(c) 


(d) The exponential model depreciates faster in the first year.

(e) Answers will vary.

**35.** $S(t) = 100(1 - e^{kt})$

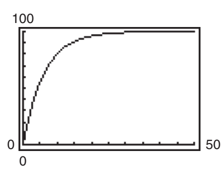
(a) $15 = 100(1 - e^{k(1)})$

$-85 = -100e^k$

$k = \ln 0.85$

$k \approx -0.1625$

$S(t) = 100(1 - e^{-0.1625t})$

(b) 


(c) $S(5) = 100(1 - e^{-0.1625(5)})$

$\approx 55.625 = 55{,}625$ units

**36.** $S = 10(1 - e^{kx})$

$x = 5$ (in hundreds)

$S = 2.5$ (in thousands)

(a) $2.5 = 10(1 - e^{k(5)})$

$0.25 = 1 - e^{5k}$

$e^{5k} = 0.75$

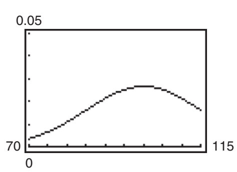
$5k = \ln 0.75$

$k \approx -0.0575$

$S = 10(1 - e^{-0.0575x})$

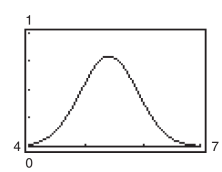
(b) When $x = 7$, $S = 10(1 - e^{-0.0575(7)}) \approx 3.314$ which corresponds to 3314 units.

**37.** $y = 0.0266e^{-(x-100)^2/450}$, $70 \le x \le 115$

(a) 


(b) Maximum point is $x = 100$, the average IQ score.

**38.** $y = 0.7979e^{-(x-5.4)^2/0.5}$

(a) 


(b) About 5.4 hours

**39.** $p(t) = \dfrac{1000}{1 + 9e^{-0.1656t}}$

(a) $p(5) = \dfrac{1000}{1 + 9e^{-0.1656(5)}} \approx 203$ animals

(b) $500 = \dfrac{1000}{1 + 9e^{-0.1656t}}$

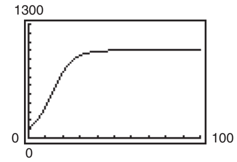
$1 + 9e^{-0.1656}t = 2$

$9e^{-0.1656t} = 1$

$e^{-0.1656t} = \dfrac{1}{9}$

$t = \dfrac{-\ln(1/9)}{0.1656} \approx 13$ months

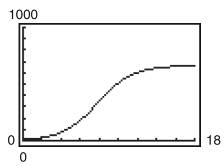
(c)




The horizontal asymptotes are $p = 0$ and $p = 1000$. The population will approach 1000 as time increases.

**40.** $y = \dfrac{663}{1 + 72e^{-0.547t}}, 0 \le t \le 18$

(a)




(b) For $t = 19$, $y \approx 662$.

For $t = 30$, $y \approx 663$.

(c) As $t \to \infty$, $y \to \dfrac{663}{1} = 663$, limiting value.

(d) Answers will vary.

**41.** $R = \log_{10}\left(\dfrac{I}{I_0}\right) = \log_{10}(I) \Longrightarrow I = 10^R$

(a) $I = 10^{6.1} \approx 1{,}258{,}925$

(b) $I = 10^{7.6} \approx 39{,}810{,}717$

(c) $I = 10^{9.0} \approx 1{,}000{,}000{,}000$

**42.** $R = \log_{10}\left(\dfrac{I}{I_0}\right) = \log_{10}(I)$

(a) $R = \log_{10}(39{,}811{,}000) \approx 7.6$

(b) $R = \log_{10}(12{,}589{,}000) \approx 7.1$

(c) $R = \log_{10}(251{,}200) \approx 5.4$

**43.** $\beta(I) = 10 \log_{10}(I/I_0)$, where $I_0 = 10^{-12}$ watt per square meter.

(a) $\beta(10^{-10}) = 10 \cdot \log_{10}\left(\dfrac{10^{-10}}{10^{-12}}\right) = 10 \log_{10} 10^2 = 20$ decibels

(b) $\beta(10^{-5}) = 10 \cdot \log_{10}\left(\dfrac{10^{-5}}{10^{-12}}\right) = 10 \log_{10} 10^7 = 70$ decibels

(c) $\beta(10^{0}) = 10 \cdot \log_{10}\left(\dfrac{10^{0}}{10^{-12}}\right) = 10 \log_{10} 10^{12} = 120$ decibels

**44.** $\beta = 10 \log_{10}\left(\dfrac{I}{I_0}\right) = 10 \log_{10}\left(\dfrac{I}{10^{-12}}\right)$

(a) $\beta(10^{-4}) = 10 \log_{10}\left(\dfrac{10^{-4}}{10^{-12}}\right) = 10 \log_{10}(10^8) = 80$ decibels

(b) $\beta(10^{-3}) = 90$ decibels

(c) $\beta(10^{-2}) = 100$ decibels

**45.** $$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

$$10^{\beta/10} = \frac{I}{I_0}$$

$$I = I_0 10^{\beta/10}$$

$$\%\text{ decrease} = \frac{I_0 10^{8.8} - I_0 10^{7.2}}{I_0 10^{8.8}} \times 100$$

$$= 97.5\%$$

**46.** $$\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$$

$$10^{\beta/10} = \frac{I}{I_0}$$

$$I = I_0 10^{\beta/10}$$

$$\%\text{ decrease} = \frac{I_0 10^{9.3} - I_0 10^{8.0}}{I_0 10^{9.3}} \times 100 \approx 95\%$$

**47.** $\text{pH} = -\log_{10}[\text{H}^+] = -\log_{10}[2.3 \times 10^{-5}] \approx 4.64$

**48.** $$5.8 = -\log_{10}[\text{H}^+]$$

$$10^{-5.8} = [\text{H}^+]$$

$[\text{H}^+] \approx 1.58 \times 10^{-6}$ moles per liter

**49.** $$\text{pH} = -\log_{10}[\text{H}^+]$$

$$-\text{pH} = \log_{10}[\text{H}^+]$$

$$10^{-\text{pH}} = [\text{H}^+]$$

$$\frac{\text{Hydrogen ion concentration of grape}}{\text{Hydrogen ion concentration of milk of magnesia}}$$

$$= \frac{10^{-3.5}}{10^{-10.5}} = 10^7$$

**50.** $$\text{pH} - 1 = -\log_{10}[\text{H}^+]$$

$$-(\text{pH} - 1) = \log_{10}[\text{H}^+]$$

$$10^{-(\text{pH}-1)} = [\text{H}^+]$$

$$10^{-\text{pH}+1} = [\text{H}^+]$$

$$10^{-\text{pH}} \cdot 10 = [\text{H}^+]$$

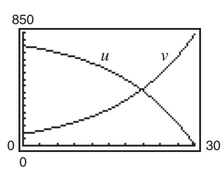
The hydrogen ion concentration is increased by a factor of 10.

**51.** (a) $P = 120{,}000$, $r = 0.075$, $M = 839.06$

$$u = M - \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}$$

$$= 839.06 - (839.06 - 750)(1 + 0.00625)^{12t}$$

$$v = (839.06 - 750)(1.00625)^{12t}$$

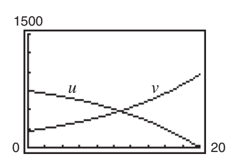



(b) In the early years, the majority of the monthly payment goes toward interest. The interest and principle are equal when $t \approx 20.729 \approx 21$ years.

(c) $P = 120{,}000$, $r = 0.075$, $M = 966.71$

$$u = 966.71 - (966.71 - 750)(1.00625)^{12t}$$

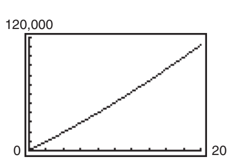
$$v = (966.71 - 750)(1.00625)^{12t}$$




$u = v$ when $t \approx 10.73$ years.

**52.** $$u = 120{,}000\left[\frac{0.075t}{1 - \left(\frac{1}{1 + 0.075/12}\right)^{12t}} - 1\right]$$

(a)




(b) From the graph, when $u = 120{,}000$, $t \approx 21.2$ years. Yes, a mortgage of approximately 37.6 years will result in about \$240,000 of interest.

**53.** $t = -10\ln\left(\dfrac{T - 70}{98.6 - 70}\right)$

At 9:00 A.M. we have
$t = -10\ln[(85.7 - 70)/(98.6 - 70)] \approx 6$ hours.

Thus, we can conclude that the person died 6 hours before 9 A.M., or 3:00 A.M.

**54.** $t = -5.05\ln\left(\dfrac{T - 40}{0 - 40}\right)$ ($t = 0$ is 11 A.M.)

$$7 = -5.05\ln\left(\frac{T - 40}{0 - 40}\right)$$

$$\frac{-7}{5.05} = \ln\left(\frac{T - 40}{-40}\right)$$

$$\left(\frac{T - 40}{-40}\right) = e^{-7/5.05}$$

$$T = 40 - 40e^{-7/5.05} \approx 29.998 \approx 30 < 32$$

Hence, the steaks do not thaw out in time.

**55.** False. The domain could be all real numbers.

**56.** False. See Example 5, page 380.

**57.** True. For the Gaussian model, $y > 0$.

**58.** True. See page 379.

**59.** $4x - 3y - 9 = 0 \implies y = \frac{1}{3}(4x - 9)$

Slope: $\frac{4}{3}$

Matches (a).

Intercepts: $(0, -3), \left(\frac{9}{4}, 0\right)$

**60.** Line with intercepts (5, 0) and (0, 2).

Matches (b).

**61.** $y = 25 - 2.25x$

Slope: $-2.25$

Matches (d).

Intercepts: $(0, 25), \left(\frac{100}{9}, 0\right)$

**62.** Line with intercepts (2, 0) and (0, 4).

Matches (c).

**63.** $f(x) = 2x^3 - 3x^2 + x - 1$

The graph falls to the left and rises to the right.

**64.** $f(x) = -4x^4 - x^2 + 5$

Falls to the left and falls to right

**65.** $g(x) = -1.6x^5 + 4x^2 - 2$

The graph rises to the left and falls to the right.

**66.** $g(x) = 7x^6 + 9.1x^5 - 3.2x^4 + 25x^3$

Rises to left and rises to right

**67.**

$$\begin{array}{r|rrrr} 4 & 2 & -8 & 3 & -9 \\ & & 8 & 0 & 12 \\ \hline & 2 & 0 & 3 & 3 \end{array}$$

$$\frac{2x^3 - 8x^2 + 3x - 9}{x - 4} = 2x^2 + 3 + \frac{3}{x - 4}$$

**68.**

$$\begin{array}{r|rrrrr} -5 & 1 & 0 & 0 & -3 & 1 \\ & & -5 & 25 & -125 & 640 \\ \hline & 1 & -5 & 25 & -128 & 641 \end{array}$$

$$\frac{x^4 - 3x + 1}{x + 5} = x^3 - 5x^2 + 25x - 128 + \frac{641}{x + 5}$$

**69.** Answers will vary.

## Section 3.6 Nonlinear Models

- You should be able to use a graphing utility to find nonlinear models, including:
  (a) Quadratic models
  (b) Exponential models
  (c) Power models
  (d) Logarithmic models
  (e) Logistic models
- You should be able to use a scatter plot to determine which model is best.
- You should be able to determine the sum of squared differences for a model.

### Vocabulary Check

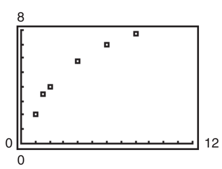
**1.** $y = ax + b$ **2.** quadratic **3.** $y = ax^b$

**4.** sum, squared differences **5.** $y = ab^x, ae^{cx}$

**1.** Logarithmic model **2.** Linear model **3.** Quadratic model **4.** Exponential model

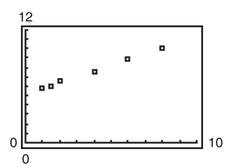
**5.** Exponential model **6.** Logistic model **7.** Quadratic model **8.** Linear model

**9.**



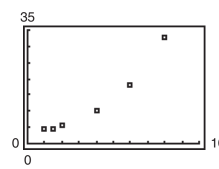

Logarithmic model

**10.**



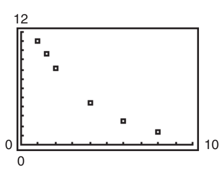

Linear model

**11.**



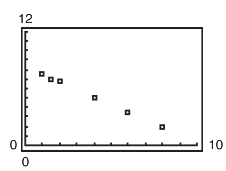

Exponential model

**12.**



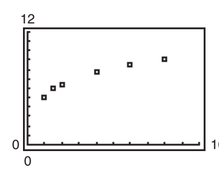

Exponential model

**13.**




Linear model

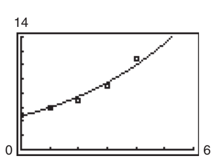
**14.**




Logarithmic model

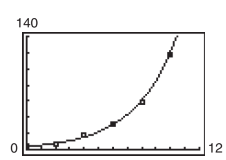
**15.** $y = 4.752(1.2607)^x$

Coefficient of determination: 0.96773



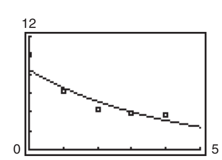

**16.** $y = 3.964(1.4084)^x$

Coefficient of determination: 0.99495



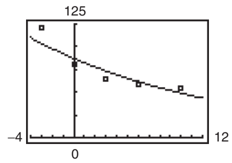

**17.** $y = 8.463(0.7775)^x$

Coefficient of determination: 0.86639



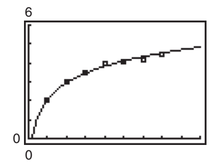
**18.** $y = 87.262(0.9438)^x$

Coefficient of determination: 0.85030



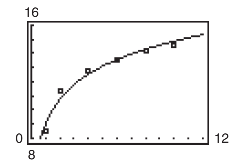

**19.** $y = 2.083 + 1.257 \ln x$

Coefficient of determination: 0.98672



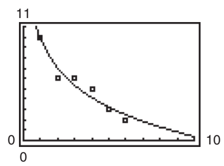

**20.** $y = 9.027 + 2.537 \ln x$

Coefficient of determination: 0.96884



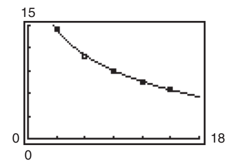

**21.** $y = 9.826 - 4.097 \ln x$

Coefficient of determination: 0.93704



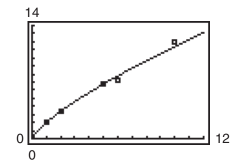

**22.** $y = 20.076 - 5.027 \ln x$

Coefficient of determination: 0.99977



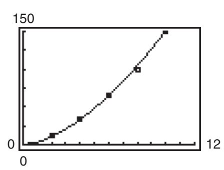

**23.** $y = 1.985x^{0.760}$

Coefficient of determination: 0.99686



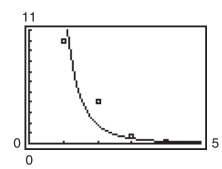

**24.** $y = 3.397x^{1.650}$

Coefficient of determination: 0.99788



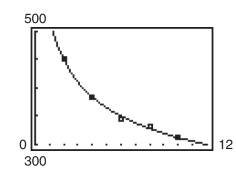

**25.** $y = 16.103x^{-3.174}$

Coefficient of determination: 0.88161




**26.** $y = 525.428x^{-0.226}$

Coefficient of determination: 0.99549

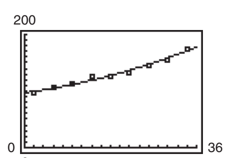


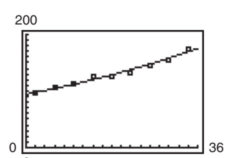

**27.** (a) Quadratic model: $R = 0.031t^2 + 1.13t + 97.1$

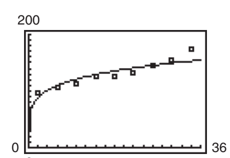
Exponential model: $R = 94.435(1.0174)^t$

Power model: $R = 77.837t^{0.1918}$

(b)




(c) The exponential model fits best. Answers will vary.

(d) For 2008, $t = 38$ and $R \approx 181.9$ million.

For 2012, $t = 42$ and $R \approx 194.9$ million.

Answers will vary.

**28.** (a) Quadratic model: $R = -0.0136t^2 + 0.396t + 1.01$

Coefficient of determination: 0.97730

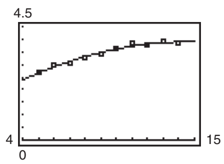
Exponential model: $R = 2.296(1.0425)^t$

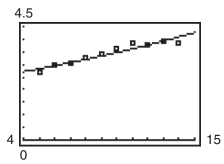
Coefficient of determination: 0.90739

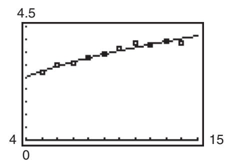
Power model: $R = 1.480t^{0.3791}$

Coefficient of determination: 0.96052

(b)




(c) The quadratic model fits best.

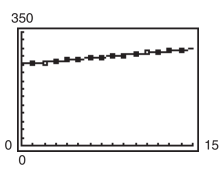
(d) Using the quadratic model:

| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Price | 3.89 | 3.86 | 3.81 | 3.73 | 3.62 | 3.49 |

Answers will vary.

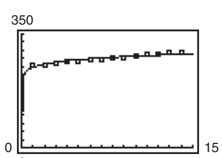
**29.** (a) Linear model: $P = 3.11t + 250.9$

Coefficient of determination: 0.99942



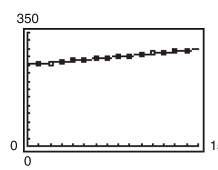

(b) Power model: $P = 246.52t^{0.0587}$

Coefficient of determination: 0.90955



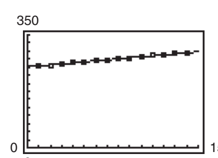

(c) Exponential model: $P = 251.57(1.0114)^t$

Coefficient of determination: 0.99811




(d) Quadratic model: $P = -0.020t^2 + 3.41t + 250.1$

Coefficient of determination: 0.99994




(e) The quadratic model is best because its coefficient of determination is closest to 1.

**—CONTINUED—**

**29. —CONTINUED—**

(f) Linear model:

| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Population (in millions) | 297.6 | 300.7 | 303.8 | 306.9 | 310.0 | 313.1 |

Power model:

| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Population (in millions) | 289.0 | 290.1 | 291.1 | 292.1 | 293.0 | 293.9 |

Exponential model:

| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Population (in millions) | 298.2 | 301.6 | 305.0 | 308.5 | 312.0 | 315.6 |

Quadratic model:

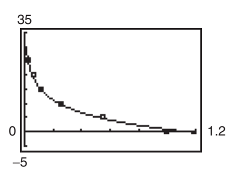
| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|---|---|---|
| Population (in millions) | 296.8 | 299.5 | 302.3 | 305.0 | 307.7 | 310.3 |

(g) and (h) Answers will vary.

**30.** (a) $h = 0$ is not in the domain of the logarithmic function.

(b) $h = 0.863 - 6.447 \ln p$

(c)

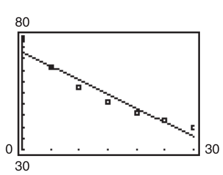



(d) For $p = 0.75$, $h \approx 2.71$ km.

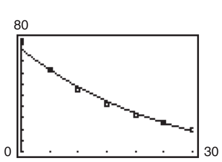
(e) For $h = 13$, $p \approx 0.15$ atmospheres.

**31.** (a) $T = -1.239t + 73.02$

No, the data does not appear linear.

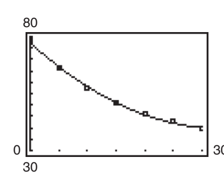



(c) Subtracting 21 from the $T$-values, the exponential model is $y = 54.438(0.9635)^t$. Adding back 21, $T = 54.438(0.9635)^t + 21$.



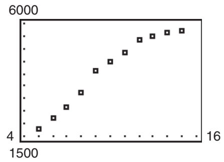

(b) $T = 0.034t^2 - 2.26t + 77.3$

Yes, the data appears quadratic. But, for $t = 60$, the graph is increasing, which is incorrect.

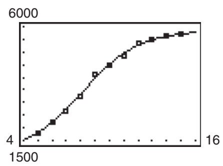



(d) Answers will vary.

**32.** (a)




(b)



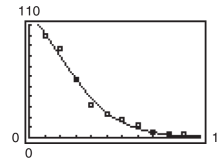

This model is a good fit.

(c) $S = 1018.4 + \dfrac{4827.2}{1 + e^{-(t-8.1391)/1.9372}}$

Using a graphing utility, $t \approx 15.7$ or 2005.

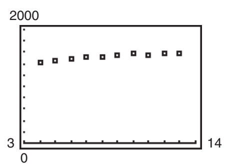
**33.** (a) $P = \dfrac{162.4}{1 + 0.34e^{0.5609x}}$

(b)




The model is a good fit.

**34.** (a)




(b) Linear model: $y = 18.5x + 1365$

Quadratic model: $-2.10x^2 + 54.2x + 1230$

Cubic model: $y = -0.071x^3 - 0.30x^2 + 39.9x + 1265$

Power model: $y = 1239.7 \cdot x^{0.0985}$

Exponential model: $y = 1370.4(1.012)^x$

(c)

| Year | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 |
|---|---|---|---|---|---|---|---|---|---|---|
| Linear | 1439 | 1458 | 1476 | 1495 | 1513 | 1532 | 1550 | 1569 | 1587 | 1606 |
| Quadratic | 1413 | 1449 | 1480 | 1507 | 1529 | 1548 | 1562 | 1572 | 1578 | 1580 |
| Cubic | 1415 | 1448 | 1478 | 1505 | 1529 | 1548 | 1563 | 1573 | 1578 | 1577 |
| Power | 1421 | 1453 | 1479 | 1502 | 1522 | 1539 | 1555 | 1570 | 1584 | 1596 |
| Exponential | 1437 | 1455 | 1472 | 1490 | 1508 | 1526 | 1544 | 1563 | 1581 | 1600 |

Answers will vary.

(d) For 2015, $x = 25$ and $y \approx 1273$ million metric tons.

**35.** (a) Linear model: $y = 15.71t + 51.0$

Logarithmic model: $y = 134.67 \ln t - 97.5$

Quadratic model: $y = -1.292t^2 + 38.96t - 45.0$

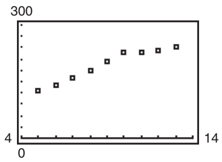
Exponential model: $y = 85.97(1.091)^t$

Power model: $y = 37.27t^{0.7506}$

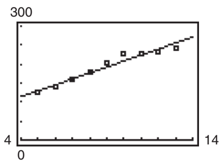
**—CONTINUED—**

**35. —CONTINUED—**

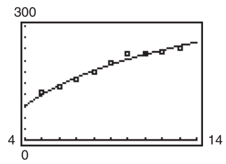
(b)



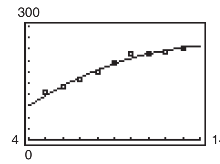

Linear model:



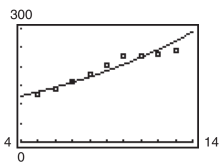

Logarithmic model:



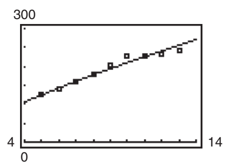

Quadratic model:




Exponential model:




Power model:




(c) Linear: 803.9

Logarithmic: 411.7

Quadratic: 289.8 (Best)

Exponential: 1611.4

Power: 667.1

(d) Linear: 0.9485

Logarithmic: 0.9736

Quadratic: 0.9814 (Best)

Exponential: 0.9274

Power: 0.9720

(e) Quadratic model is best.

**36.** Answers will vary.

**37.** True

**38.** False. Write $b$ as $b = e^{\ln b}$.
Then,
$y = ab^x = ae^{(\ln b)x} = ae^{cx}$.

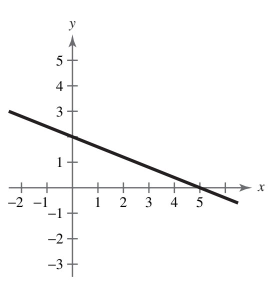
**39.** $2x + 5y = 10$

$$5y = -2x + 10$$

$$y = -\tfrac{2}{5}x + 2$$

Slope: $-\frac{2}{5}$

$y$-intercept: $(0, 2)$

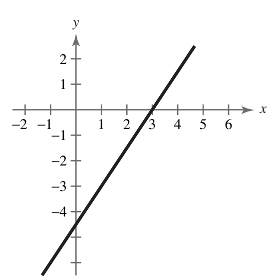


**40.** $3x - 2y = 9$

$$y = \tfrac{3}{2}x - \tfrac{9}{2}$$

Slope: $\frac{3}{2}$

$y$-intercept: $\left(0, -\frac{9}{2}\right)$

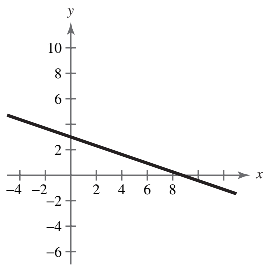


**41.** $1.2x + 3.5y = 10.5$

$$35y = -12x + 105$$
$$y = -\frac{12}{35}x + \frac{105}{35}$$
$$= -\frac{12}{35}x + 3$$

Slope: $-\frac{12}{35}$

$y$-intercept: $(0, 3)$

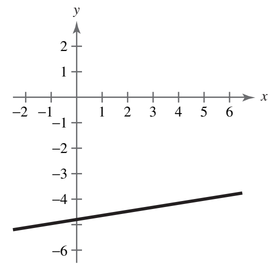



**42.** $0.4x - 2.5y = 12.0$

$$25y = 4x - 120$$
$$y = \frac{4}{25}x - \frac{24}{5}$$

Slope: $\frac{4}{25} = 0.16$

$y$-intercept: $\left(0, -\frac{24}{5}\right) = (0, -4.8)$




## Review Exercises for Chapter 3

**1.** $(1.45)^{2\pi} \approx 10.3254$

**2.** $7^{-\sqrt{11}} \approx 0.002$

**3.** $60^{2(-1.1)} = 60^{-2.2}$

$\approx 0.0001225 \approx 0.0$

**4.** $25^{-3(3/2)} \approx 5.12 \times 10^{-7} \approx 0$

**5.** $e^8 \approx 2980.958$

**6.** $5e^{\sqrt{5}} \approx 46.7823$

**7.** $e^{-(-2.1)} \approx e^{2.1} \approx 8.1662$

**8.** $-4e^{(-3/5)} \approx -2.1952$

**9.** $f(x) = 4^x$

Intercept: $(0, 1)$

Horizontal asymptote: $x$-axis

Increasing on: $(-\infty, \infty)$

Matches graph (c).

**10.** $f(x) = 4^{-x}$

Intercept: $(0,1)$

Horizontal asymptote: $x$-axis

Decreasing on: $(-\infty, \infty)$

Matches graph (d).

**11.** $f(x) = -4^x$

Intercept: $(0, -1)$

Horizontal asymptote: $x$-axis

Decreasing on: $(-\infty, \infty)$

Matches graph (b).

**12.** $f(x) = 4^x + 1$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 1$

Increasing on: $(-\infty, \infty)$

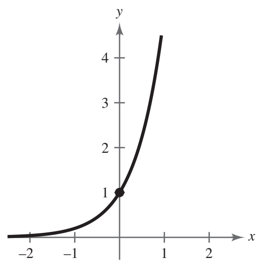
Matches graph (a).

**13.** $f(x) = 6^x$

Intercept: $(0, 1)$

Horizontal asymptote: $x$-axis

Increasing on: $(-\infty, \infty)$

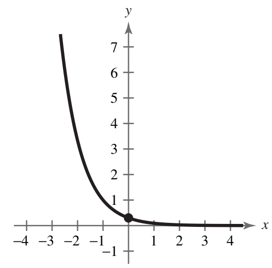


**14.** $f(x) = 0.3^{x+1} = \left(\frac{3}{10}\right)^{x+1}$

Horizontal asymptote: $y = 0$

Intercept: $(0, 0.3)$

Decreasing on $(-\infty, \infty)$

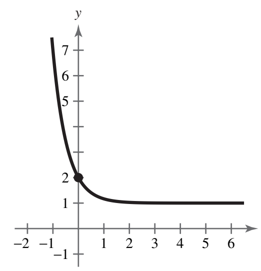


**15.** $g(x) = 1 + 6^{-x}$

Intercept: $(0, 2)$

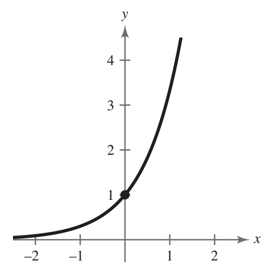
Horizontal asymptote: $y = 1$

Decreasing on: $(-\infty, \infty)$

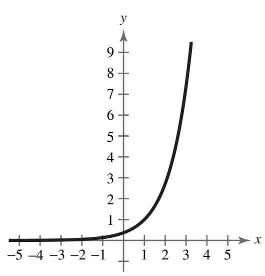


**16.** $g(x) = 0.3^{-x}$

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 0.09 | 0.3 | 1 | $3\frac{1}{3}$ | $11\frac{1}{9}$ |



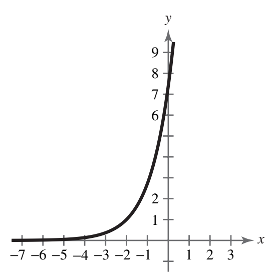
**17.**



$h(x) = e^{x-1}$

Horizontal asymptote: $y = 0$

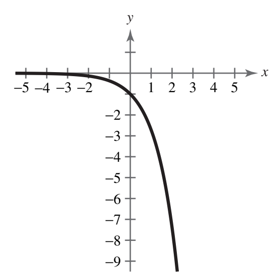
**18.**



$f(x) = e^{x+2}$

Horizontal asymptote: $y = 0$

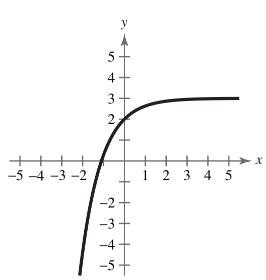
**19.**



$h(x) = -e^x$

Horizontal asymptote: $y = 0$

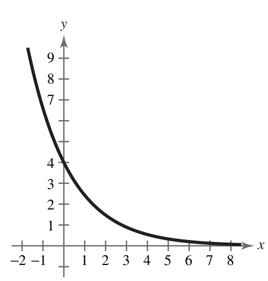
**20.**



$f(x) = 3 - e^{-x}$

Horizontal asymptote: $y = 3$

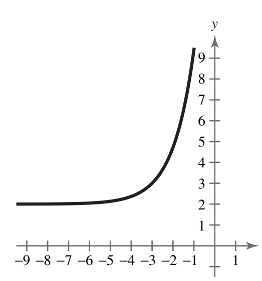
**21.**



$f(x) = 4e^{-0.5x}$

Horizontal asymptote: $y = 0$

**22.**

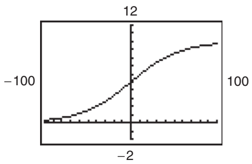


$f(x) = 2 + e^{x+3}$

Horizontal asymptote: $y = 2$

**23.** $f(x) = \dfrac{10}{1 + 2e^{-0.05x}}$

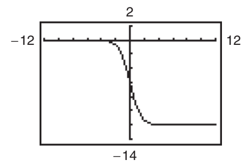
(a)




(b) Horizontal asymptotes: $y = 0, y = 10$

**24.** $f(x) = \dfrac{-12}{1 + 4^{-x}}$

(a)




(b) Horizontal asymptotes: $y = 0, y = -12$

**25.** $A = Pe^{rt} = 10{,}000e^{0.08t}$

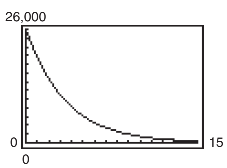
| $t$ | 1 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $A$ | 10,832.87 | 22,255.41 | 49,530.32 | 110,231.76 | 245,325.30 | 545,981.50 |

**26.** $r = 3\% = 0.03,\ A = 10{,}000e^{0.03t}$

| $t$ | 1 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $A$ | 10,305 | 13,499 | 18,221 | 24,596 | 33,201 | 44,817 |

**27.** $V(t) = 26{,}000\left(\frac{3}{4}\right)^t$

(a)




(b) For $t = 2$, $V(2) = \$14{,}625$.

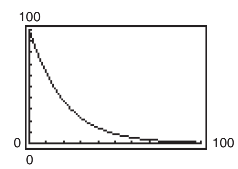
(c) The car depreciates most rapidly at the beginning, which is realistic.

**28.** $Q = 100\left(\frac{1}{2}\right)^{t/14}$

(a) When $t = 0$, $Q = 100$ grams.

(b) When $t = 10$, $Q = 100\left(\frac{1}{2}\right)^{10/14} \approx 60.95$ grams.

(c)




**29.** $\log_5 125 = 3$

$5^3 = 125$

**30.** $\log_6 36 = 2$

$6^2 = 36$

**31.** $\log_{64} 2 = \frac{1}{6}$

$64^{1/6} = 2$

**32.** $\log_{10}\left(\frac{1}{100}\right) = -2$

$10^{-2} = \frac{1}{100}$

**33.** $\ln e^4 = 4$

$e^4 = e^4$

**34.** $\ln\sqrt{e^3} = \frac{3}{2}$

$e^{3/2} = \sqrt{e^3}$

**35.** $4^3 = 64$

$\log_4 64 = 3$

**36.** $3^5 = 243$

$\log_3 243 = 5$

**37.** $25^{3/2} = 125$

$\log_{25} 125 = \frac{3}{2}$

**38.** $12^{-1} = \frac{1}{12}$

$\log_{12}\left(\frac{1}{12}\right) = -1$

**39.** $\left(\frac{1}{2}\right)^{-3} = 8$

$\log_{1/2} 8 = -3$

**40.** $\left(\frac{2}{3}\right)^{-2} = \frac{9}{4}$

$\log_{2/3}\left(\frac{9}{4}\right) = -2$

**41.** $e^7 = 1096.6331\ldots$

$\ln 1096.6331\ldots = 7$

**42.** $e^{-3} = 0.0497\ldots$

$\ln 0.0497\ldots = -3$

**43.** $\log_6 216 = \log_6 6^3$

$= 3\log_6 6$

$= 3$

**44.** $\log_7 1 = 0$

**45.** $\log_4\left(\frac{1}{4}\right) = \log_4(4^{-1})$
$= -\log_4 4$
$= -1$

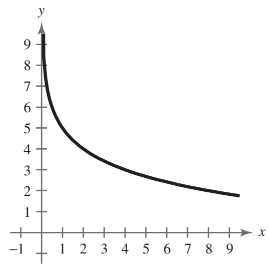
**46.** $\log_{10} 0.001 = \log_{10} 10^{-3}$
$= -3$

**47.** $g(x) = -\log_2 x + 5 = 5 - \dfrac{\ln x}{\ln 2}$

Domain: $x > 0$

Vertical asymptote: $x = 0$

$x$-intercept: $(32, 0)$



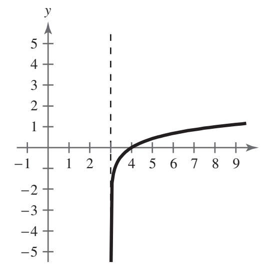
**48.** $g(x) = \log_5(x - 3)$

Vertical asymptote: $x = 3$

Intercept: $(4, 0)$

Domain: $x > 3$

| $x$ | 3.2 | 4 | 8 | 28 |
|---|---|---|---|---|
| $y$ | $-1$ | 0 | 1 | 2 |

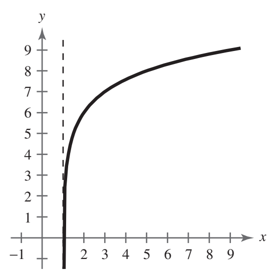


**49.** $f(x) = \log_2(x - 1) + 6 = 6 + \dfrac{\ln(x - 1)}{\ln(2)}$

Domain: $x > 1$

Vertical asymptote: $x = 1$

$x$-intercept: $(1.016, 0)$



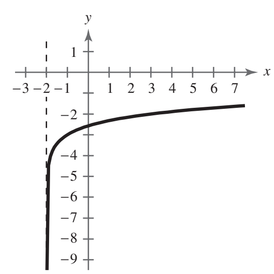
**50.** $f(x) = \log_5(x + 2) - 3$

Vertical asymptote: $x = -2$

Intercept: $(123, 0)$

Domain: $x > -2$

| $x$ | $-1.8$ | $-1$ | 3 | 23 |
|---|---|---|---|---|
| $y$ | $-4$ | $-3$ | $-2$ | $-1$ |



**51.** $\ln(21.5) \approx 3.068$

**52.** $\ln(0.98) \approx -0.020$

**53.** $\ln\sqrt{6} \approx 0.896$

**54.** $\ln\left(\frac{2}{5}\right) \approx -0.916$

**55.** $\log_5 3 = \log_5 x$
$3 = x$

**56.** $\log_2 8 = x$
$2^x = 8$
$x = 3$

**57.** $\log_9 x = \log_9 3^{-2}$
$x = 3^{-2} = \frac{1}{9}$

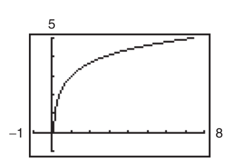
**58.** $\log_4 4^3 = x$
$4^x = 4^3$
$x = 3$

**59.** $f(x) = \ln x + 3$

Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

$x$-intercept: $(0.05, 0)$

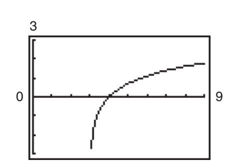



**60.** $f(x) = \ln(x - 3)$

Domain: $(3, \infty)$

Vertical asymptote: $x = 3$

Intercept: $(4, 0)$

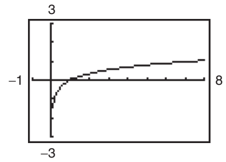


**61.** $h(x) = \frac{1}{2} \ln x$

Domain: $x > 0$

Vertical asymptote: $x = 0$

$x$-intercept: $(1, 0)$

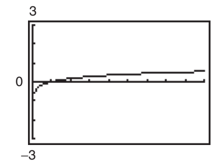



**62.** $f(x) = \frac{1}{4} \ln x$

Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

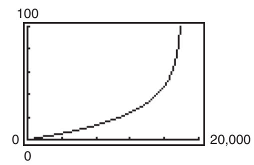
Intercept: $(1, 0)$




**63.** $t = 50 \log_{10} \dfrac{18{,}000}{18{,}000 - h}$

(a) $0 \le h < 18{,}000$

(b)




Vertical asymptote: $h = 18{,}000$

(c) The plane climbs at a faster rate as it approaches its absolute ceiling.

(d) If $h = 4000$, $t = 50 \log_{10} \dfrac{18{,}000}{18{,}000 - 4000} \approx 5.46$ minutes.

**64.** $t = 12.542 \ln\left(\dfrac{x}{x - 1000}\right), x > 1000$

(a) For $x = 1254.68$, $t \approx 20$ years.

(b) For $x = 1254.68$, $t = 20$, the total amount paid is $(1254.68)(20)(12) = \$301{,}123.20$.

The interest is $301{,}123.20 - 150{,}000.00 = \$151{,}123.20$.

**65.** $\log_4 9 = \dfrac{\log_{10} 9}{\log_{10} 4} \approx 1.585$

$\log_4 9 = \dfrac{\ln 9}{\ln 4} \approx 1.585$

**66.** $\log_{1/2} 5 = \dfrac{\log_{10} 5}{\log_{10}(1/2)} \approx -2.322$

$\log_{1/2} 5 = \dfrac{\ln 5}{\ln(1/2)} \approx -2.322$

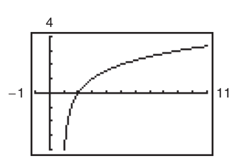
**67.** $\log_{12} 200 = \dfrac{\log_{10} 200}{\log_{10} 12} \approx 2.132$

$\log_{12} 200 = \dfrac{\ln 200}{\ln 12} \approx 2.132$

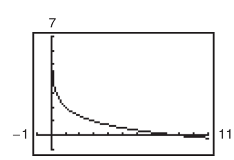
**68.** $\log_3 0.28 = \dfrac{\log_{10} 0.28}{\log_{10} 3} \approx -1.159$

$\log_3 0.28 = \dfrac{\ln 0.28}{\ln 3} \approx -1.159$

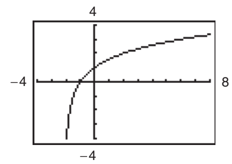
**69.** $f(x) = \log_2(x - 1) = \dfrac{\ln(x - 1)}{\ln 2}$




**70.** $f(x) = 2 - \log_3 x = 2 - \dfrac{\ln x}{\ln 3}$

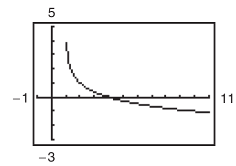


**71.** $f(x) = -\log_{1/2}(x + 2) = -\dfrac{\ln(x + 2)}{\ln(1/2)} = \dfrac{\ln(x + 2)}{\ln 2}$




**72.** $f(x) = \log_{1/3}(x - 1) + 1$

$= \dfrac{\ln(x - 1)}{\ln(1/3)} + 1 = -\dfrac{\ln(x - 1)}{\ln 3} + 1$




**73.** $\log_b 9 = \log_b 3^2$

$= 2 \log_b 3$

$= 2(0.5646)$

$= 1.1292$

**74.** $\log_b\left(\frac{4}{9}\right) = \log_b 2^2 - \log_b 3^2$

$= 2 \log_b 2 - 2 \log_b 3$

$= 2(0.3562) - 2(0.5646)$

$= -0.4168$

**75.** $\log_b \sqrt{5} = \log_b 5^{1/2}$

$= \frac{1}{2} \log_b 5$

$= \frac{1}{2}(0.8271)$

$= 0.41355$

**76.** $\log_b 50 = \log_b[2 \cdot 5^2]$

$= \log_b 2 + 2 \log_b 5$

$= 0.3562 + 2(0.8271)$

$= 2.0104$

**77.** $\ln(5e^{-2}) = \ln 5 + \ln e^{-2}$

$= \ln 5 - 2 \ln e$

$= \ln 5 - 2$

**78.** $\ln \sqrt{e^5} = \ln e^{5/2}$

$= \frac{5}{2} \ln e = \frac{5}{2}$

**79.** $\log_{10} 200 = \log_{10}(2 \cdot 100)$

$= \log_{10} 2 + \log_{10} 10^2$

$= \log_{10} 2 + 2$

**80.** $\log_{10} 0.002 = \log_{10}(2 \cdot 10^{-3})$

$= \log_{10} 2 + \log_{10} 10^{-3}$

$= \log_{10} 2 - 3$

**81.** $\log_5 5x^2 = \log_5 5 + \log_5 x^2 = 1 + 2 \log_5 x$

**82.** $\log_4(3xy^2) = \log_4 3 + \log_4 x + 2 \log_4 y$

**83.** $\log_{10} \dfrac{5\sqrt{y}}{x^2} = \log_{10} 5\sqrt{y} - \log_{10} x^2$

$= \log_{10} 5 + \log_{10}\sqrt{y} - \log_{10} x^2$

$= \log_{10} 5 + \dfrac{1}{2} \log_{10} y - 2 \log_{10} x$

**84.** $\ln \dfrac{\sqrt{x}}{4} = \ln x^{1/2} - \ln 4 = \dfrac{1}{2}\ln x - \ln 4$

$= \dfrac{1}{2}\ln x - 2 \ln 2$

**85.** $\ln\left(\dfrac{x + 3}{xy}\right) = \ln(x + 3) - \ln(xy)$

$= \ln(x + 3) - \ln x - \ln y$

**86.** $\ln \dfrac{xy^5}{\sqrt{z}} = \ln x + \ln y^5 - \ln z^{1/2}$

$= \ln x + 5 \ln y - \dfrac{1}{2}\ln z$

**87.** $\log_2 5 + \log_2 x = \log_2 5x$

**88.** $\log_6 y - 2 \log_6 z = \log_6 y - \log_6 z^2$

$= \log_6 \dfrac{y}{z^2}$

**89.** $\dfrac{1}{2} \ln(2x - 1) - 2 \ln(x + 1) = \ln\sqrt{2x - 1} - \ln(x + 1)^2$

$= \ln \dfrac{\sqrt{2x - 1}}{(x + 1)^2}$

**90.** $5\ln(x-2) - \ln(x+2) - 3\ln(x) = \ln(x-2)^5 - \ln(x+2) - \ln(x)^3$

$$= \ln\left(\frac{(x-2)^5}{(x+2)x^3}\right)$$

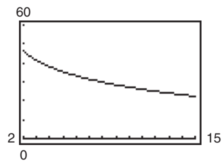
**91.** $\ln 3 + \frac{1}{3}\ln(4-x^2) - \ln x = \ln\left[\frac{3(4-x^2)^{1/3}}{x}\right] = \ln\left[\frac{3\sqrt[3]{4-x^2}}{x}\right]$

**92.** $3[\ln x - 2\ln(x^2+1)] + 2\ln 5 = \ln x^3 - \ln(x^2+1)^6 + \ln 5^2$

$$= \ln\frac{25x^3}{(x^2+1)^6}$$

**93.** $s = 25 - \dfrac{13\ln(h/12)}{\ln 3}$

(a)




(b)

| $h$ | 4 | 6 | 8 | 10 | 12 | 14 |
|---|---|---|---|---|---|---|
| $s$ | 38 | 33.2 | 29.8 | 27.2 | 25 | 23.2 |

(c) As the depth increases, the number of miles of roads cleared decreases.

**94.**

$$f(t) = 85 - 14\log_{10}(t+1)$$
$$71 = 85 - 14\log_{10}(t+1)$$
$$\log_{10}(t+1) = 1$$
$$t = 9 \text{ months}$$

**95.** $8^x = 512 = 8^3 \implies x = 3$

**96.** $3^x = 729 = 3^6 \implies x = 6$

**97.** $6^x = \frac{1}{216} = \frac{1}{6^3} = 6^{-3} \implies x = -3$

**98.** $6^{x-2} = 1296 = 6^4 \implies x - 2 = 4 \implies x = 6$

**99.**

$$2^{x+1} = \frac{1}{16}$$
$$2^{x+1} = 2^{-4}$$
$$x + 1 = -4$$
$$x = -5$$

**100.**

$$4^{x/2} = 64$$
$$4^{x/2} = 4^3$$
$$\frac{x}{2} = 3$$
$$x = 6$$

**101.** $\log_7 x = 4 \implies x = 7^4 = 2401$

**102.** $\log_x 243 = 5 \implies x^5 = 243 = 3^5 \implies x = 3$

**103.**

$$\log_2(x-1) = 3$$
$$2^3 = x - 1$$
$$x = 9$$

**104.**

$$\log_5(2x+1) = 2$$
$$5^2 = 2x + 1$$
$$2x = 24$$
$$x = 12$$

**105.**

$$\ln x = 4$$
$$x = e^4 \approx 54.598$$

**106.** $\ln x = -3$

$x = e^{-3} \approx 0.0498$

**107.** $\ln(x - 1) = 2$

$e^2 = x - 1$

$x = 1 + e^2$

**108.** $\ln(2x + 1) = -4$

$e^{-4} = 2x + 1$

$x = \dfrac{e^{-4} - 1}{2}$

$= \dfrac{1 - e^4}{2e^4}$

**109.** $3e^{-5x} = 132$

$e^{-5x} = 44$

$-5x = \ln 44$

$x = -\dfrac{\ln 44}{5} \approx -0.757$

**110.** $14e^{3x+2} = 560$

$e^{3x+2} = 40$

$\ln e^{3x+2} = \ln 40$

$3x + 2 = \ln 40$

$x = \dfrac{(\ln 40) - 2}{3} \approx 0.563$

**111.** $2^x + 13 = 35$

$2^x = 22$

$x \ln 2 = \ln 22$

$x = \dfrac{\ln 22}{\ln 2} \approx 4.459$

**112.** $6^x - 28 = -8$

$6^x = 20$

$x \ln 6 = \ln 20$

$x = \dfrac{\ln 20}{\ln 6} \approx 1.672$

**113.** $-4(5^x) = -68$

$5^x = 17$

$x \ln 5 = \ln 17$

$x = \dfrac{\ln 17}{\ln 5} \approx 1.760$

**114.** $2(12^x) = 190$

$12^x = 95$

$x \ln 12 = \ln 95$

$x = \dfrac{\ln 95}{\ln 12} \approx 1.833$

**115.** $2e^{x-3} - 1 = 4$

$2e^{x-3} = 5$

$e^{x-3} = \dfrac{5}{2}$

$x - 3 = \ln\left(\dfrac{5}{2}\right)$

$x = 3 + \ln\left(\dfrac{5}{2}\right) \approx 3.916$

**116.** $-e^{x/2} + 1 = \dfrac{1}{2}$

$e^{x/2} = \dfrac{1}{2}$

$\dfrac{x}{2} = \ln\left(\dfrac{1}{2}\right) = -\ln 2$

$x = -2 \ln 2$

$= -\ln 4 \approx -1.386$

**117.** $e^{2x} - 7e^x + 10 = 0$

$(e^x - 5)(e^x - 2) = 0$

$e^x = 5 \implies x = \ln 5 \approx 1.609$

$e^x = 2 \implies x = \ln 2 \approx 0.693$

**118.** $e^{2x} - 6e^x + 8 = 0$

$(e^x - 4)(e^x - 2) = 0$

$e^x = 4$ or $e^x = 2$

$x = \ln 4$ or $x = \ln 2$

$x \approx 1.386$ $\quad x \approx 0.693$

**119.** $\ln 3x = 8.2$

$3x = e^{8.2}$

$x = \dfrac{e^{8.2}}{3} \approx 1213.650$

**120.** $\ln 5x = 7.2$

$5x = e^{7.2}$

$x = \dfrac{1}{5}e^{7.2} \approx 267.886$

**121.** $\ln x - \ln 3 = 2$

$\ln \dfrac{x}{3} = 2$

$\dfrac{x}{3} = e^2$

$x = 3e^2 \approx 22.167$

**122.** $\ln x - \ln 5 = 4$

$\ln\left(\dfrac{x}{5}\right) = 4$

$e^4 = \dfrac{x}{5}$

$x = 5e^4 \approx 272.991$

**123.** $\ln\sqrt{x + 1} = 2$

$\dfrac{1}{2}\ln(x + 1) = 2$

$\ln(x + 1) = 4$

$x + 1 = e^4$

$x = e^4 - 1$

$\approx 53.598$

**124.** $\ln\sqrt{x+8} = 3$

$\frac{1}{2}\ln(x+8) = 3$

$\ln(x+8) = 6$

$x + 8 = e^6$

$x = e^6 - 8 \approx 395.429$

**125.** $\log_4(x-1) = \log_4(x-2) - \log_4(x+2)$

$\log_4(x-1) = \log_4\left(\frac{x-2}{x+2}\right)$

$x - 1 = \frac{x-2}{x+2}$

$(x-1)(x+2) = x - 2$

$x^2 + x - 2 = x - 2$

$x^2 = 0$

$x = 0$ (extraneous)

No solution

**126.** $\log_5(x+2) - \log_5(x) = \log_5(x+5)$

$\log_5\left(\frac{x+2}{x}\right) = \log_5(x+5)$

$\frac{x+2}{x} = x + 5$

$x + 2 = x^2 + 5x$

$x^2 + 4x - 2 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(-2)}}{2} = -2 \pm \sqrt{6}$

$x = -2 + \sqrt{6} \approx 0.449$

(Other zero is extraneous.)

**127.** $\log_{10}(1-x) = -1$

$10^{-1} = 1 - x$

$x = 1 - 10^{-1} = 0.9$

**128.** $\log_{10}(-x-4) = 2$

$-x - 4 = 10^2 = 100$

$-x = 104$

$x = -104$

**129.** $xe^x + e^x = 0$

$(x+1)e^x = 0$

$x + 1 = 0$ (since $e^x \neq 0$)

$x = -1$

**130.** $2xe^{2x} + e^{2x} = 0$

$(2x+1)e^{2x} = 0$

$2x + 1 = 0$ (since $e^{2x} \neq 0$)

$x = -\frac{1}{2}$

**131.** $x\ln x + x = 0$

$x(\ln x + 1) = 0$

$\ln x + 1 = 0$ (since $x > 0$)

$\ln x = -1$

$x = e^{-1} = \frac{1}{e} \approx 0.368$

**132.** $\frac{1 - \ln x}{x^2} = 0$

$1 - \ln x = 0$ (since $x > 0$)

$\ln x = 1$

$x = e \approx 2.718$

**133.** $3(7550) = 7550e^{0.0725t}$

$3 = e^{0.0725t}$

$\ln 3 = 0.0725t$

$t = \frac{\ln 3}{0.0725} \approx 15.2$ years

**134.** $p = 500 - 0.5e^{0.004x}$

(a)
$$\begin{aligned} p &= 450 \\ 450 &= 500 - 0.5e^{0.004x} \\ 0.5e^{0.004x} &= 50 \\ e^{0.004x} &= 100 \\ 0.004x &= \ln 100 \\ x &\approx 1151 \text{ units} \end{aligned}$$

(b)
$$\begin{aligned} p &= 400 \\ 400 &= 500 - 0.5e^{0.004x} \\ 0.5e^{0.004x} &= 100 \\ e^{0.004x} &= 200 \\ 0.004x &= \ln 200 \\ x &\approx 1325 \text{ units} \end{aligned}$$

**135.** $y = 3e^{-2x/3}$

Decreasing exponential

Matches graph (e).

**136.** $y = 4e^{2x/3}$

Intercept: $(0, 4)$

Increasing

Matches (b).

**137.** $y = \ln(x + 3)$

Logarithmic function shifted to left

Matches graph (f).

**138.** $y = 7 - \log(x + 3)$

Vertical asymptote: $x = -3$

Decreasing

Matches (d).

**139.** $y = 2e^{-(x+4)^2/3}$

Gaussian model

Matches graph (a).

**140.** $y = \dfrac{6}{1 + 2e^{-2x}}$

Logistic model

Matches (c).

**141.**
$$\begin{aligned} y &= ae^{bx} \\ 2 &= ae^{b(0)} \implies a = 2 \\ 3 &= 2e^{b(4)} \\ 1.5 &= e^{4b} \end{aligned}$$

$\ln 1.5 = 4b \implies b \approx 0.1014$

Thus, $y = 2e^{0.1014x}$.

**142.** $y = ae^{bx}$

$2 = ae^{b(0)} \implies a = 2$

$1 = 2e^{b(5)} \implies \frac{1}{2} = e^{5b} \implies 5b = \ln \frac{1}{2} \implies b = \frac{1}{5} \ln \frac{1}{2} = -\frac{1}{5} \ln 2 \approx -0.1386$

$y = 2e^{-0.1386x}$

**143.**
$$\begin{aligned} y &= ae^{bx} \\ \frac{1}{2} &= ae^{b(0)} \implies a = \frac{1}{2} \\ 5 &= \frac{1}{2}e^{b(5)} \\ 10 &= e^{5b} \end{aligned}$$

$\ln 10 = 5b \implies b \approx 0.4605$

Thus, $y = \frac{1}{2}e^{0.4605x}$.

**144.**
$$\begin{aligned} y &= ae^{bx} \\ 4 &= ae^{b(0)} = a \implies a = 4 \\ \frac{1}{2} &= 4e^{b(5)} \\ \frac{1}{8} &= e^{5b} \end{aligned}$$

$\ln \frac{1}{8} = 5b \implies b = -\frac{\ln 8}{5} \approx -0.4159$

$\implies y = 4e^{-0.4159x}$

**145.** $P = 361e^{kt}$

$t = 0$ corresponds to 2000.

$(-20, 215)$:

$$215 = 361e^{k(-20)}$$

$$\frac{215}{361} = e^{-20k}$$

$$-20k = \ln\left(\frac{215}{361}\right)$$

$$k = -\frac{1}{20}\ln\left(\frac{215}{361}\right) = \frac{1}{20}\ln\left(\frac{361}{215}\right) \approx 0.02591$$

$P = 361e^{0.02591t}$

For 2020, $P(20) = 361e^{0.02591(20)} \approx 606.1$ or 606,100 population in 2020.

**146.** $\frac{1}{2}P = Pe^{k(245{,}500)}$

$$\ln\left(\frac{1}{2}\right) = 245{,}500k \implies k = \frac{-\ln 2}{245{,}500}$$

$$\approx -2.8234 \times 10^{-6}$$

After 5000 years,

$A = e^{k(5000)} \approx 0.98598$ or 98.6% remains.

**147.** (a) $20{,}000 = 10{,}000e^{r(12)}$

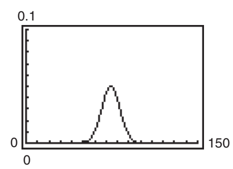
$$2 = e^{12r}$$

$$\ln 2 = 12r$$

$$r = \frac{\ln 2}{12} \approx 0.0578 \text{ or } 5.78\%$$

(b) $10{,}000e^{0.0578(1)} \approx \$10{,}595.03$

**148.** (a)




$y = 0.0499e^{-(x-74)^2/128}$

(b) The average score corresponds to the maximum, 74.

**149.** (a)

$$50 = \frac{158}{1 + 5.4e^{-0.12t}}$$

$$1 + 5.4e^{-0.12t} = \frac{158}{50}$$

$$5.4e^{-0.12t} = \frac{108}{50}$$

$$e^{-0.12t} = \frac{108}{50(5.4)}$$

$$-0.12t = \ln\frac{108}{270}$$

$$t = \frac{\ln(108/270)}{-0.12} \approx 7.6 \text{ weeks}$$

(b) Similarly:

$$75 = \frac{158}{1 + 5.4e^{-0.12t}}$$

$$1 + 5.4e^{-0.12t} = \frac{158}{75}$$

$$e^{-0.12t} = 0.20494$$

$$-0.12t = \ln(0.20494)$$

$$t \approx 13.2 \text{ weeks}$$

**150.** $R = \log_{10}\left(\frac{I}{I_0}\right) = \log_{10}(I) \implies I = 10^R$

(a) $I = 10^{8.4} \approx 251{,}188{,}643$

(b) $I = 10^{6.85} \approx 7{,}079{,}458$

(c) $I = 10^{9.1} \approx 1{,}258{,}925{,}412$

**151.** Logistic model

**152.** Linear model

**153.** Logarithmic model

**154.** Quadratic model (or exponential)

**155.** (a) Linear model: $y = 297.8t + 739$; 0.97653

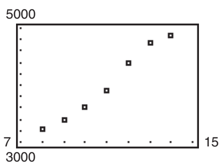
Quadratic model: $y = 11.79t^2 + 38.5t + 2118$; 0.98112

Exponential model: $y = 1751.5(1.077)^t$; 0.98225

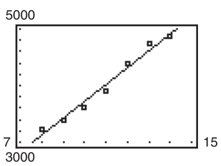
Logarithmic model: $y = 3169.8 \ln t - 3532$; 0.95779

Power model: $y = 598.1t^{0.7950}$; 0.97118

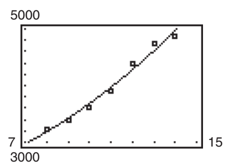
(b)



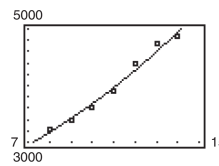

Linear model:



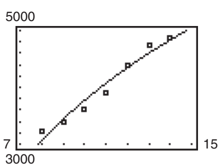

Quadratic model:



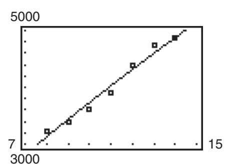

Exponential model:




Logarithmic model:




Power model:




(c) The exponential model is best because its coefficient of determination is closest to 1. Answers will vary.

(d) For 2010, $t = 20$ and $y \approx \$7722$ million.

(e) $y = 5250$ when $t \approx 14.8$, or 2004.

**156.** (a) Linear model: $y = 82.9t + 1825$; 0.96953

Quadratic model: $y = -3.35t^2 + 133.2t + 1691$; 0.98982

Exponential model: $y = 1865(1.0354)^t$; 0.95481

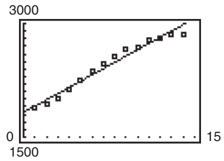
Logarithmic model: $y = 1663 + 435.5 \ln t$; 0.91664

Power model: $y = 1733t^{0.1862}$; 0.93412

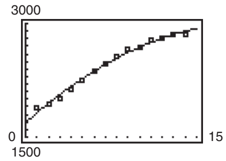
**—CONTINUED—**

**156. —CONTINUED—**

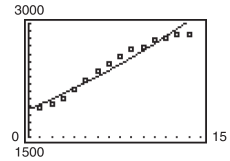
(b) Linear model:



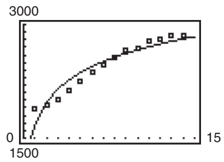

Quadratic model:



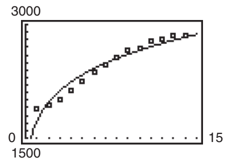

Exponential model:




Logarithmic model:




Power model:



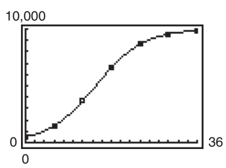

(c) The quadratic model is best because its coefficient of determination is closest to 1.

(d) For 2010, $t = 20$ and $y \approx 3015$ thousand.

(e) $y = 3000$ when $t \approx 17.8$, or 2007.

**157.** (a) $P = \dfrac{9999.887}{1 + 19.0e^{-0.2x}}$

(b)




(c) The model is a good fit.

(d) The limiting size is $\dfrac{9999.887}{1 + 0} \approx 10{,}000$ fish.

**158.** $P = 56.8e^{0.001603t}$

(a)

| Year | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
|---|---|---|---|---|---|---|
| $P$ | 56.8 | 56.9 | 57 | 57.1 | 57.2 | 57.3 |

| Year | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
|---|---|---|---|---|---|---|
| $P$ | 57.3 | 57.4 | 57.5 | 57.6 | 57.7 | 57.8 |

| Year | 2002 | 2003 | 2004 | 2005 |
|---|---|---|---|---|
| $P$ | 57.9 | 58.0 | 58.1 | 58.2 |

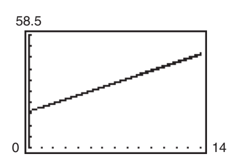
(b) $\text{Slope} = \dfrac{58.2 - 56.8}{15 - 0} \approx 0.09$

$y - 56.8 = 0.09(t - 0)$

$y = 0.09t + 56.8$, Linear model

(c) Slope is 0.09. The population increases by 90,000 people each year.

(d)




Answers will vary.

**159.** True; by the Inverse Properties, $\log_b b^{2x} = 2x$.

**160.** $e^{x-1} = e^x \cdot e^{-1} = \frac{e^x}{e}$

True (by Properties of exponents).

**161.** False; $\ln x + \ln y = \ln(xy) \neq \ln(x + y)$

**162.** $\ln(x + y) = \ln(x \cdot y)$

False

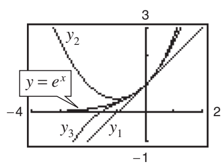
$\ln(x \cdot y) = \ln x + \ln y \neq \ln(x + y)$

**163.** False. The domain of $f(x) = \ln(x)$ is $x > 0$.

**164.** True. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

**165.** Since $1 < \sqrt{2} < 2$, $2^1 < 2^{\sqrt{2}} < 2^2 \implies 2 < 2^{\sqrt{2}} < 4$.

**166.** (a)




(b) Pattern $\sum_{i=0}^{n} \frac{x^i}{i!}$

$y_4 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

The graph of $y_4$ closely approximates $y = e^x$ near $(0, 1)$.

## Chapter 3 Practice Test

**1.** Solve for $x$: $x^{3/5} = 8$

**2.** Solve for $x$: $3^{x-1} = \frac{1}{81}$

**3.** Graph $f(x) = 2^{-x}$ by hand.

**4.** Graph $g(x) = e^x + 1$ by hand.

**5.** If \$5000 is invested at 9% interest, find the amount after three years if the interest is compounded
(a) monthly. (b) quarterly. (c) continuously.

**6.** Write the equation in logarithmic form: $7^{-2} = \frac{1}{49}$

**7.** Solve for $x$: $x - 4 = \log_2 \frac{1}{64}$

**8.** Given $\log_b 2 = 0.3562$ and $\log_b 5 = 0.8271$, evaluate $\log_b \sqrt[4]{8/25}$.

**9.** Write $5 \ln x - \frac{1}{2} \ln y + 6 \ln z$ as a single logarithm.

**10.** Using your calculator and the change of base formula, evaluate $\log_9 28$.

**11.** Use your calculator to solve for $N$: $\log_{10} N = 0.6646$

**12.** Graph $y = \log_4 x$ by hand.

**13.** Determine the domain of $f(x) = \log_3(x^2 - 9)$.

**14.** Graph $y = \ln(x - 2)$ by hand.

**15.** True or false: $\dfrac{\ln x}{\ln y} = \ln(x - y)$

**16.** Solve for $x$: $5^x = 41$

**17.** Solve for $x$: $x - x^2 = \log_5 \frac{1}{25}$

**18.** Solve for $x$: $\log_2 x + \log_2(x - 3) = 2$

**19.** Solve for $x$: $\dfrac{e^x + e^{-x}}{3} = 4$

**20.** Six thousand dollars is deposited into a fund at an annual percentage rate of 13%.
Find the time required for the investment to double if the interest is compounded continuously.

**21.** Use a graphing utility to find the points of intersection of the graphs of $y = \ln(3x)$ and $y = e^x - 4$.

**22.** Use a graphing utility to find the power model $y = ax^b$ for the data (1, 1), (2, 5), (3, 8), and (4, 17).