CALCULUS PROBLEM BOOK

P. 104 Exc 879–910

879 $\int -2x\sqrt{9-x^2}\,dx = \frac{2}{3}\cdot(9-x^2)^{3/2} + C$

880 $\int x(4x^2+3)^3\,dx = \frac{1}{8}\int 8x(4x^2+3)^3\,dx =$

$\frac{1}{8}\cdot\frac{(4x^2+3)^4}{4} = \frac{(4x^2+3)^4}{32} + C$

881 $\int \frac{x^2}{(1+x^3)^2}\,dx = \frac{1}{3}\int \frac{3x^2}{(1+x^3)^2}\,dx$

$= \frac{1}{3}\cdot\frac{(1+x^3)^{-1}}{-1} = \frac{-1}{3(1+x^3)} + C$

882 $\int\left(x^2+\frac{1}{9x^2}\right)dx = \int x^2\,dx + \frac{1}{9}\int x^{-2}\,dx$

$= \frac{x^3}{3} + \frac{1}{9}\cdot\frac{x^{-1}}{-1} = \frac{x^3}{3} - \frac{1}{9x} + C$

883 $\int \frac{x^2+3x+7}{\sqrt{x}}\,dx = \int x^{3/2}\,dx + 3\int x^{1/2}\,dx + 7\int x^{-1/2}\,dx$

$= \frac{2}{5}\cdot x^{5/2} + 3\cdot\frac{2}{3}x^{3/2} + 7\cdot\frac{2}{1}x^{1/2} = \frac{2}{5}x^{5/2} + 2x^{3/2} + 14x^{1/2} + C$

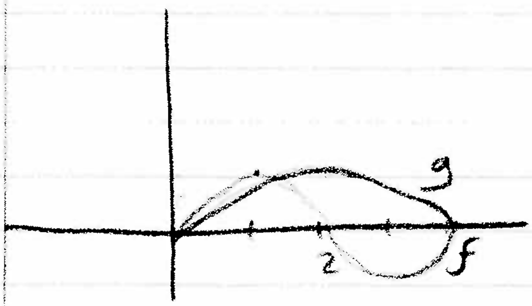
884 $\int\left(\frac{t^3}{3} + \frac{1}{4t^2}\right)dt = \frac{1}{3}\int t^3\,dt + \frac{1}{4}\int t^{-2}\,dt$

$= \frac{1}{3}\cdot\frac{t^4}{4} + \frac{1}{4}\cdot\frac{t^{-1}}{-1} = \frac{t^4}{12} - \frac{1}{4t} + C$

(93) $g(x) = 0$

$(0,2)$ INCREASING , MAX @ 2

$(2,4)$ DECREASING



$g(0)=0$

$g(1)=\frac{1}{2}$

$g(2)=1$

$g(3)=\frac{1}{2}$

$g(4)=0$

885 $\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \frac{1}{2}\cos 2x + C$

886 $\int \cos 6x\,dx = \frac{1}{6}\int 6\cos 6x\,dx = \frac{1}{6}\sin 6x + C$

887 $\int \tan^4\theta \sec^2\theta\,d\theta$ LET $u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\int u^4\,du = \frac{u^5}{5} + C = \frac{\tan^5\theta}{5} + C$

888 $\int \frac{\sin\theta}{\cos^2\theta}\,d\theta$ LeT $u = \cos\theta$

$du = -\sin\theta\,d\theta$

$-du = \sin\theta\,d\theta$

$\int \frac{-du}{u^2} = -\int u^{-2}\,du = -1\cdot\frac{u^{-1}}{-1}$

$= \frac{1}{u} = \frac{1}{\cos\theta} + C$

889 $\int \cos\left(\frac{\theta}{2}\right)\,d\theta = 2\int \frac{1}{2}\cos\left(\frac{\theta}{2}\right)\,d\theta$

$= 2\cdot\ \sin\left(\frac{\theta}{2}\right) = 2\sin\left(\frac{\theta}{2}\right) + C$

890 $\int x\sqrt{2x+1}\,dx$ LET $u = 2x+1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$x = \frac{u-1}{2}$

$\int \left(\frac{u-1}{2}\right)\sqrt{u}\cdot\frac{du}{2}$

$\frac{1}{4}\int u^{3/2} - u^{1/2}\,du = \frac{1}{4}\left[\frac{2}{5}\cdot u^{5/2} - \frac{2}{3}u^{3/2}\right] = \frac{1}{4}\left[\frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2}\right] + C$

(891) $\int x^2\sqrt{1-x}\,dx$ Let $u = 1-x$

$du = -1\,dx$

$-du = dx$

$-x = u-1$

$x = 1-u$

$\int -(1-u)^2 \cdot u^{1/2}\,du$

$-\int (1-2u+u^2)\,u^{1/2}\,du$

$-\int (u^{1/2} - 2u^{3/2} + u^{5/2})\,du = -\left[\frac{2}{3}\cdot u^{3/2} - 2\cdot\frac{2}{5}\cdot u^{5/2} + \frac{2}{7}\cdot u^{7/2}\right]$

$= -\frac{2}{3}(1-x)^{3/2} + \frac{4}{5}(1-x)^{5/2} - \frac{2}{7}(1-x)^{7/2} + C$

(892) $\int \sqrt{4x-3}\,dx$ $u = 4x-3$

$du = 4\,dx$

$\frac{du}{4} = dx$

$\int \sqrt{u}\,\frac{du}{4} = \frac{1}{4}\cdot\frac{2}{3}u^{3/2} = \frac{1}{6}(4x-3)^{3/2} + C$

(893) $\int x^4\sqrt{3x^5-4}\,dx = \frac{1}{15}\int 15x^4\sqrt{3x^5-4}\,dx$

$= \frac{1}{15}\cdot\frac{2}{3}(3x^5-4)^{3/2} = \frac{2}{45}(3x^5-4)^{3/2} + C$

894 $\int \frac{3x^6}{(2x^7-1)^5}\,dx = \frac{1}{14\cdot 3}\int \frac{14\cdot 3x^6}{(2x^7-1)^5}\,dx$

$\frac{1}{42}\cdot\frac{(2x^7-1)^{-4}}{-4} = -\frac{1}{168}(2x^7-1)^{-4}+C$

895 $\int 4x\sqrt{5x-2}\,dx$

$u = 5x-2$
$du = 5\,dx$
$\frac{du}{5} = dx$
$x = \frac{u+2}{5}$

$\int 4\left(\frac{u+2}{5}\right)\sqrt{u}\,\frac{du}{5} = \frac{4}{25}\int (u^{3/2}+2u^{1/2})\,du$

$\frac{4}{25}\cdot\frac{2}{5}u^{5/2}+2\cdot\frac{2}{3}u^{3/2} = \frac{8}{25}\left[\frac{(5x-2)^{5/2}}{5}+\frac{2}{3}(5x-2)^{3/2}\right]+C$

896 $\int 12x^2\sin(4x^3)\,dx = -\cos(4x^3)+C$

897 $\int 4e^x\cos(4e^x)\,dx = \sin(4e^x)+C$

898 $\int 3^{3t}\ln 3\,dt =$ Let $u = 3t$
$du = 3\,dt$
$\frac{du}{3} = dt$

$\frac{\ln 3}{3}\int 3^u\,du = \frac{\cancel{\ln 3}}{3}\cdot\frac{1}{\cancel{\ln 3}}3^{3t}$ $\boxed{\frac{3^{3t}}{3}}$

(899) $\int 6^{2x^2-3} x \ln 6 = \ln 6 \int 6^{2x^2-3} \cdot x \, dx$

$u = 2x^2-3$
$du = 4x\,dx$
$\frac{du}{4} = x\,dx$

$= \frac{\ln 6}{4} \int 6^u dx = \frac{\cancel{\ln 6}}{4} \cdot \frac{6^{2x^2-3}}{\cancel{\ln 6}}$

$= \frac{6^{2x^2-3}}{4} + C$

(900) $\int 2^{5x} dx = \frac{1}{5} \int 5 \cdot 2^{5x} = \frac{2^{5x}}{5 \ln 2}$

$u = 5x$
$du = 5dx$
$\frac{du}{5} = dx$

$\frac{1}{5} \int 2^u dx = \frac{1}{5} \cdot \frac{1}{\ln 2} \cdot 2^{5x} = \frac{2^{5x}}{5 \ln 2} + C$

OR

SHORT CUT

$(2^{5x})' = (\ln 5) \cdot 2^{5x} \cdot 5$

$\frac{1}{5 \cdot \ln 5} \int 2^{5x} = \frac{2^{5x}}{5 \ln 5} + C$

(901) $\int \frac{1}{\sqrt{5x+4}} dx = \frac{1}{5} \int \frac{5}{\sqrt{5x+4}} dx$

$= \frac{1}{5} \cdot \frac{2}{1} u^{\frac{1}{2}} = \frac{2}{5} (5x+4)^{1/2} + C$

902 $\int 3y\sqrt{7-3y^2}\,dy = -\frac{1}{2}\int -2\cdot 3y\sqrt{7-3y^2}\,dy$

$= -\frac{1}{2}\cdot\frac{2}{3}(7-3y^2)^{3/2} = -\frac{1}{3}(7-3y^2)^{3/2} + C$

903 $\int \cos(3z+4)\,dz = \frac{1}{3}\int 3\cos(3z+4)\,dz$

$\frac{1}{3}\cdot\sin(3z+4) + C$

* 904 $\int \frac{1}{t^2}e^{1/t}\,dt = -e^{1/t}$

* 905 $\int \sec(x+\pi/2)\tan(x+\pi/2)\,dx = \sec(x+\frac{\pi}{2})$

906 $\int -\csc^2\theta\sqrt{\cot\theta}\,d\theta = \frac{2}{3}(\cot\theta)^{3/2} + C$

907 $\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx =$

$u = x^2+4$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\cdot\ln(x^2+4)$

908 $\int \frac{1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int \frac{2}{\sqrt{1-(2x)^2}} = \frac{1}{2}\arcsin\frac{2x}{1}$

909 $\int \frac{e^x}{1+(e^x)^2}\,dx = \boxed{\tan^{-1} e^x}$

910 $\int \frac{1}{x}\,dx = \ln x$