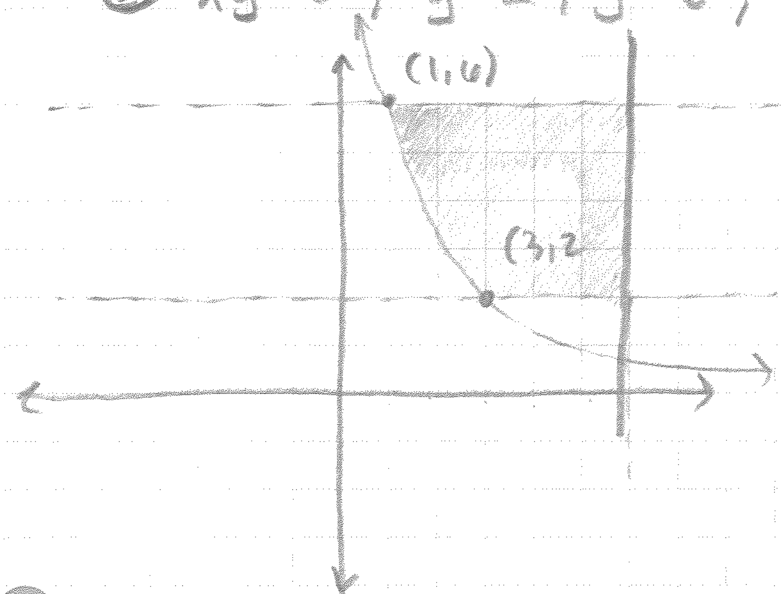
ASSIGNMENT 69

SECTION 7-2 EXC 22, 40
SECTION 7-1 EXC 67, 71
SECTION 4-4 EXC 25, 45, 46
SECTION 4-5 EXC 26, 48
CPB EXC 1111, 1116, 1119

(22) $xy = 6, \quad y = 2, \quad y = 6, \quad x = 6$



$xy = 6 \implies y = \frac{6}{x}$

$\frac{6}{x} = 2 \implies x = 3$

$$A = \pi \int_2^6 \left(6 - \frac{6}{y}\right)^2 dy$$

$$= \pi \int_2^6 \left[36 - \frac{72}{y} + \frac{36}{y^2}\right] dy$$

$$= 36\pi \int_2^6 \left(1 - \frac{2}{y} + \frac{1}{y^2}\right) dy$$

$$= 36\pi \left[y - 2\ln y - \frac{1}{y} \right]_2^6$$

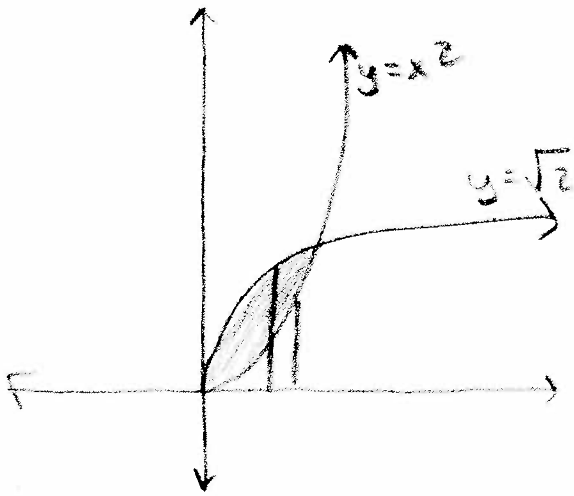
$$= 36\pi \left[\left(6 - 2\ln 6 - \frac{1}{6}\right) - \left(2 - 2\ln 2 - \frac{1}{2}\right) \right]$$

$$= 36\pi \left(\left[\frac{35}{6} - 2\ln 6\right] - \left[\frac{3}{2} - 2\ln 2\right] \right)$$

$$= 36\pi \left(\frac{13}{3} - 2\ln 6 + 2\ln 2 \right) = 36\pi \left(\frac{13}{3} + \ln 4 - \ln 36 \right)$$

$$= \boxed{36\pi \left(\frac{13}{3} + \ln(1/9) \right)}$$

(40) $y = \sqrt{2x}$, $y = x^2$



$x^2 = \sqrt{2x}$
$x^4 = 2x$
$x^4 - 2x = 0$
$x(x^3 - 2) = 0$
$x = 0 \quad x = \sqrt[3]{2}$

$$V = \pi \int_0^{\sqrt[3]{2}} \left[(\sqrt{2x})^2 - (x^2)^2 \right] dx$$

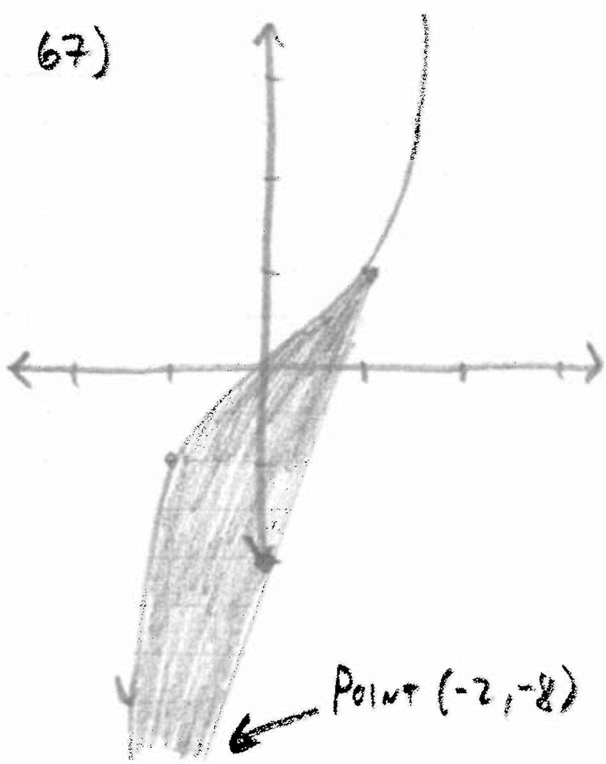
$$= \pi \int_0^{\sqrt[3]{2}} (2x - x^4)\, dx$$

$$= \pi \left[\frac{2x^2}{2} - \frac{x^5}{5} \right]_0^{\sqrt[3]{2}}$$

$$= \pi \left[\left((2^{1/3})^2 - \frac{(2^{1/3})^5}{5} \right) - (0) \right]$$

$$\boxed{= \pi \left(2^{2/3} - \frac{2^{5/3}}{5} \right)}$$

67)



$f(x) = x^3$
$f'(x) = 3x^2$
$f'(1) = 3$

Tangent line
$m = 3 \quad (1,1)$
$y - y_1 = m(x - x_1)$
$y - 1 = 3(x - 1)$
$y = 3x - 2$

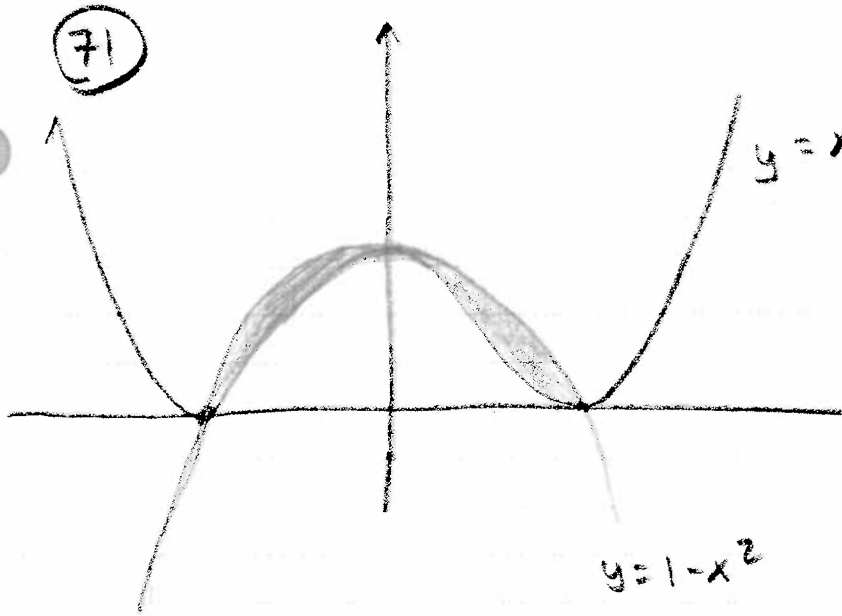
$x^3 = 3x - 2$
$x = -2$

$$A = \int_{-2}^{1} [x^3 - (3x - 2)]\, dx$$

$$A = \left[\frac{x^4}{4} - \frac{3x^2}{2} - 2x \right]_{-2}^{1}$$

$$= \boxed{\frac{27}{4}}$$

(71)

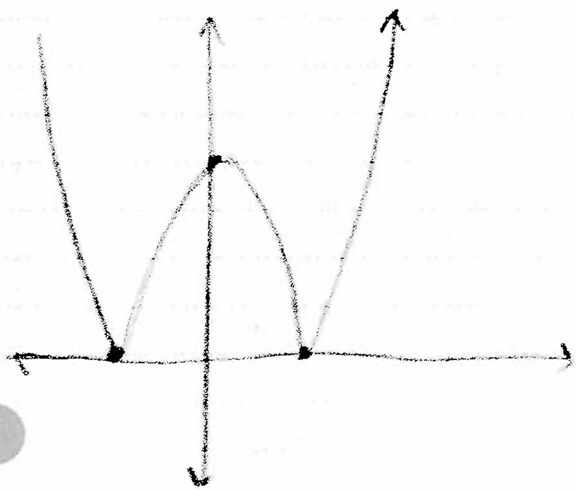


SINCE THE UPPER BOUND REMAINS CONSTANT FROM [-1,1], WE ONLY NEED ONE INTEGRAL

$$A = \int_{-1}^{1} \left[(1-x^2) - (x^4 - 2x^2 + 1) \right] dx$$

$$= \int_{-1}^{1} (-x^4 + x^2)\, dx = \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_{-1}^{1} = \boxed{\frac{4}{15}}$$

(25) $\int_0^3 |x^2 - 4|\, dx$



$$A = \int_0^2 -(x^2 - 4)\, dx + \int_2^3 (x^2 - 4)\, dx$$

$$= \left[4x - \frac{x^3}{3} \right]_0^2 + \left[\frac{x^3}{3} - 4x \right]_2^3$$

$$= \boxed{\frac{23}{3}}$$

(45) $f(x) = 2\sec^2 x$, $[-\pi/4, \pi/4]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{\frac{\pi}{4}+\frac{\pi}{4}} \cdot 2\int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = \frac{4}{\pi}\Big[\tan x\Big]_{-\pi/4}^{\pi/4} = \frac{4}{\pi}\left(\tan(+\pi/4) - \tan(-\tfrac{\pi}{4})\right)$$

$$\frac{4}{\pi}\left(1 + 1\right) = \frac{8}{\pi}$$

FIND C

$f(c) = \frac{8}{\pi}$

$2\sec^2 c = \frac{8}{\pi}$

$\sec^2 c = \frac{4}{\pi}$

$\sec c = \pm\frac{2}{\sqrt{\pi}}$

$c = \pm\sec^{-1}\left(\frac{2}{\sqrt{\pi}}\right) = \pm .4817$

(46) $f(x) = \cos x$, $[-\pi/3, \pi/3]$

$$f(c) = \frac{1}{\frac{\pi}{3}+\frac{\pi}{3}} \cdot \int_{-\pi/3}^{\pi/3} \cos x\,dx = \frac{3}{2\pi}\Big[\sin x\Big]_{-\pi/3}^{\pi/3} = \frac{3}{2\pi}\Big[\sin(\pi/3) - \sin(-\pi/3)\Big]$$

$$\frac{3}{2\pi}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = \frac{3}{2\pi}\cdot\sqrt{3}$$

$$= \frac{3\sqrt{3}}{2\pi}$$

$f(c) = \frac{3\sqrt{3}}{2\pi}$

$\cos c = \frac{3\sqrt{3}}{2\pi}$

$\boxed{c = \arccos(3\sqrt{3}/2\pi)}$

26 $\int\left[x^2+\frac{1}{(3x)^2}\right]dx = \int\left(x^2+\frac{x^{-2}}{9}\right)dx$

$= \boxed{\frac{x^3}{3}+\frac{x^{-1}}{-9}+c}$

48 $\int x\cdot\sin x^2\,dx = \frac{1}{2}\int 2x\sin x^2\,dx = \boxed{\frac{1}{2}\cdot -\cos x^2 + c}$

1111 $\int_0^1 \frac{e^x}{(3-e^x)^2}\,dx = -\int_0^1 \frac{-e^x}{(3-e^x)^2} = \left[\frac{(3-e^x)^{-1}}{-1}\right]_0^1$

$= \left.\frac{-1}{(3-e^x)}\right]_0^1 = \left(\frac{-1}{(3-e^1)} - \frac{-1}{(3-e^0)}\right)$

$= \frac{2}{2}\cdot\frac{-1}{3-e} - \frac{1}{2}\frac{(3-e)}{(3-e)}$

$= \frac{2-3+e}{2(3-e)} = \boxed{\frac{e-1}{2(3-e)}}$

1116 $\int_0^{\pi/2} \cos^2 x\sin x\,dx = -\int_0^{\frac{\pi}{2}} -\cos^2 x\sin x\,dx$

$= -\left.\frac{(\cos^3 x)}{3}\right]_0^{\pi/2} = \frac{-\cos^3(\pi/2)}{3} + \frac{\cos^3(0)}{3}$

$0 + \frac{1}{3} = \boxed{1/3}$

1119 $\int_0^1 (2-3x)^5\,dx = -\frac{1}{3}\int_0^1 -3(2-3x)^5\,dx$

$= -\frac{1}{3}\left.\frac{(2-3x)^6}{6}\right]_0^1 = \left[-\frac{(2-3x)^6}{18}\right]_0^1 = \boxed{\frac{7}{2}}$