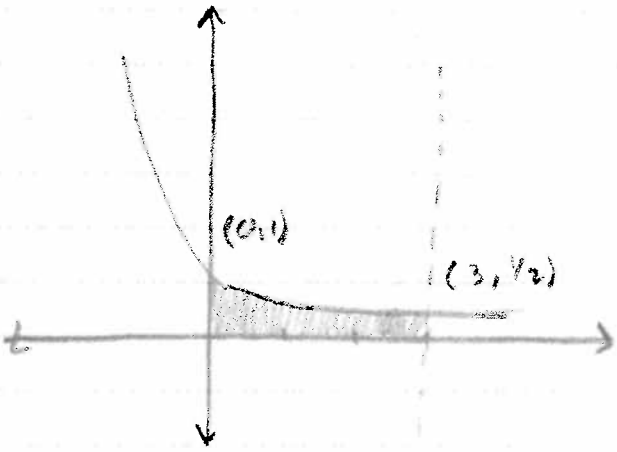
Assignment 68

Section 7-2 Exc 23, 24, 26
Section 7-1 Exc 33, 35, 47
Section 4-6 Exc 11, 14 (trapezoid only)
Section 4-5 Exc 37, 38

(23) $y = \dfrac{1}{\sqrt{x+1}}$, $y = 0$, $x = 0$, $x = 3$



$$A = \pi \int_0^3 \left(\frac{1}{\sqrt{x+1}}\right)^2 dx$$

$$= \pi \int_0^3 \frac{1}{x+1}$$

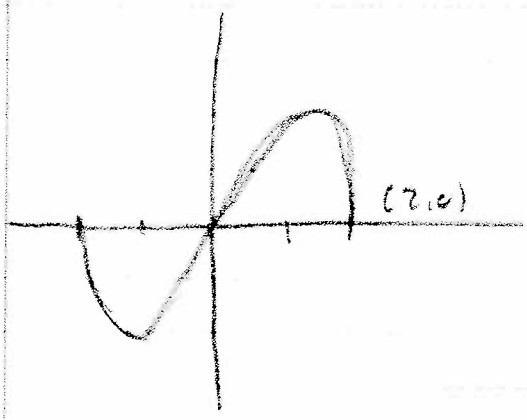
$$= \pi \ln(x+1) \Big]_0^3$$

$$= \pi [\ln(3+1) - \ln(0+1)]$$

$$\pi [\ln(4) - \ln(1)]$$

$$\pi \ln 4$$

(24) $y = x\sqrt{4-x^2}$, $y = 0$



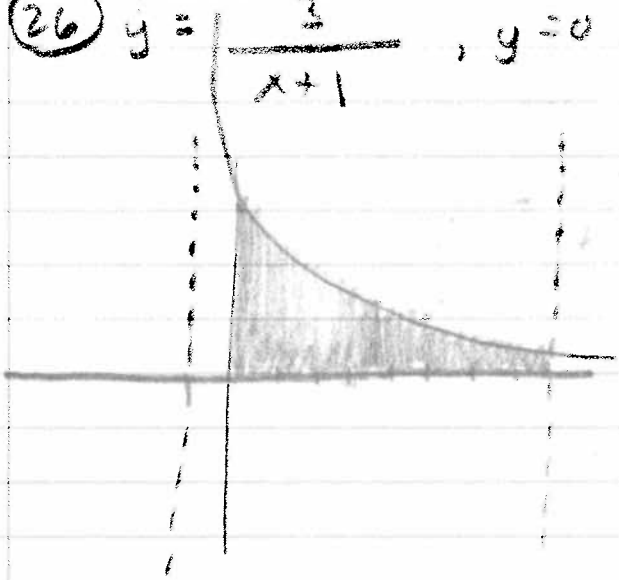
$$A = 2\pi \int \left(x\sqrt{4-x^2}\right) dx$$

$$= 2\pi \int [x^2(4-x^2)]\, dx$$

$$= 2\pi \int_0^2 (4x^2 - x^4)\, dx$$

$$= 2\pi \left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2 = \frac{128\pi}{15}$$

(26) $y = \frac{3}{x+1}$, $y=0$, $x=0$, $x=8$



$$A = \pi \int_0^8 \left(\frac{3}{x+1}\right)^2 dx$$

$$= \pi \int_0^8 \frac{9}{(x+1)^2} dx$$

$$= 9\pi \int_0^8 (x+1)^{-2} dx$$

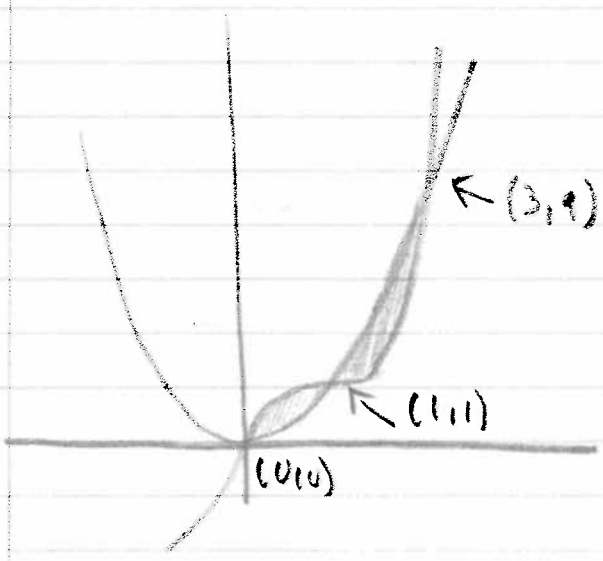
$$= 9\pi(-1)(x+1)^{-1} \Big]_0^8$$

$$-9\pi \left[\frac{1}{(8+1)} - \frac{1}{0+1}\right]$$

$$-9\pi \left[1/9 - 1\right] =$$

$$-9\pi \cdot \frac{-8}{9} = 8\pi$$

(33) $f(x) = x(x^2-3x+3)$, $g(x) = x^2$



$$A = \int_0^1 \left[(x^3-3x^2+3x) - x^2\right] dx$$

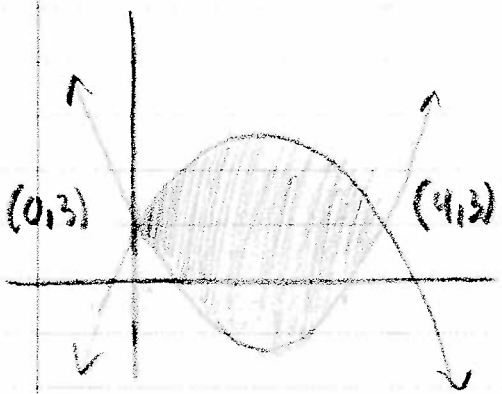
$$+$$

$$\int_1^3 \left[x^2 - (x^3-3x^2+3x)\right] dx$$

$$A = \int_0^1 (x^3 - 4x^2 + 3x)\,dx + \int_1^3 (-x^3 + 4x^2 - 3x)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_0^1 + \left[-\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2}\right]_1^3 = \boxed{\frac{37}{12}}$$

(35) $y = x^2 - 4x + 3$, $y = 3 + 4x - x^2$

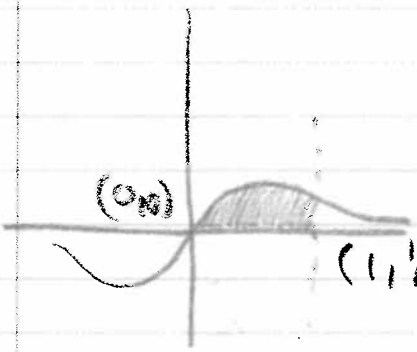


$$A = \int_0^4 [(3 + 4x - x^2) - (x^2 - 4x + 3)]\,dx$$

$$= \int_0^4 (-2x^2 + 8x)\,dx$$

$$= \left[-\frac{2x^3}{3} + \frac{8x^2}{2}\right]_0^4 = \boxed{\frac{64}{3}}$$

(47) $f(x) = xe^{-x^2}$, $y = 0$, $0 \le x \le 1$



$$A = \int_0^1 xe^{-x^2}\,dx = -\frac{1}{2}\int -2xe^{-x^2}\,dx$$

$$= -\frac{1}{2} \cdot e^{-x^2}\Big]_0^1 =$$

$$-\frac{1}{2}\left(\left[e^{-(1)^2}\right] - \left[e^{-(0)^2}\right]\right)$$

$$-\frac{1}{2}\left(\frac{1}{e} - 1\right) = \boxed{\frac{1}{2} - \frac{1}{2e}}$$

(11) $\int_0^2 \sqrt{1+x^2}\,dx$

$\approx \frac{2-0}{2(4)}\left[\sqrt{1+(0)^3} + 2\sqrt{1+(1/2)^3} + 2\sqrt{1+(1)^3} + 2\sqrt{1+(3/2)^3} + \sqrt{1+2^3}\right]$

$\approx \frac{1}{4}\left[1 + 2\sqrt{1+(1/8)} + 2\sqrt{2} + 2\sqrt{1+(27/8)} + 3\right]$

$\approx \boxed{3.283}$

$n=4$
$\Delta x = \frac{\pi}{8}$

(14) $\int_{\pi/2}^{\pi} \sqrt{x}\sin x\,dx$

$\approx \frac{\pi - \frac{\pi}{2}}{2(4)}\left[\sqrt{\frac{\pi}{2}}\sin\frac{\pi}{2} + \sqrt{\frac{5\pi}{8}}\sin\frac{5\pi}{8} + \sqrt{\frac{3\pi}{4}}\sin\frac{3\pi}{4} + \sqrt{\frac{7\pi}{8}}\sin\frac{7\pi}{8} + \sqrt{\pi}\sin\pi\right]$

$\approx \boxed{1.430}$

(37) $\frac{du}{dx} = \frac{x+1}{(x^2+2x-3)^2}$

$\int \frac{x+1}{(x^2+2x-3)^2}\,dx = \frac{1}{2}\int \frac{2x+2}{(x^2+2x-3)^2}\,dx$

$= \frac{1}{2}\cdot\frac{(x^2+2x-3)^{-1}}{-1} = \boxed{\frac{-1}{2(x^2+2x-3)} + C}$

(38) $\frac{dy}{dx} = \frac{x-4}{\sqrt{x^2-8x+1}}$

$$\int \frac{x-4}{\sqrt{x^2-8x+1}}\,dx = \frac{1}{2}\int \frac{2(x-4)}{\sqrt{x^2-8x+1}}\,dx$$

$$= \frac{1}{2}\cdot\frac{2}{1}\left(x^2-8x+1\right)^{1/2} = \left(x^2-8x+1\right)^{1/2} + C$$