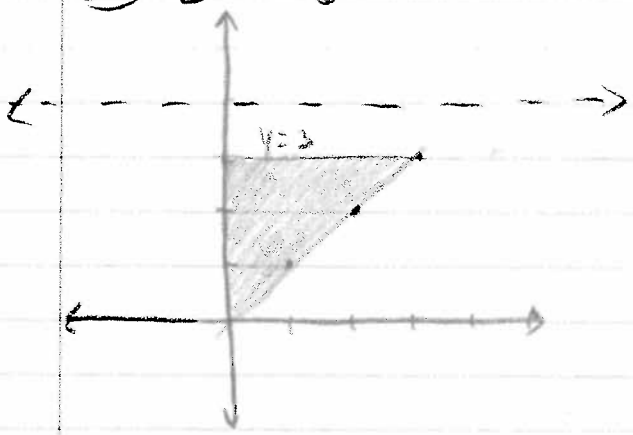
Assignment 67

Section 7-2 P. 463 Exc 15, 16; 20 & 21
Section 7-1 P. 453 Exc 43, 44
CPB P. 109 Exc 967, 972, 978; P. 110 Exc 1001

(15) $y = x$ $y = 3$ $x = 0$



$$V = \pi \int_0^3 [(4-x)^2 - 1^2]\,dx$$

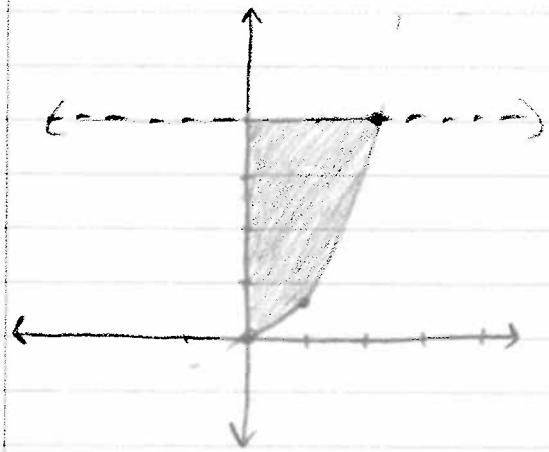
$$= \pi \int_0^3 (16 - 8x + x^2 - 1)\,dx$$

$$= \pi \left[15x - \frac{8x^2}{2} + \frac{x^3}{3}\right]_0^3$$

$$= \pi \left[15(3) - 4(3)^2 + \frac{(3)^3}{3}\right]$$

$$= \pi [45 - 36 + 9] = \boxed{18\pi}$$

(16) $y = \frac{1}{2}x^3$, $y = 4$, $x = 0$



$$V = \pi \int_0^2 (4 - \tfrac{1}{2}x^3)^2\,dx$$

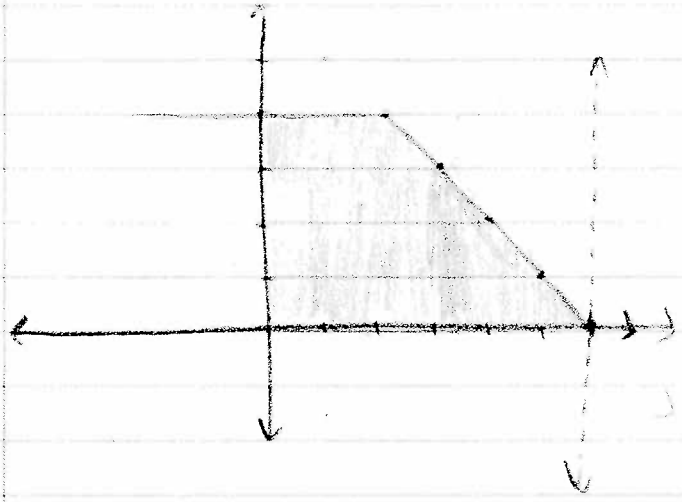
$$= \pi \int_0^2 \left[16 - 4x^3 + \frac{x^6}{4}\right] dx$$

$$= \pi \left[16x - \frac{4x^4}{4} + \frac{x^7}{28}\right]_0^2$$

$$= \pi \left[16(2) - (2)^4 + \frac{(2)^7}{28}\right]$$

$$\pi \left[32 - 16 + \frac{32}{7}\right] = \boxed{\frac{144\pi}{7}}$$

(20) $y = 6 - x$, $y = 0$, $y = 4$, $x = 0$



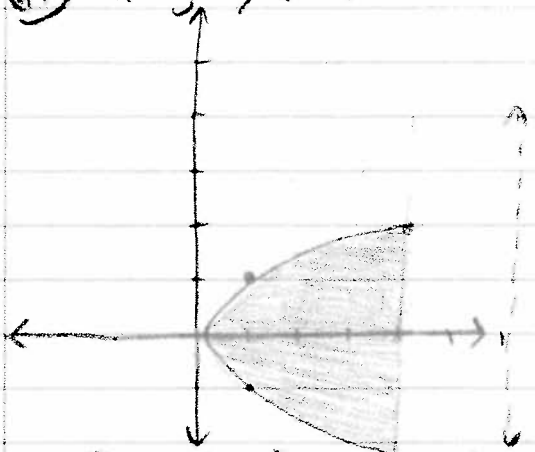
$R(y) = 6$

$r(y) = 6 - (6 - y) = y$

$$V = \pi \int_0^4 [(6)^2 - (y)^2]\, dy = \pi \left[36y - \frac{y^3}{3}\right]_0^4$$

$$= \pi \left[36(4) - \frac{(4)^3}{3}\right] = \frac{368\pi}{3}$$

(21) $x = y^2$, $x = 4$



$R(y) = 6 - y^2$, $r(y) = 2$

$$V = \pi \int_{-2}^{2} [(6 - y^2)^2 - 2^2]\, dy$$

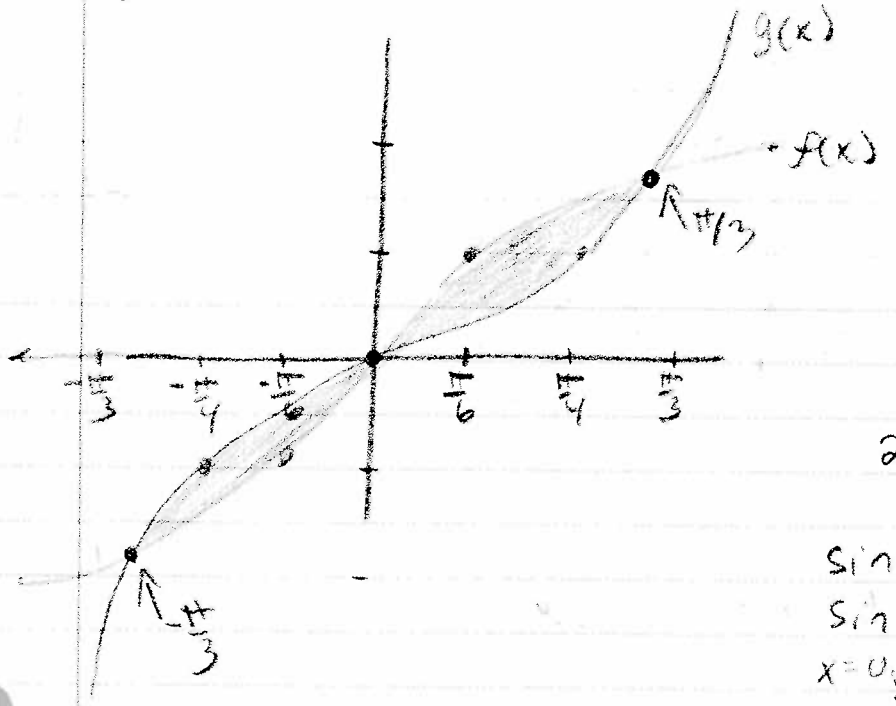
$$= \pi \int_{-2}^{2} (36 - 12y^2 + y^4 - 4)\, dy$$

$$= \pi \left[32y - \frac{12y^3}{3} + \frac{y^5}{5}\right]_{-2}^{2}$$

$$= \pi \left[\left(32(2) - 4(2)^3 + \frac{(2)^5}{5}\right) - \left(32(-2) - \frac{4(-2)^3}{3} + \frac{(-2)^5}{5}\right)\right]$$

$$\frac{384\pi}{5}$$

(43) $f(x) = 2\sin x$, $g(x) = \tan(x)$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$



INTERSECTION

$2\sin x = \tan x$

$2\sin x = \frac{\sin x}{\cos x}$

$2\sin x - \frac{\sin x}{\cos x} = 0$

$\sin x(2 - \sec(x)) = 0$

$\sin x = 0$ $\quad 2 - \sec x = 0$

$x = 0, \pi$ $\quad \sec x = 2$

$\cos x = 1/2$

$x = \pi/3, -\pi/3$

$$A = 2\int_{-\pi/3}^{0} (\tan x - 2\sin x)\,dx$$

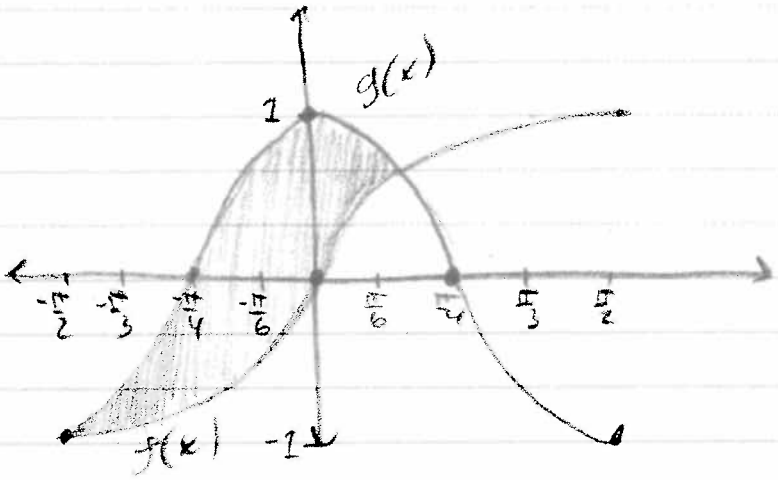
$$= 2\left[-\ln|\cos x| + 2\cos x\right]_{-\pi/3}^{0}$$

$$= 2\left[\left(-\ln|\cos 0| + 2\cos(0)\right) - \left(-\ln\left|\cos\tfrac{-\pi}{3}\right| + 2\cos\tfrac{-\pi}{3}\right)\right]$$

$$= 2\left[(-\ln 1 + 2) - \left(-\ln\tfrac{1}{2} + 2\cdot\tfrac{1}{2}\right)\right]$$

$$2\left[2 + \ln\tfrac{1}{2} - 1\right] = \boxed{2\ln\tfrac{1}{2} + 2}$$

(44) $f(x) = \sin x \quad g(x) = \cos 2x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{6}$



$2x = \theta$

$x = \frac{1}{2}\theta$

0	1
$\frac{\pi}{4}$	0
$\pi/2$	-1
$\frac{3\pi}{4}$	0
π	1

θ	$\cos\theta$
0	1
$\pi/2$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

$$A = \int_{-\pi/2}^{\pi/6} (\cos 2x - \sin x)\,dx$$

$$= \left[\frac{1}{2}\sin 2x + \cos x\right]_{-\frac{\pi}{2}}^{\pi/6}$$

$$\left[\frac{1}{2}\sin 2\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right)\right] - \left[\frac{1}{2}\sin 2\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)\right]$$

$$\left[\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] - \left[0 + 0\right]$$

$$\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} = \boxed{\frac{3\sqrt{3}}{4}}$$

(967) $\int_0^1 x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int_0^1 -2x\sqrt{1-x^2}\,dx$

$= \left[-\frac{1}{2}\cdot\frac{2}{3}(1-x^2)^{3/2}\right]_0^1 = \left[-\frac{1}{3}(1-x^2)^{3/2}\right]_0^1 =$

$\left[-\frac{1}{3}(1-1^2)^{3/2}\right] - \left[-\frac{1}{3}(1-0^2)^{3/2}\right]$

$0 + \frac{1}{3} = \boxed{\frac{1}{3}}$

(972) $\int_1^2 (x-1)\sqrt{2-x}\,dx$ $u = 2-x$

$du = -dx$

$-du = dx$

$x = 2-u$

NEW LIMITS

$u = 2-x$

$u = 2-1 = 1$

$u = 2-2 = 0$

$-\int_1^0 (2-u-1)(u)^{1/2}\,du$

$= -\int_1^0 (1-u)(u)^{1/2}\,du = -\int_1^0 (u^{1/2} - u^{3/2})\,du$

$= -\left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_1^0 = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big]_1^0$

$0 - \left[\frac{2}{5} - \frac{2}{3}\right] = \boxed{\frac{4}{15}}$

(978) $y = 2\sin x + \sin(2x)$ $[0, \pi]$

$$A = \int_0^{\pi} (2\sin x + \sin(2x))\,dx$$

$$= \left[-2\cos x - \frac{1}{2}\cos 2x\right]_0^{\pi}$$

$$= \left[-2\cos\pi - \frac{1}{2}\cos 2(\pi)\right] - \left[-2\cos(0) - \frac{1}{2}\cos 2(0)\right]$$

$$= \left(-2(-1) - \frac{1}{2}(1)\right) - \left(-2(1) - \frac{1}{2}(1)\right)$$

$$\left(2 - \frac{1}{2}\right) - \left(-2 - \frac{1}{2}\right)$$

$$\left(\frac{3}{2}\right) - \left(-\frac{5}{2}\right) = \boxed{4}$$

(1001) $$\int 3^x\,dx = \frac{1}{\ln(3)} \cdot 3^x = \boxed{\frac{3^x}{\ln 3} + C}$$