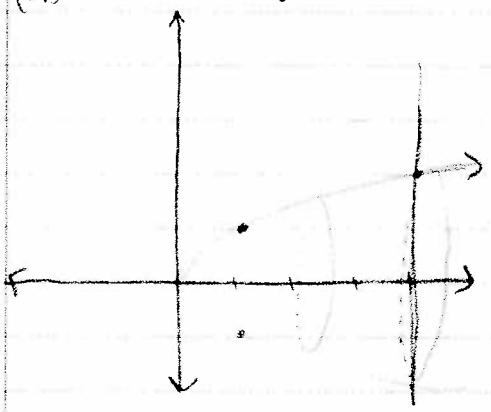
Assignment 6b

Section 7-2 Exc 11-14; Section 7-4 Exc 22, 25, 26
CPB p.107 938, 948
CPB p.109 Exc 959, 963

(11) $y = \sqrt{x}$, $y = 0$, $x = 4$

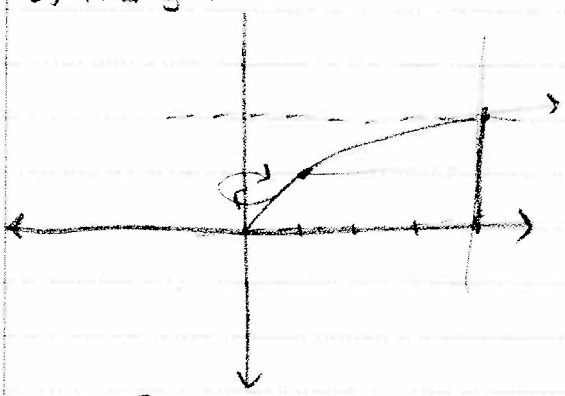
(a) The x-axis



$$V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \frac{x^2}{2}\Big]_0^4$$

$$\pi\left[\frac{(4)^2}{2}\right] = \boxed{8\pi}$$

(b) The y-axis



$$V = \pi \int_0^2 [4^2 - (y^2)^2]\, dy$$

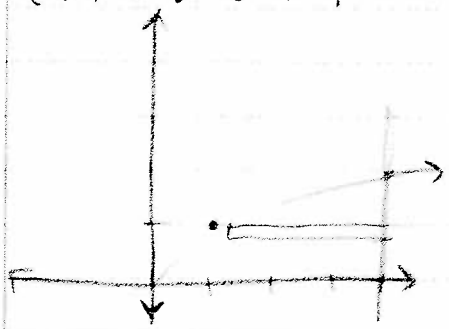
$$= \pi\left[16y - \frac{y^5}{5}\right]_0^2$$

$$\pi\left[16(2) - \frac{(2)^5}{5}\right]$$

$$= \pi\left[\frac{128}{5}\right] = \boxed{\frac{128\pi}{5}}$$

$x = y^2$
$x = 4$

(c) The line $x = 4$

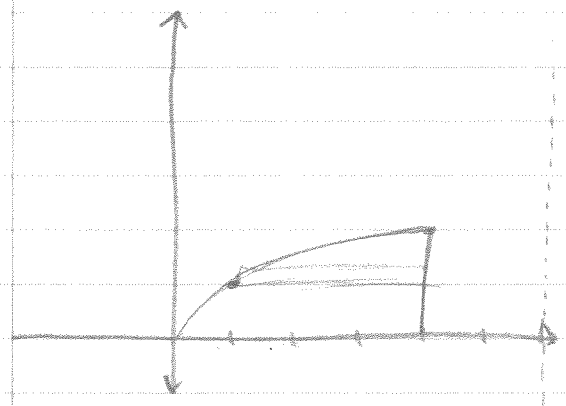


$$\pi \int_0^2 (4 - y^2)^2 dy = \pi \int (16 - 8y^2 + y^4) dy$$

$$\pi\left[16y - \frac{8y^3}{3} + \frac{y^5}{5}\right]_0^2$$

$$\pi\left[16(2) - \frac{8(2)^3}{3} + \frac{(2)^5}{5}\right] = \frac{256\pi}{15}$$

(c) THE LINE $x = 6$

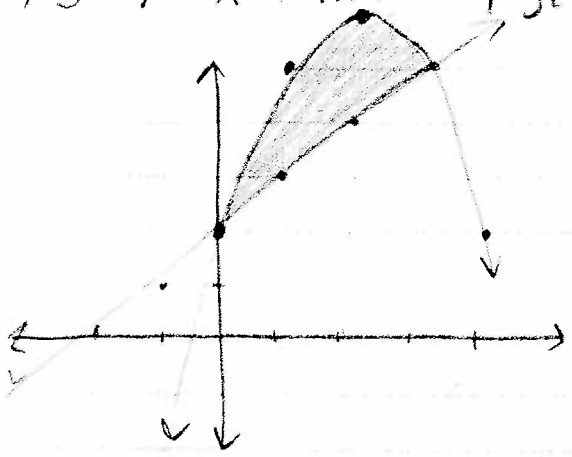


$$V = \pi \int_0^2 [(6-y^2)^2 - 4]\,dy = \pi \int_0^2 (36 - 12y^2 + y^4 - 4)\,dy$$

$$= \pi \left[32y - \frac{12y^3}{3} + \frac{y^5}{5} \right]_0^2$$

$$= \pi \left[32(2) - 4(2)^3 + \frac{(2)^5}{5} \right] = \frac{192\pi}{5}$$

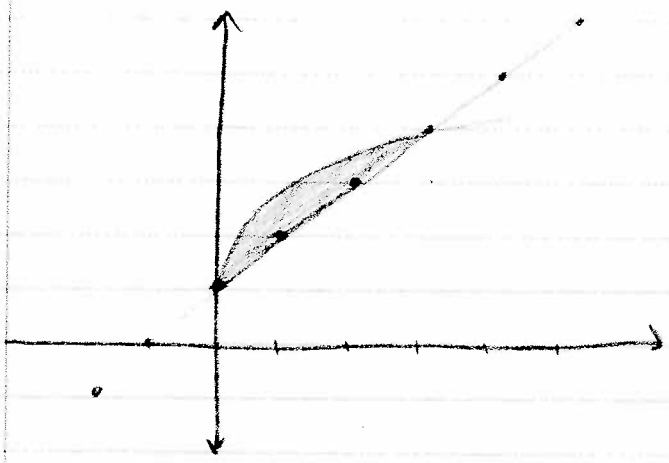
22) $f(x) = -x^2 + 4x + 2$, $g(x) = x + 2$



$$A = \int_0^3 \left[(-x^2 + 4x + 2) - (x + 2)\right] dx = \int_0^3 (-x^2 + 3x)\, dx$$

$$= \left. -\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^3 = \boxed{\frac{9}{2}}$$

25) $f(x) = \sqrt{3x} + 1$, $g(x) = x + 1$



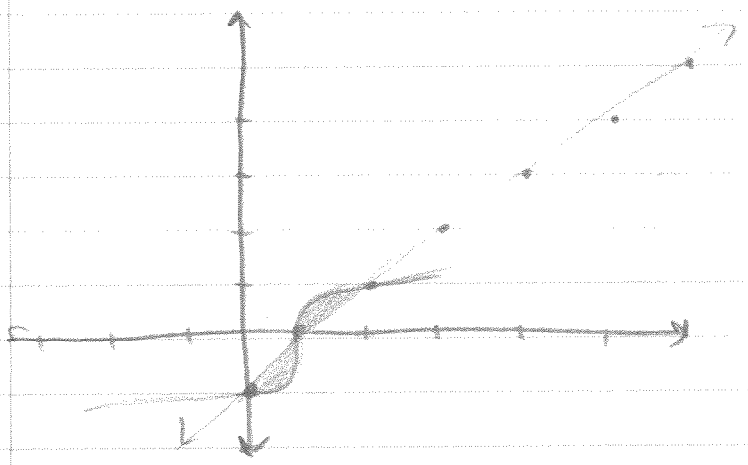
$$A = \int_0^3 \left[(\sqrt{3x} + 1) - (x + 1)\right] dx$$

$$= \int_0^3 (\sqrt{3x} - x)\, dx$$

$$= \left. \frac{1}{3} \cdot \frac{2}{3} (3x)^{3/2} - \frac{x^2}{2} \right]_0^3$$

$$= \frac{2}{9}(9)^{3/2} - \frac{(3)^2}{2} = \boxed{\frac{3}{2}}$$

26) $f(x) = \sqrt[3]{x-1}$, $g(x) = x-1$



$$A = 2\int_0^1 \left[(x-1) - (x-1)^{1/3}\right] dx = 2\left[\frac{x^2}{2} - x - \frac{3}{4}(x-1)^{4/3}\right]_0^1$$

$$= 2\left[\left(\frac{1}{2} - 1 - \frac{3}{4}(1-1)^{4/3}\right) - \left(\frac{0^2}{2} - (0) - \frac{3}{4}(0-1)^{4/3}\right)\right]$$

$$= \boxed{\frac{1}{2}}$$

(938) $\frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{1}{\frac{\pi}{4}+\frac{\pi}{4}}\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx = \frac{2}{\frac{\pi}{2}}\int_{-\pi/4}^{\pi/4}\sec^2 x\,dx$$

$$= \frac{4}{\pi}\left[\tan x\right]_{-\pi/4}^{\pi/4} = \frac{4}{\pi}\left[\tan(\pi/4) - \tan(-\pi/4)\right]$$

$$= \frac{4}{\pi}\left[1 - (-1)\right] = \boxed{\frac{8}{\pi}}$$

(948) $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx = \int_1^2 (3x^{-2} - 1)\, dx =$

$$\frac{3x^{-1}}{-1} - x \Big]_1^2 = \left[\frac{-3}{2} - 2\right] - \left[\frac{-3}{1} - 1\right] = \boxed{\frac{1}{2}}$$

(959) $\int \sin(5\theta)\cos(5\theta)\, d\theta \qquad u = \cos(5\theta)$

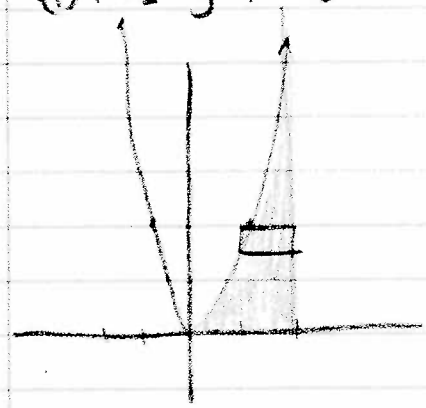
$du = -5\sin(5\theta)\, d\theta$

$$-\frac{1}{5}\int u\, du = -\frac{1}{5}\cdot\frac{u^2}{2} \rightarrow -\frac{(\sin 5\theta)^2}{10} + C \qquad -\frac{du}{5} =$$

(963) $\int e^x (e^x - 1)^7\, dx = \frac{(e^x - 1)^8}{8} + C$

(12) $y = 2x^2$, $y = 0$, $x = 2$

(a) THE y-AXIS



$y = 2x^2 \Rightarrow x = \sqrt{y/2}$

$$V = \pi \int_0^8 \left[2^2 - \left(\sqrt{\frac{y}{2}}\right)^2\right] dy$$

$$= \pi \int_0^8 \left(4 - \frac{y}{2}\right) dy$$

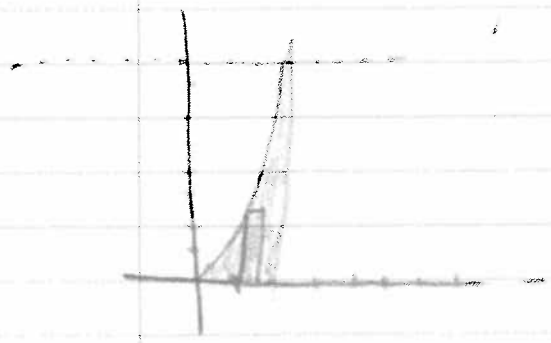
$$\pi \left[4y - \frac{y^2}{4}\right]_0^8 = 16\pi$$

(b) THE x-AXIS

$$V = \pi \int_0^2 [2x^2]^2 dx = \pi \int_0^2 4x^4$$

$$= \pi \left[\frac{4x^5}{5}\right]_0^2 = \frac{128\pi}{5}$$

(c) THE LINE $y = 8$



$$V = \int_0^2 64 - (8 - 2x^2)^2$$