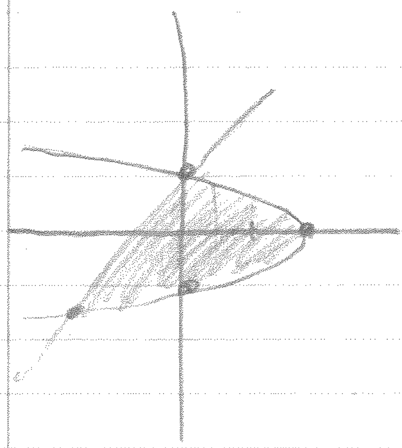
ASSIGNMENT 65

SECTION 7-1 P. 452 EXC 13, 14, 17, 19

CPBP. 107 EXC 934, 936, 941, 950-953

(13) $x = 4 - y^2$

$x = y - 2$



(b) $4 - y^2 = y - 2$

$0 = y^2 + y - 6$

$0 = (y+3)(y-2)$

$y = -3, 2$

$$\int_{-3}^{2} [(y-2) - (y - y^2)]\,dy$$

$$\int_{-3}^{2} (y^2 + 2y - 2)\,dy = \left.\frac{y^2}{3} + \frac{2y^2}{2} - 2y\right]_{-3}^{2}$$

$$\left[\frac{(2)^2}{3} + (2)^2 - 2(2)\right] - \left[\frac{(-3)^2}{3} + (-3)^2 - 2(-3)\right] = \frac{125}{6}$$

(a) $y = -3, 2$

$x = 4 - (-3)^2$ $\quad x = y - 2$ $\quad x = 4 - y^2 \Rightarrow y = \sqrt{4-x}$

$x = 4 - 9 = -5$ $\quad x = 2 - 2 = 0$ $\quad x = y - 2 \Rightarrow y = x + 2$

$(-5, -3)$ $\quad (0, 2)$

$$\int_{-5}^{0} [(x+2) - \sqrt{4-x}\,]\,dx + \int_{0}^{2} 2\sqrt{4-x}\,dx$$

$$\left.\frac{x^2}{2} + 2x + \frac{2}{3}(4-x)^{3/2}\right]_{-5}^{0} + \left.\frac{-4}{3}(4-x)^{3/2}\right]_{0}^{2}$$

$$\frac{61}{6} + \frac{32}{3} = \frac{125}{6}$$

SIDE NOTE

$\int -\sqrt{4-x}$

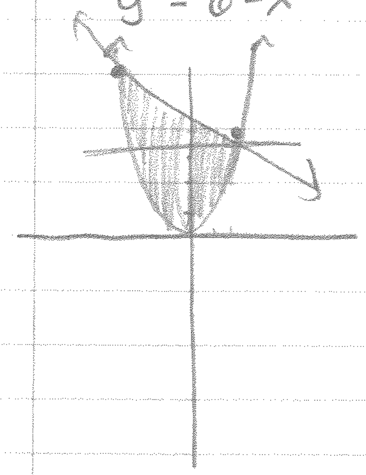
$= \frac{2}{3}(4-x)^{3/2}$

$-2 \int -\sqrt{4-x}$

$-2 \cdot \frac{2}{3}(4-x)^{3/2}$

(14) $y = x^2$

$y = 6 - x$



$x^2 = 6 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2)$

$x = 3, -2$

(a) $\int_{-2}^{3} [(6-x) - (x)^2]\, dx$

$\frac{-x^3}{3} - \frac{x^2}{2} + 6x \Big]_{-2}^{3}$

$\left[\frac{-(3)^3}{3} - \frac{(3)^2}{2} + 6(3)\right] - \left[\frac{-(-2)^3}{3} - \frac{(-2)^2}{2} + 6(-2)\right]$

$= \frac{125}{6}$

b) $y = x^2 \Rightarrow x = \sqrt{y}$

$y = 6 - x \Rightarrow x = 6 - y$

$\sqrt{y} = 6 - y$

$y = (6-y)^2$

$y = 36 - 12y + y^2$

$0 = y^2 - 11y + 36$

$0 = (y-9)(y-4)$

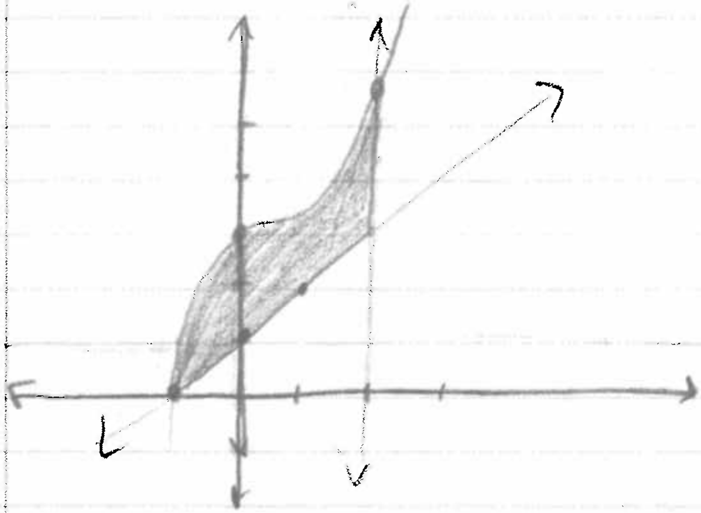
$y = 9, 4$

$\int_0^4 2\sqrt{y}\, dy + \int_4^9 [(6-y) - (-\sqrt{y})]\, dy$

$= 2 \cdot \frac{2}{3}(y)^{3/2} \Big]_0^4 + 6y - \frac{y^2}{2} + \frac{2}{3}y^{3/2} \Big]_4^9$

$\frac{32}{3} + \frac{61}{6} = \frac{125}{6}$

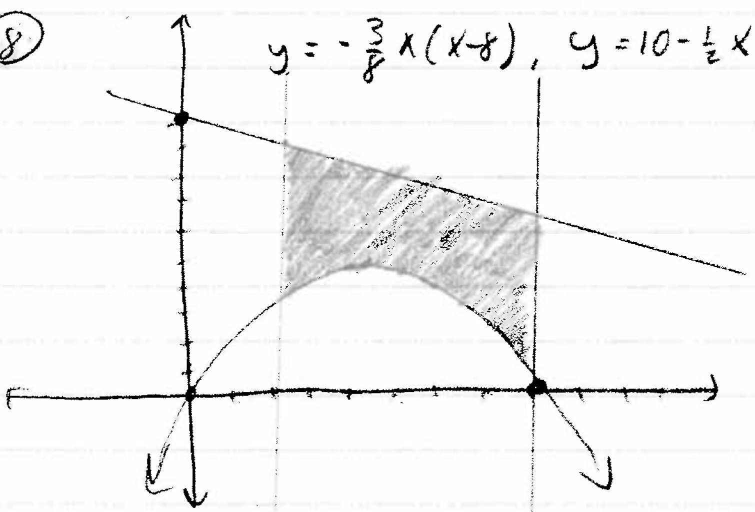
(17) $y = \frac{1}{2}x^3 + 2$, $y = x+1$, $x = 0$, $x = 2$



$$\int_0^2 \left[\left(\frac{1}{2}x^3+2\right) - (x+1)\right] dx = \frac{1}{2} \cdot \frac{x^4}{4} - \frac{x^2}{2} + x \Big]_0^2$$

$$\frac{x^4}{8} - \frac{x^2}{2} + x \Big]_0^2 = \boxed{2}$$

(18) $y = -\frac{3}{8}x(x-8)$, $y = 10 - \frac{1}{2}x$, $x = 2$, $x = 8$



$$\int_2^8 \left[\left(10 - \frac{1}{2}x\right) - \left(-\frac{3}{8}x(x-8)\right)\right] dx$$

$$\int_2^8 \left(10 - \frac{1}{2}x + \frac{3}{8}x^2 - 3x\right) dx$$

$$\int_2^8 \left(\frac{3}{8}x^2 - \frac{7}{2}x + 10\right) dx$$

$$\frac{3}{8}\frac{x^3}{3} - \frac{7}{2}\frac{x^2}{2} + 10x \Big]_2^8$$

$$\frac{x^3}{8} - \frac{7}{4}x^2 + 10x \Big]_2^8 = 18$$

934 $\int_0^{x^3} \sin(t^2)\,dt = \sin(x^6)\cdot 6x^5 + C$

936 AVG VALUE $\frac{1}{b-a}\int_a^b f(x)\,dx$

$\frac{1}{2}\int_0^2 (x - 2\sqrt{x})\,dx$

$= \frac{1}{2}\left[\frac{x^2}{2} - 2\cdot\frac{2}{3}x^{3/2}\right]_0^2$

$= \frac{x^2}{4} - \frac{2}{3}x^{3/2}\Big]_0^2$

$\frac{(2)^2}{4} - \frac{2}{3}(2)^{3/2} = 1 - \frac{2}{3}\cdot 2\sqrt{2} = 1 - \frac{4}{3}\sqrt{2}$

941 $\int_0^{\pi/3} 2\sec^2 x\,dx = 2\tan x\Big]_0^{\pi/3}$

$2\tan\frac{\pi}{3} - 2\tan 0$

$2\sqrt{3} - 0 = 2\sqrt{3}$

950 $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta = 4\sec\theta\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$

$4\sec\frac{\pi}{3} - 4\sec(-\pi/3)$

$4(2) - 4(2) = 0$

951 $\int_0^2 3^x \ln 3\,dx = = \ln 3\int_0^2 3^x = \frac{\ln 3\, 3^x}{\ln 3}\Big]_0^2$

$= 3^x\Big]_0^2 = 3^2 - 3^0 = 9 - 1 = 8$

952 $\int_0^{\ln 5} e^x \, dx = e^x \Big]_0^{\ln 5}$

$e^{\ln 5} - e^0$

$5 - 1 = 4$

953 $\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$