Calculus AB ANSWERS, Part A

1) $f'(x) = \ln x(2x) + x^2\cdot\frac{1}{x} = \underline{2x\ln x + x}$
(E)

2) $\int_1^8 x^{-2/3}dx = 3\left[x^{1/3}\right]_1^8 = 3\left[(8)^{1/3} - 1^{1/3}\right] = \underline{3}$ ($2-1$)
(E)

3) $f'(x) = -e^{-x} + \cos x + \sin x$; $f''(x) = e^{-x} - \sin x + \cos x$
$f''(0) = 1 - 0 + 1 = \underline{2}$
(E)

4) $6x - 2[y(1) + xy'] + 2yy' = 0$ @ $(1,-2)$
(D) $6 - 2(-2 + y') - 4y' = 0$; $6 + 4 - 2y' - 4y' = 0$
$10 = 6y'$ $\boxed{y' = \frac{5}{3}}$

5) $\frac{1}{\sec x}\cdot\sec x\tan x = \boxed{\tan x}$
(B)

6) $\frac{d}{dx}(2^{\cos x}) = \ln 2(2^{\cos x})(-\sin x)$
(C) $\frac{d}{dx}a^u = \ln a(a^u)u'$

7) $f''(x) = (x+2)^2(3x^2) + x^3(2)(x+2)$
(D) $= x^2(x+2)[3x+6+2x] = x^2(x+2)(5x+6)$
$x = 0, -2, -\frac{6}{5}$. 0 is not because of x^2

8) $\lim_{x\to-\infty}\frac{2x+3}{\sqrt{x^2+x+1}} = -2$ top is 2 bottom is −
(A)

9) $\lim_{x\to5}\frac{2x^2-50}{x^2-15x+50} = \frac{0}{0}$ $\lim_{x\to5}\frac{2\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x-10)} = \frac{2(5+5)}{(5-10)}$
(A) $\frac{20}{-5} = -4$

10) $f(0) > 0$ means function is above x axis
This eliminates B & C. $f'(0)=0$ eliminates E
(A) This leave A & D. $f''(0) < 0$ means function is concave down so A.

11) (B) $\int_a^b f(x)dx = \int_a^0 f(x)dx + \int_0^b f(x)dx$, so
$3 = 5 + \int_0^b f(x)dx \Rightarrow -2 = \int_0^b f(x)dx$

12) $g'(x) = \frac{e^{3x}(6x) - 3x^2(e^{3x})(3)}{e^{6x}} = \frac{e^{3x}(6x-9x^2)}{e^{6x}}$. Set this $= 0$
(C) $e^{3x}(6x-9x^2) = 0$. $e^{3x} = 0$ (never happens). $6x - 9x^2 = 0$
$3x(2-3x) = 0 \Rightarrow x = 0, 2/3$. Now test intervals. $(-\infty,0), (0,\frac{2}{3}), (\frac{2}{3},\infty)$. Use 1st der. test. When $x=-1$, $\frac{e^{-3}(-6-9)}{e^{-6}} = -\#$
when $x=\frac{1}{2}$, $\frac{e^{3/2}(3-\frac{9}{4})}{e^3} = +\#$, when $x=4$, $-\#$

13) $y' = 3x^2+6x-45$.
(C) $y'' = 6x+6$ $0 = 6(x+1)$ $x=-1$

14) Need to do 2 things. Find y and find y'! Put $x=2$ into original eq. Get $4y + y^2 + 4 = 0$; $y^2+4y+4=0$.
(E) Factor. $(y+2)^2 = 0$. $y=-2$. Take der. implicitely.
$y(2x) + x^2y' + 2yy' = 0$. Put 2 in for x and −2 in for y.
$-8 + 4y' - 4y' = 0$ $-8 \neq 0$ Not so.
no solution.

15) use $(-3,4)$ and $m=2$ to write eq. of tangent line. $y - 4 = 2(x+3)$. Now put -3.1 in for x.
(A) $y = 2(-3.1+3)+4$ and simplify. $y = 2(-.1)+4 = \underline{3.8}$

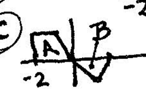
16) $\frac{d}{dx}\tan u = \sec^2 u\,u'$ so $y' = \sec^2(2x)\cdot 2$. put in $\frac{\pi}{8}$ for x
(E) $y' = 2(\sec\frac{\pi}{4})^2$. The $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so $\sec\frac{\pi}{4} = \frac{2}{\sqrt{2}}$
$y' = 2(\frac{2}{\sqrt{2}})^2 = 2(\frac{4}{2}) = 4$

17) $2x - 2yy' = 0$ $y' = \frac{x}{y}$. Take der. of this. $y'' = \frac{y(1) - xy'}{y^2}$.
(D) Use $(3,2)$ so $y' = \frac{3}{2}$ and $y'' = \frac{2 - 3(\frac{3}{2})}{4} = \frac{4-9}{8} = -\frac{5}{8}$

18) $v'(t) = a(t) = 3t^2 - \cos t$. put 2π in for t.
(C) $a(2\pi) = 3(4\pi^2) - \cos 2\pi = 12\pi^2 - 1$

19) Since this is a der. graph, max and mins on this graph would
(B) be where poi's occur on original graph. f' changes from dec to inc.

20) Now look on this der. graph and count the # of zeros. Make sure there is a change in sign from
(A) (+) to (−) for maxima. Happens one time at $x=0$.

21) FTC. $\int_{-2}^2 f'(x)dx = f(2) - f(-2)$. So, $f(-2) = f(2) - \int_{-2}^2 f'(x)dx$
(C) 

$A = \frac{1}{2}(1)(2+1) = \frac{3}{2}$. $B = -\frac{1}{2}(1)(2) = -1$
$f(-2) = 1 - (\frac{3}{2} - 1)$
$f(-2) = 1 - \frac{1}{2} = \frac{1}{2}$

22) u-substitution. let $u = x^2+1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$
(C) when $x=0$, $u=1$
when $x=\sqrt{e-1}$, $u=e$
$\frac{1}{2}\int_1^e \frac{1}{u}du = \frac{1}{2}[\ln|u|]_1^e$
$\frac{1}{2}[\ln e - \ln 1] = \frac{1}{2}$

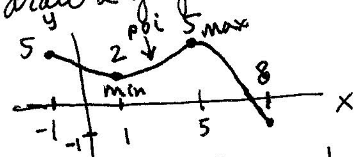
23) $f(x) = e^{2\ln x} = e^{\ln x^2} = x^2$. $f'(x) = 2x$, $f'(3) = 6$
(A)

24) $y' = 2x$, $y' = \frac{1}{2\sqrt{x}}$ $2x = \frac{1}{2\sqrt{x}}$ $= 4x^{3/2} = 1$; $x^{3/2} = \frac{1}{4}$ so
(B) $x = \frac{1}{4^{2/3}}$ Rationalize den. $\frac{1}{4^{2/3}}\cdot\frac{4^{1/3}}{4^{1/3}} = \frac{4^{1/3}}{4} = \frac{\sqrt[3]{4}}{4}$

25) FTC. substitute x^2 in for (t) and then take
(D) der. of x^2 $\cos(x^2)^3(2x) = 2x\cos x^6$

26) put in 2 for x $\frac{e^4-e^4}{2-2} = \frac{0}{0}$. Use L'Hopital's Rule.
(E) $\lim_{x\to2}\frac{2e^{2x}}{1} = 2e^4$

27) $\frac{dx}{dt} = 2\frac{cm}{sec}$ $\frac{dA}{dt} = ?$ $V = 27cm^3$ $27 = x^3$, $3 = x$
(E) $SA = 6x^2$ $\frac{dA}{dt} = 12x\frac{dx}{dt} = 12(3)(2) = 72$

28) draw a graph. Connect dots
(A) 

A) True
B) not necessarily
C) not nec.
D) not nec.
E) not nec.

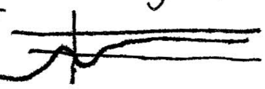
must have at least one zero, one min, one poi & one max

* used calculator

Calculus AB Answers, Part B

* (29) (calc) put $\tan^{-1}(x^3-x)$ in $y=$. Also graph $y=2$ (der of $2x$)

(A) you get



no intersections

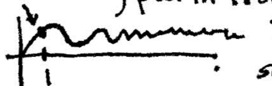
* (30) (calc) Graph each one.

(C) I. II. III

not diff @ $x=0$ not diff @ $x=0$ diff @ $x=0$

* (31) (calc) graph $\sqrt{x}-\cos(e^x)$. Have window $x[0,10]$

(D) hit trace, put in 1 for x.



This is a vel graph, vis (+) so function is always moving up. Since vel graph is increasing, acc. will be positive.

(32) If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

(A) In the problem, $h'(x)=\frac{f(x)[g(x)-g'(x)]}{[g(x)]^2}$.

This means $f(x)=f'(x)$. The only function whose der. is equal to itself is $\underline{e^x}$

* calc

(33) 2 ways to do this. With the 89 calc.

(D) set $f'(x)=.2$ and solve.

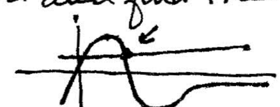
get $x=.1199$ and $x=1.4177$ OR

graph f' which is $e^{-x}(2x)-x^2(e^{-x})$ in y_1 and in y_2 put $.2$ and find the intersections.

window $x[0,10]$

$x\,scl=1$

$y[-.5,1]$ $y\,scl=.1$



(34) Ave. value is $\frac{1}{b-a}\int_a^b f(x)dx$, so $f(x)=x^3$ gives

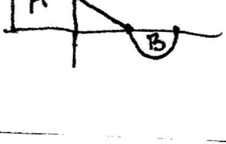
(E) $\frac{1}{b-a}\int_a^b x^3dx \Rightarrow \frac{1}{b-a}\left[\frac{x^4}{4}\right]_a^b=\frac{1}{4}\left(\frac{1}{b-a}\right)[b^4-a^4]$

(35) Graph is f'! On f', from $(-\infty,-4)$, f' ~~should~~ is neg.

(B) f should dec. On f' from $(-4,4)$, f' is pos. f should inc on $(-4,4)$. On f' from $(4,\infty)$, f' is neg. f should dec. That's graph B, and A. Now look at f' as it increases and dec. f' inc from $(-4,-2)$ so f'' will be (+) so f will be con. up. f' dec. from $(-2,0)$, so f'' will be (-), so f will be con. down. This is B only.

(36) Since $g(x)=\int_{-4}^x f(t)dt$, $g(4)=\int_{-4}^4 f(t)dt$. Use

(C) geometry for this one.

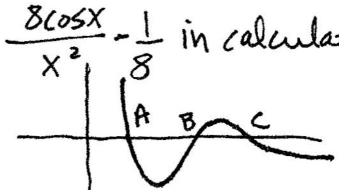


$A=\frac{1}{2}(6)(6)=18$ $B=-\frac{\pi(1)^2}{2}=-\frac{\pi}{2}$

$\boxed{18-\frac{\pi}{2}}$

* (37) (calc) put $\frac{8\cos x}{x^2}-\frac{1}{8}$ in calculator. $x[1,10]$

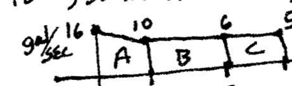
(C) Get $y[-1,2]$ $y\,scl=.1$



Since this is the der. graph and we need to know on f how many max & min, we look at the ZEROS and check + to − or − to +. At A, + to −, so max. At B, − to +, so min. At C, + to −, so max. 2 max, 1 min.

(38) Sketch first.

(D) make traps.



$\frac{1}{2}(3)[16+10]+\frac{1}{2}(5)[10+6]+\frac{1}{2}(2)[6+5]$

$39+40+11=90$ gal.

(39) Hope you didn't miss this one. 'A' is false because

(B) there's a hole there which means DNE. 'C' is false because $\lim_{x\to b} f(x)$ DNE since 2 different values which is explained in 'D'. $\lim_{x\to b^-} f(x)=2$. $\lim_{x\to b^+} f(x)=1$

'E' is false because @ $x=0$ there is a hole.

(40) $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ $\frac{dV}{dt}=\frac{6\,in^3}{deg}$ so for 2 deg. $=12$

(C) $36\pi=\frac{4}{3}\pi r^3$ $12=4\pi(3)^2\frac{dr}{dt}$ $\frac{12}{36\pi}=\frac{dr}{dt}=\frac{1}{3\pi}$ in/min.

$3=r$ Tricky part is knowing to multiply 6×2.

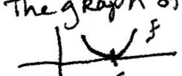
(41) Implicit diff. TI 89 calc will do this problem. By hand

(E) $1+\cos y\,y'=\frac{1}{y}y' \Rightarrow 1=\frac{1}{y}y'-\cos y\,y' \Rightarrow 1=y'\left(\frac{1}{y}-\cos y\right)$

$\frac{1}{\frac{1}{y}-\cos y}\cdot\frac{y}{y}=\frac{y}{1-y\cos y}$

(42) The graph of f must look something like this:

(C)



$f(c)=0$ because x-int.

$f'(c)<0$ because f is dec

$f''(c)>0$ because f is con. ↑.

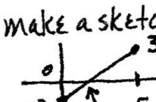
* calc $f'<f<f''$

* (43) FTC $\int_1^4 f'(x)dx=f(4)-f(1)$ so $f(4)=f(1)+\int_1^4\sqrt{1+x^3}dx$

(D) put $\int_1^4\sqrt{1+x^3}dx$ in calc. get 12.871 don't pick this because you still need to add $f(1)$ to this $f(1)=.5$ so ans. is $\underline{13.371}$

(44) make a sketch. Doesn't have to be a straight line connecting, could have hills & valleys.

(E)



I. True by MVT $\frac{f(5)-f(0)}{5-0}=\frac{3+2}{5}=1=f'(c)$

II. True. Somewhere (at arrow) there has to be a ZERO. IVT

III. True. Another int. value th. IVT. Since y's go from $[-2,3]$ −1 is in this

(45) This is a graph of f. For f' to be neg., f must be

(C) $\underline{\text{dec}}$ and for f'' to be neg, f must be con ↓.

This happens @ C.

done