

ASSIGNMENT 7

SECTION 7-2 EXC 17 #33

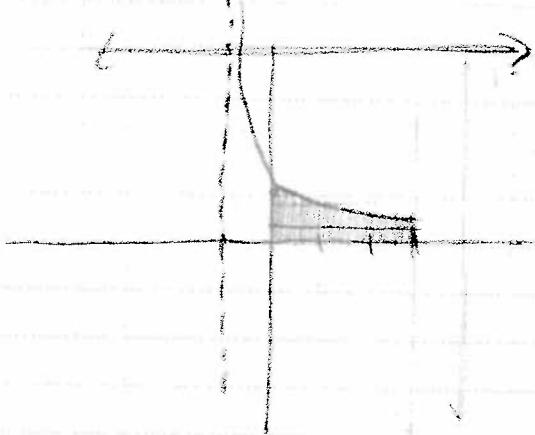
SECTION 7-1 EXC 52 & 51

SECTION 4-4 EXC 24, 31, 44

SECTION 4-5 EXC 70, 74

SECTION 4-6 EXC 16

(17) $y = \frac{1}{1+x}$, $y=0$, $x=0$, $x=3$



$$A = \pi \int_0^3 \left[y^2 - \left(\frac{1}{1+x} \right)^2 \right] dx$$

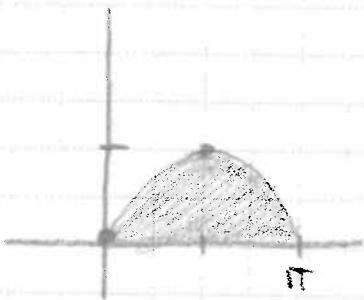
$$= \pi \int_0^3 \left(16 - \frac{1}{1+2x+x^2} \right) dx$$

$$R(y) = 4 \quad = \pi \left[16x + \frac{1}{(1+x)} \right]_0^3$$

$$r(y) = 4 - \left(\frac{1}{1+x} \right) \quad = \pi \left[\left(16(3) + \frac{1}{4} \right) - \left(16(0) + 1 \right) \right]$$

$$= \frac{19}{4} + 1 = \boxed{\frac{23}{4}}$$

(33) $y = \sin x$, $y=0$, $x=0$, $x=\pi$



$$A = \pi \int_0^\pi \sin^2 x dx = \pi \int_0^\pi \left(\frac{1 - \cos 2x}{2} \right) dx$$

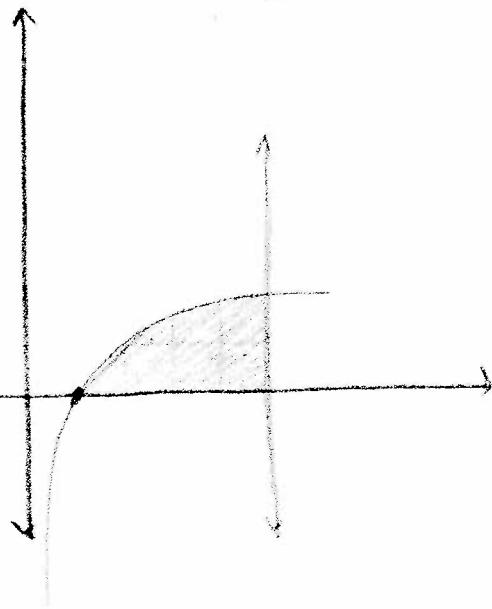
$$= \pi \left(\frac{x}{2} - \frac{1}{4} \sin 2x \right)_0^\pi$$

$$\int -\frac{\cos 2x}{2} dx = \frac{\pi}{2} \left[\left(\pi - \frac{1}{2} \sin 2\pi \right) - \left(0 - \frac{1}{2} \sin 0 \right) \right]$$

$$-\frac{1}{2} \cdot \frac{1}{2} \int 2 \cos 2x dx = \frac{\pi}{2} \left(\pi - 0 \right) = \boxed{\frac{\pi^2}{2}}$$

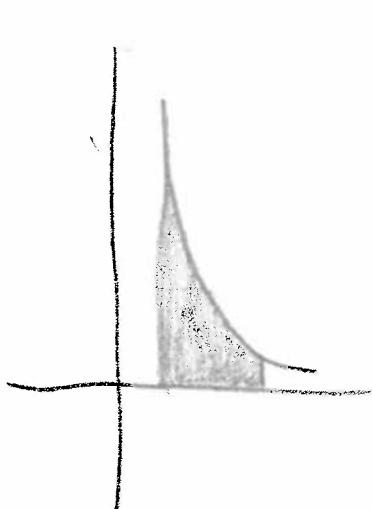
$$= -\frac{1}{4} \sin 2x$$

⑤2) $g(x) = \frac{4 \ln x}{x}$, $y=0$, $x=5$



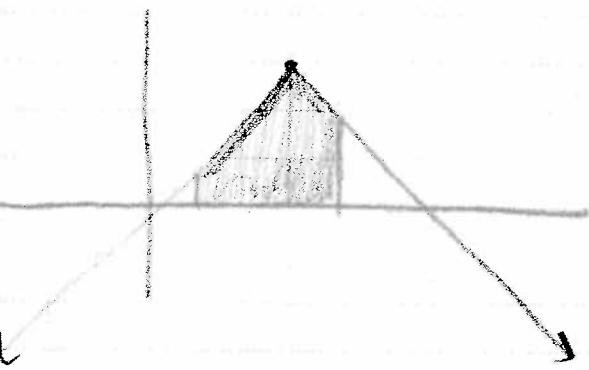
$$A = \int_1^5 \frac{4 \ln x}{x} dx = \int_1^5 2^2 \ln x dx$$

⑤1) $f(x) = \frac{1}{x^2} e^{4x}$, $y=0$, $1 \leq x \leq 3$



$$\int_1^3 \frac{e^{4x}}{x^2} dx = \left[-\frac{e^{4x}}{x^2} \right]_1^3 = \left[-e^{4x} \right]_1^3 = \boxed{\frac{-e^{-4} + e^4}{1}}$$

(24) $\int_1^4 (3 - |x-3|) dx$

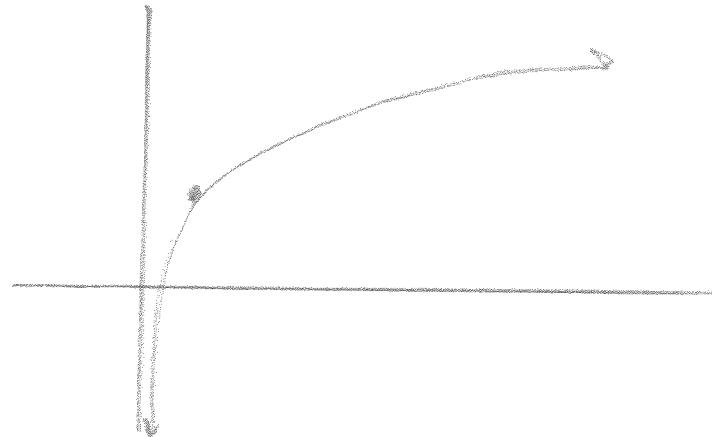


$$\begin{aligned}
 A &= \int_0^3 x dx + \int_3^4 (-x+6) dx \\
 &= \left[\frac{x^2}{2} \right]_0^3 + \left[-\frac{x^2}{2} + 6x \right]_3^4 \\
 &= \frac{9}{2} + \left[\left(-\frac{(4)^2}{2} + 6(4) \right) - \left(-\frac{(3)^2}{2} + 6(3) \right) \right] \\
 &= \boxed{\frac{13}{2}}
 \end{aligned}$$

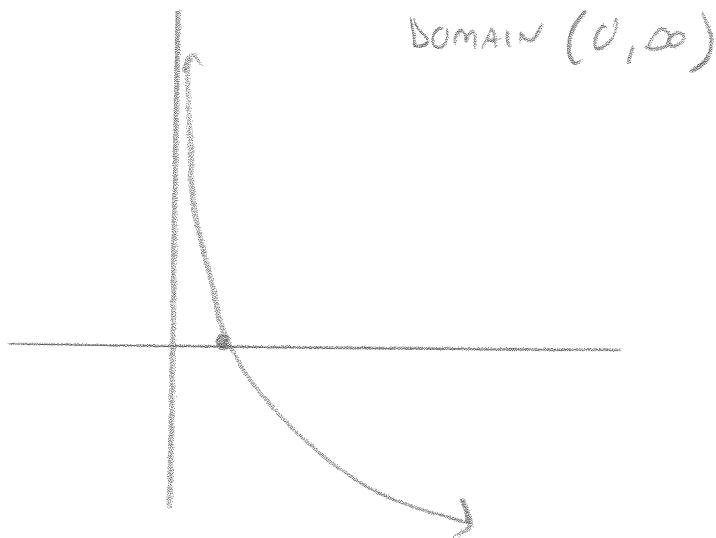


ASSIGNMENT +3
 SECTION 5.1 EXC 7, 12, 23, 34
 SECTION 7.1 EXC 19
 SECTION 7.2 EXC 14
 SECTION 3.2 EXC 14, 28, 43
 CPB EXC 1102, 1107

⑦ $f(x) = \ln x + 2$
 VERTICAL SHIFT UP 2 UNITS



⑫ $f(x) := -2 \ln x$



⑬ $\ln \sqrt[3]{a^2+1} = \frac{1}{3} \ln(a^2+1)$

⑭ $\frac{3}{2} [\ln(x^2+1) - \ln(x+1) - \ln(x-1)]$

$$⑯ \frac{3}{2} [\ln(x^2+1) - \ln(x+1) - \ln(x-1)]$$

$$= \frac{3}{2} \left[\ln\left(\frac{x^2+1}{x+1}\right) - \ln(x-1) \right]$$

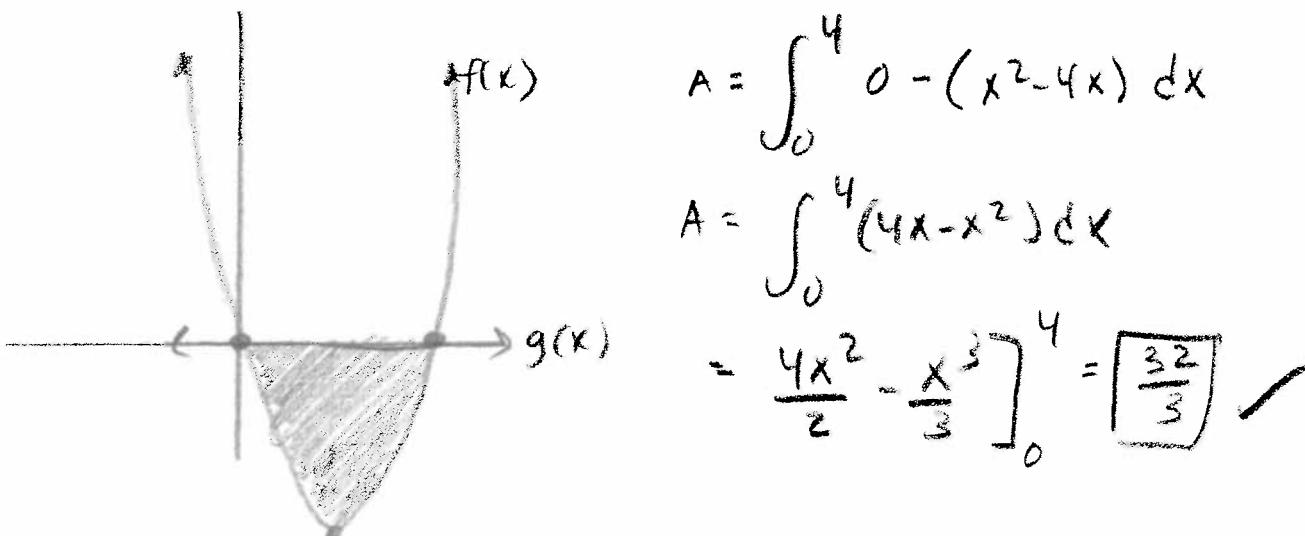
$$= \frac{3}{2} \left[\ln\left(\frac{x^2+1}{(x+1)(x-1)}\right) \right]$$

$$= \frac{3}{2} \ln\left(\frac{x^2+1}{x^2-1}\right)$$

$$= \ln\left(\frac{x^2+1}{x^2-1}\right)^{3/2}$$

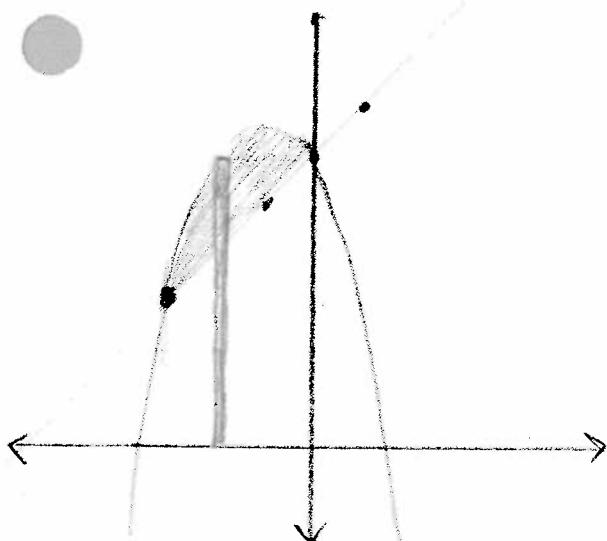
$$= \ln \sqrt{\left(\frac{x^2+1}{x^2-1}\right)^3}$$

$$⑯ f(x) = x^2 - 4x, g(x) = 0$$



(14)

$$y = 6 - 2x - x^2, \quad y = x + 6$$

INTERSECTION

$$6 - 2x - x^2 = x + 6$$

$$-x^2 - 3x = 0$$

$$x(x+3) = 0$$

$$x = 0, x = -3$$

$$(0, 6), (-3, 3)$$

(a) X-AXIS

$$V = \pi \int_{-3}^0 [(6 - 2x - x^2)^2 - (x + 6)^2] dx$$

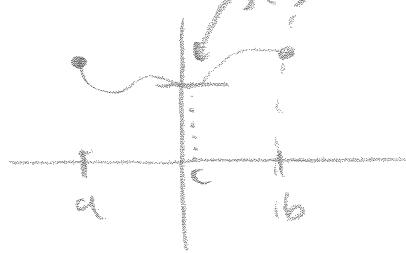
$$\begin{aligned} & (6 - 2x - x^2)(6 - 2x - x^2) \\ & 36 - 12x - 6x^2 \\ & -12x + 4x^2 + 2x^3 \\ & \hline & (36 - 24x - 8x^2 + 4x^3 + x^4) - (x^2 + 12x + 36) \end{aligned}$$

$$V = \pi \int_{-3}^0 (x^4 + 4x^3 - 9x^2 - 36x) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{4x^4}{4} - \frac{9x^3}{3} - \frac{36x^2}{2} \right]_{-3}^0$$

$$= \pi \left[\frac{x^5}{5} + x^4 - 3x^3 - 18x^2 \right]_{-3}^0 = \boxed{\frac{243\pi}{5}}$$

⑯ REVIEW ROLLE'S THEOREM



$$f(x) = (x-3)(x+1)^2 \quad [-1, 3]$$

$$\begin{aligned} f(-1) &= (-4)(0) = 0 & f(a) = f(b) & \checkmark \\ f(3) &= (0)(4)^2 = 0 & \text{CONTINUOUS} \end{aligned}$$

$$\begin{aligned} f'(x) &= (x+1)^2 + (x-3) \cdot 2(x+1) \\ &= (x^2 + 2x + 1) + 2(x^2 - 2x - 3) \\ &= x^2 + 2x + 1 + 2x^2 - 4x - 6 \\ &= 3x^2 - 2x - 5 \\ &= (3x - 5)(x + 1) = 0 \\ x &= \frac{5}{3} \quad x = -1 \end{aligned}$$

$$\boxed{x = 5/3}$$

⑰ $f(x) = \frac{x}{2} - \sin \frac{\pi x}{6}, \quad [-1, 0]$

$$f(-1) = f(0) = 0 \quad \& \text{CONTINUOUS}$$

$$f'(x) = \frac{1}{2} - \left(\cos \frac{\pi x}{6}\right) \frac{\pi}{6}$$

$$\frac{1}{2} - \frac{\pi}{6} \cdot \cos \frac{\pi x}{6} = 0$$

$$-\frac{\pi}{6} \cos \frac{\pi}{6} x = -\frac{1}{2}$$

$$\cos \frac{\pi x}{6} = \frac{3}{\pi}$$

$$\cos^{-1} \left(\cos \frac{\pi x}{6} \right) = \cos^{-1} \left(\frac{3}{\pi} \right)$$

$$\frac{\pi x}{6} = \cos^{-1} \left(\frac{3}{\pi} \right)$$

$$x = \frac{6}{\pi} \cos^{-1} \left(\frac{3}{\pi} \right)$$

$$x \approx -5.76$$

(43) $f(x) = \sqrt{2-x}$, $[-7, 2]$ REVIEW MEAN VALUE THM

SINCE $f(x)$ IS CONTINUOUS &
DIFFERENTIABLE ON $[-7, 2]$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

FIND AVG SLOPE

$$\frac{\sqrt{2-2} - \sqrt{2-(-7)}}{2 - (-7)}$$

$$-\frac{\sqrt{9}}{9} = -\frac{3}{9} = -\frac{1}{3}$$

$$f'(x) = \frac{1}{2}(2-x)^{-1/2} = -\frac{1}{2}$$

$$\frac{1}{2\sqrt{2-x}} = -\frac{1}{3}$$

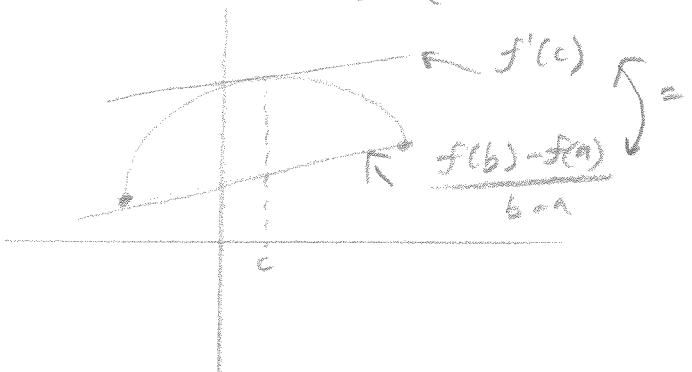
$$3 = -2\sqrt{2-x}$$

$$-\frac{3}{2} = \sqrt{2-x}$$

$$\frac{9}{4} = 2-x$$

$$\frac{1}{4} = -x$$

$$\boxed{-\frac{1}{4} = x}$$



1102

$$\int_0^b |2x| dx$$

$$\boxed{b|b|}$$

CASE 1

$$\int_0^2 2x dx = \left[\frac{2x^2}{2} \right]_0^2 = 4$$

$$\int_0^{-2} |2x| dx = \int_{-2}^0 -2x dx = \left[\frac{-2x^2}{2} \right]_{-2}^0 = -4$$

$$\textcircled{107} \int_{\pi/6}^{\pi/2} \cot x dx = \left[\ln |\sin x| \right]_{\pi/6}^{\pi/2}$$

$$= \ln |\sin \frac{\pi}{2}| - \ln |\sin \frac{\pi}{6}|$$

$$\ln 1 - \ln \frac{1}{2}$$

$$0 - \ln \frac{1}{2}$$

$$\ln \left(\frac{1}{2}\right)^{-1}$$

$$\boxed{\ln 2}$$

ASSIGNMENT #4

SECTION 5.1 EXC 41, 43, 49, 52, 55

SECTION 7.1 EXC 31

SECTION 7.2 63 a, b, c

CPB P. 128-129 EXC 1-6

$$\textcircled{41} \quad y = \ln x^3 @ (1, 0)$$

$$y' = 3 \cdot \frac{1}{x} \quad y - y_1 = m(x - x_1)$$

$$@ (1, 0) \quad y' = 3$$

$$\begin{cases} y - 0 = 3(x - 1) \\ y = 3x - 3 \end{cases} \quad \checkmark$$

$$\textcircled{43} \quad y = \ln x^2$$

$$y' = 2 \cdot \frac{1}{x} \quad y - y_1 = m(x - x_1)$$

$$@ (1, 0) = 2$$

$$\begin{cases} y - 0 = 2(x - 1) \\ y = 2x - 2 \end{cases} \quad \checkmark$$

$$\textcircled{49} \quad y = \ln(x\sqrt{x^2-1}) = \ln x + \frac{1}{2}\ln(x^2-1)$$

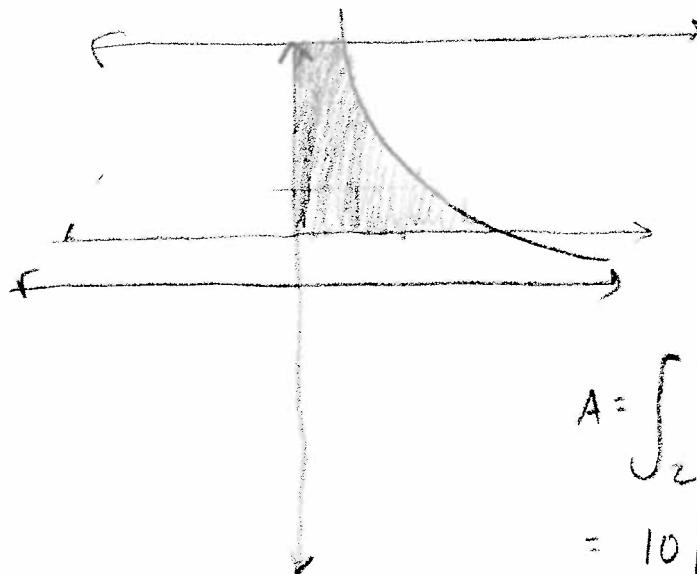
$$y' = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2-1} = \boxed{\frac{1}{x} + \frac{x}{x^2-1}} \quad \checkmark$$

$$\textcircled{52} \quad f(x) = \ln\left(\frac{2x}{x+3}\right) = \ln(2x) - \ln(x+3)$$

$$f'(x) = \frac{2}{2x} - \frac{1}{x+3} = \boxed{\frac{1}{x} - \frac{1}{x+3}}$$

$$\textcircled{55} \quad y = \ln(\ln x^2)$$

31) $f(x) = \frac{10}{x}$, $x=0$, $y=2$, $y=10$



$$y = \frac{10}{x}$$

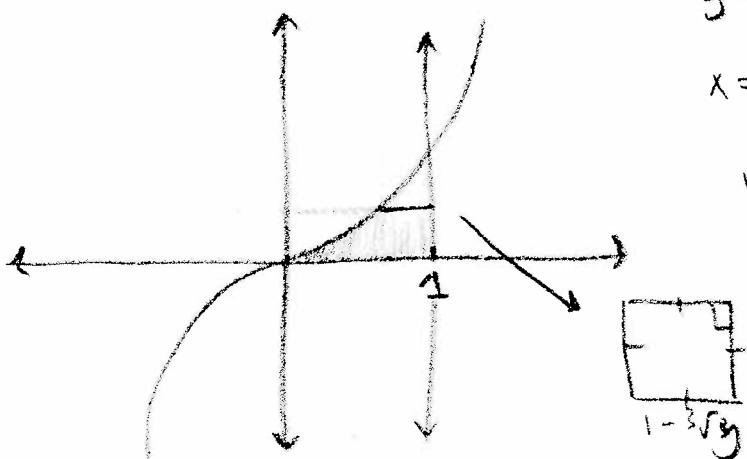
$$xy = 10$$

$$x = \frac{10}{y}$$

$$A = \int_2^{10} \frac{10}{y} dy = 10 \ln y \Big|_2^{10}$$

$$= 10 [\ln 10 - \ln 2] = \boxed{10 \ln 5}$$

63) a) CROSS SECTION SQUARES



$$y = x^3 \quad y=0 \quad x=1$$

$$x = \sqrt[3]{y}$$

$$V = \int_0^1 (1 - \sqrt[3]{y})^2 dy$$

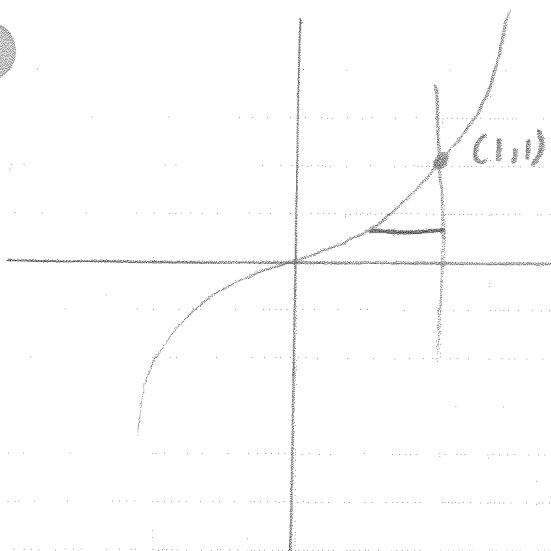
$$= \int_0^1 [1 - 2y^{2/3} + y^{4/3}] dy$$

$$= \left[y - 2 \cdot \frac{3}{4} y^{4/3} + \frac{3}{5} y^{5/3} \right]_0^1$$

$$[1 - \frac{3}{2} + \frac{3}{5}] = \boxed{\frac{1}{10}}$$

63 b

SEMICIRCLES



$$\text{SEMI } O = \frac{1}{2} \pi r^2$$

$$DO = (1 - \sqrt[3]{y})$$

$$rO = \frac{1}{2} (1 - \sqrt[3]{y})$$



$$r = \frac{1}{2} (1 - \sqrt[3]{y})$$

$$V = \frac{\pi}{2} \int_0^1 \left(\frac{1 - \sqrt[3]{y}}{2}\right)^2 dy$$

$$V = \frac{\pi}{8} \int_0^1 (1 - 2y^{1/3} + y^{2/3}) dy$$

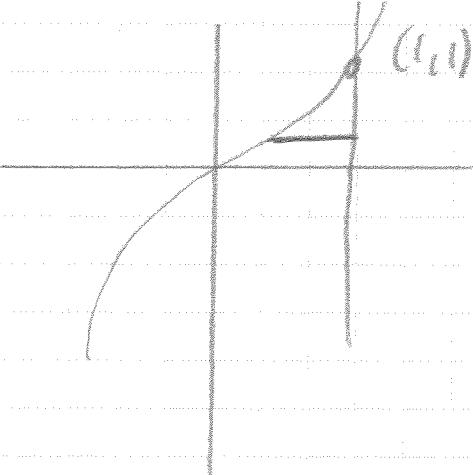
$$\frac{\pi}{8} \left[y - \frac{3}{2} y^{4/3} + \frac{3}{5} y^{5/3} \right]_0^1$$

$$\frac{\pi}{8} \left[1 - \frac{3}{2} + \frac{3}{5} \right] = \boxed{\frac{\pi}{80}}$$

63 c

EQUILATERAL Δ'S

$$A \Delta = \frac{\sqrt{3}}{4} a^2$$



$$V = \int_0^1 \frac{\sqrt{3}}{4} (1 - \sqrt[3]{y})^2 dy$$

$$= \frac{\sqrt{3}}{4} \int_0^1 (1 - 2y^{1/3} + y^{2/3}) dy$$

$$\frac{\sqrt{3}}{4} \left[y - \frac{3}{2} y^{4/3} + \frac{3}{5} y^{5/3} \right]_0^1$$

$$\frac{\sqrt{3}}{4} \left[1 - \frac{3}{2} + \frac{3}{5} \right] = \frac{\sqrt{3}}{4} \cdot \frac{1}{10} = \boxed{\frac{\sqrt{3}}{40}}$$

CPB P.128-129

① $\int \sin 3\theta d\theta = \frac{1}{3} \int 3 \sin 3\theta d\theta = \frac{1}{3} \cdot -\cos 3\theta + C$

OR

$$\boxed{-\frac{\cos 3\theta + C}{3}}$$

② $\int 3^{x^2} \cdot x dx = \frac{1}{2} \int 2x \cdot 3^{x^2} dx = \frac{1}{2} \frac{1}{\ln 3} \cdot 3^{x^2} + C$

OR

$$\boxed{\frac{3^{x^2}}{\ln 3} + C}$$

③ $\int_0^6 f(x) dx$

$$f(x) = \begin{cases} x & 0 < x \leq 2 \\ 1 & 2 < x \leq 4 \\ \frac{1}{2}x & 4 < x \leq 6 \end{cases}$$

$$\int_0^6 f(x) dx = \int_0^2 x dx + \int_2^4 dx + \int_4^6 \frac{1}{2}x dx$$

$$= \left[\frac{x^2}{2} \right]_0^2 + [x]_2^4 + \left[\frac{x^2}{4} \right]_4^6$$

$$= \left[\frac{(2)^2}{2} \right] + [4 - 2] + \left[\frac{6^2}{4} - \frac{4^2}{4} \right]$$

$$2 + 2 + (9 - 4) = \boxed{9}$$

④ $\int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2} \int_0^1 \frac{2x}{x^2+1} dx = \left[\frac{1}{2} \ln(x^2+1) \right]_0^1$

$$= \frac{1}{2} [\ln(2) - \ln(1)] = \frac{1}{2} (\ln 2) = \boxed{\ln \sqrt{2}}$$

$$\textcircled{5} \quad g(x) = (x-3)^2 \quad [1, 3]$$

AVERAGE
VALUE

$$\begin{aligned}
 f(c) &= \frac{1}{b-a} \int_a^b f(x) dx \\
 &= \frac{1}{2} \int_1^3 (x-3)^2 dx \\
 &= \frac{1}{2} \int_1^3 (x^2 - 6x + 9) dx \\
 &= \frac{1}{2} \left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x \right]_1^3 \\
 &= \frac{1}{2} \left[\left(\frac{3^3}{3} - 3(3)^2 + 9(3) \right) - \left(\frac{1^3}{3} - 3(1)^2 + 9(1) \right) \right] \\
 &= \frac{1}{2} \left(9 - \frac{19}{3} \right) = \boxed{\frac{4}{3}}
 \end{aligned}$$

$\int_a^b f(x) dx$

$$\begin{aligned}
 \textcircled{6} \quad \int_0^5 \frac{dx}{\sqrt{3x+1}} &= \frac{1}{3} \int_0^5 \frac{3}{\sqrt{3x+1}} dx = \left[\frac{1}{3} \cdot \frac{2}{1} \cdot (3x+1)^{\frac{1}{2}} \right]_0^5 \\
 \left. \frac{2}{3} (3x+1)^{\frac{1}{2}} \right|_0^5 &= \frac{2}{3} \left[\sqrt{3(5)+1} - \sqrt{3(0)+1} \right] = \frac{2}{3} (4 - 1) = \boxed{2}
 \end{aligned}$$



ASSIGNMENT 77

SECTION 5.1 EX 85, 93, 102; SECTION 5.2 EXC 25, 29,
35, 37, 46; SECTION 7.1 EXC 55; CPB EXC 1125,
1139, 1143

(85) $y = x \ln x$

$$y' = \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

$$\ln x + 1 = 0$$

$$\ln x = -1$$

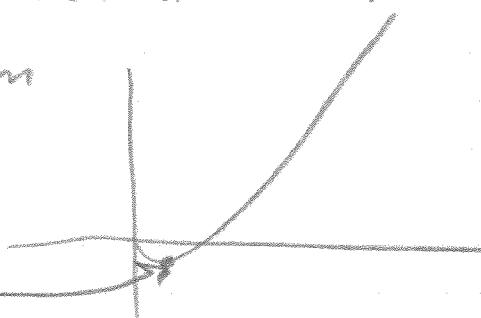
$x = e^{-1}$ \Rightarrow RELATIVE EXTREMA OCCUR AT CRITICAL POINTS

$$y'' = \frac{1}{x} > 0 \Rightarrow e^{-1} \text{ is a minimum}$$

$$y = e^{-1} \ln(e^{-1})$$

$$e^{-1} \cdot -\frac{1}{e^{-1}} = -e^{-1}$$

$$(e^{-1}, -e^{-1})$$



(93) $y = x \sqrt{x^2 - 1}$

$$\ln y = \ln(x \sqrt{x^2 - 1})$$

$$\ln y = \ln x + \frac{1}{2} \ln(x^2 - 1)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} + \frac{1}{2} \cdot \frac{2x}{x^2 - 1}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{(x^2 - 1) \cdot 1}{(x^2 - 1)x} + \frac{x \cdot x}{(x^2 - 1)x}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2x^2 - 1}{x(x^2 - 1)}$$

$$\frac{dy}{dx} = y \left[\frac{2x^2 - 1}{x(x^2 - 1)} \right]$$

$$x \sqrt{x^2 - 1} \cdot \left[\frac{2x^2 - 1}{x(x^2 - 1)} \right] = \frac{2x^2 - 1}{\sqrt{x^2 - 1}}$$

$$102 \quad f(x) = x - 2\ln x \quad [1, 3]$$

a) $f(1) = 1 - 2\ln 1 = 1$ $f(a) \neq f(b) \Rightarrow$ ROLLE'S THEOREM
 $f(2) = 3 - 2\ln 3 \approx .803$ DOES NOT APPLY

b) $f'(x) = 1 - \frac{2}{x} = 0$ IT STILL WORKS BECAUSE $f(x)$ IS CONCAVE UP. THERE EXISTS A MINIMUM
 $\frac{2}{x} = 1$
 $x = 2$

$$25 \quad \int \frac{1}{1+\sqrt{2x}} dx$$

$$u = 1 + \sqrt{2x}$$

$$du = \frac{1}{2}(2x)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{2x}} dx$$

$$\int \left(\frac{u-1}{u}\right) du$$

$$= \int \left(1 - \frac{1}{u}\right) du$$

$$u-1 = \sqrt{2x} \quad \sqrt{2x} du = dx$$

$$(u-1) du = dx$$

$$= u - \ln|u| + C$$

$$(1 + \sqrt{2x}) - \ln|1 + \sqrt{2x}| + C$$

$$29 \quad \int \frac{\cos \theta}{\sin \theta} d\theta = \ln|\sin \theta| + C$$

$$35 \quad \int \frac{\sec x + 1}{\sec x - 1} dx = \ln|\sec x - 1| + C$$

$$37 \quad \frac{dy}{dx} = \frac{3}{2-x}, (1, 0)$$

$$y = \int \frac{3}{2-x} = 3 \int \frac{1}{2-x} = -3 \int \frac{-1}{2-x} = -3 \ln|2-x| + C$$

$$0 = -3 \ln|2-1| + C$$

$$0 = -3 \cdot \ln 1 + C$$

$$0 = C$$

$$y = -3 \ln|2-x|$$

$$\textcircled{16} \quad \frac{dy}{dx} = \sec x \quad (0, 1)$$

$$y = \int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$l = \ln |\sec 0 + \tan 0| + C$$

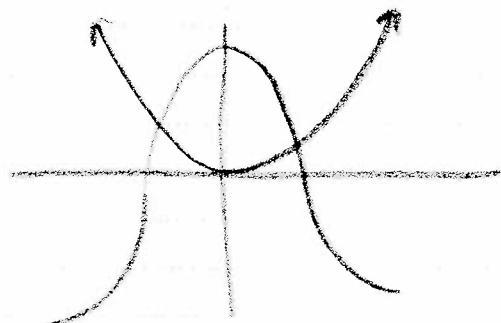
$$l = \ln |1 + 0| + C$$

$$l = \ln 1 + C$$

$$l = C$$

$$y = \ln |\sec x + \tan x| + l$$

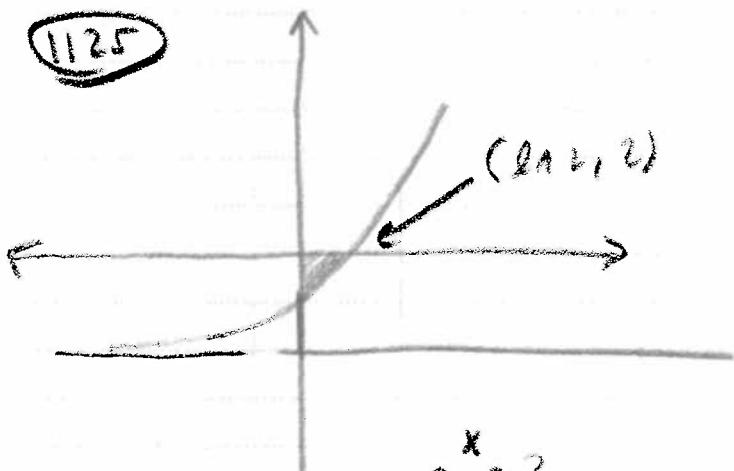
$$\textcircled{55} \quad y = x^2, \quad y = 4\cos x$$



INTERSECTIONS ARE IRRATIONAL NUMBERS

$$A = \int_{-1.20}^{1.20} (4\cos x - x^2) dx \approx 6.3043$$

\textcircled{1125}



$$\begin{aligned} e^x &= 2 \\ \ln e^x &= \ln 2 \\ x &= \ln 2 \end{aligned}$$

SEMI O'S

$$\frac{1}{2}\pi r^2$$

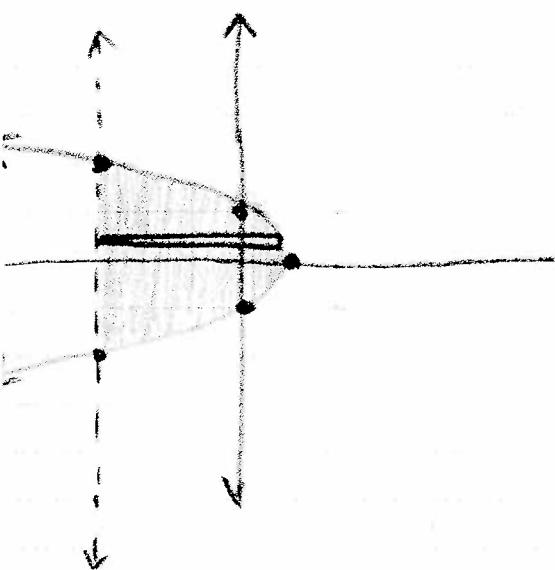
$$\frac{1}{2}\pi \left[\frac{2-e^x}{2} \right]^2$$

$$V = \frac{1}{8}\pi \int_0^{2\ln 2} (2-e^x)^2 dx$$

1139 $x = 1 - y^2$; $x = -3$; $x\text{-axis}$; $\lambda = -3$

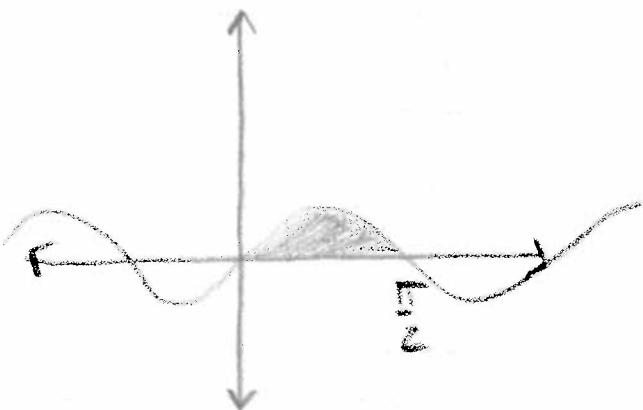
$$-x+1=y^2$$

$$y = \pm \sqrt{x-1}$$



$$V = \int_{-2}^2 ((1-y^2)+3)^2 dy = \frac{512}{15}$$

1143 $y = \sin x \cos x$; $x\text{-axis}$ $[0, \pi/2]$



$$V = \int_0^{\pi/2} (\sin x \cos x)^2 dx \approx 1.196$$

ASSIGNMENT 78

SECTION 5.3 EXC 1-2

SECTION 5.4 EXC 5, 15, 35, 38

SECTION 5.2 EXC 43, 61

SECTION 5.1 EXC 63, 68

(PPB EX 1136, 1172)

$$\textcircled{1} \quad f(x) = 5x+1 \quad g(x) = (x-1)/5$$

$$f(g(x)) = 5\left(\frac{x-1}{5}\right) + 1 = x \quad \checkmark$$

$$g(f(x)) = \frac{(5x+1)-1}{5} = x \quad \checkmark$$

$$\textcircled{2} \quad f(x) = 3 - 4x \quad g(x) = \frac{(3-x)}{4}$$

$$f(g(x)) = 3 - 4\left(\frac{3-x}{4}\right) = x \quad \checkmark$$

$$g(f(x)) = \frac{(5x+1)-1}{5} = x \quad \checkmark$$

$$\textcircled{5} \quad 9 - 2e^x = 7 \\ -2e^x = -2 \\ e^x = 1 \\ x = \ln 1 \\ \boxed{x=0}$$

$$\textcircled{13} \quad \ln \sqrt{x+2} = 1$$

$$e^{\ln \sqrt{x+2}} = e^1 \\ \sqrt{x+2} = e \\ x+2 = e^2 \\ \boxed{x = e^2 - 2}$$

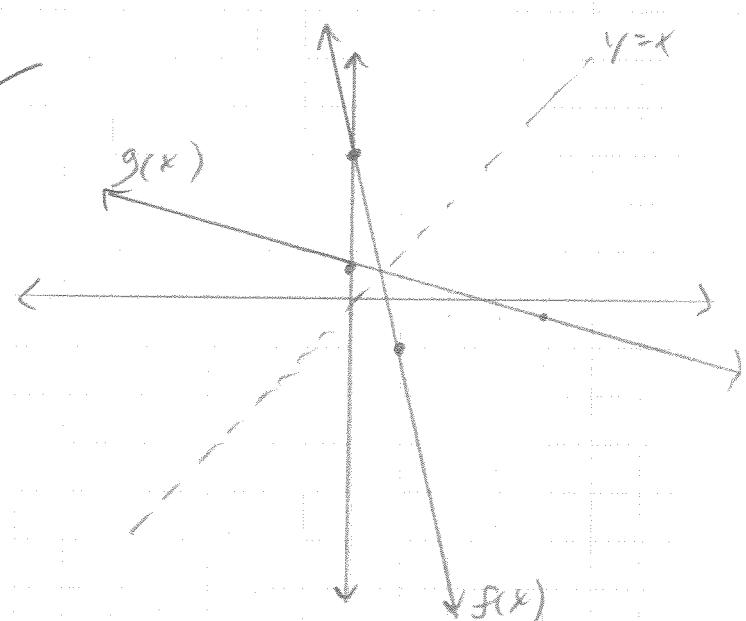
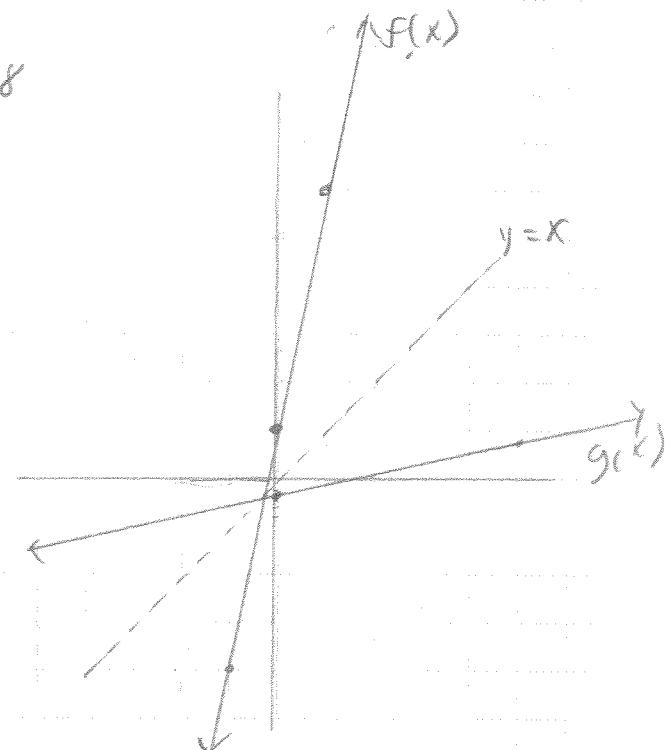
$$\textcircled{35} \quad f(x) = e^{2x} \\ f'(x) = 2e^{2x}$$

$$\textcircled{38} \quad y = x^2 e^{-x}$$

$$y' = 2xe^{-x} + x^2(e^{-x})$$

$$2xe^{-x} - x^2e^{-x}$$

$$\boxed{x e^{-x}(2-x)}$$



$$\textcircled{43} \quad \frac{dy}{dx} = \frac{1}{x+2}, \quad (0, 1)$$

$$y = \int \frac{1}{x+2} dx = \ln|x+2| + C$$

$$1 = \ln|0+2| + C$$

$$1 = \ln 2 + C$$

$$1 - \ln 2 = C$$

$$y = \ln|x+2| - \ln 2 + 1$$

$$= \boxed{\ln \left| \frac{x+2}{2} \right| + 1}$$

$$\textcircled{61} \quad \int_1^x \frac{1}{t} dt$$

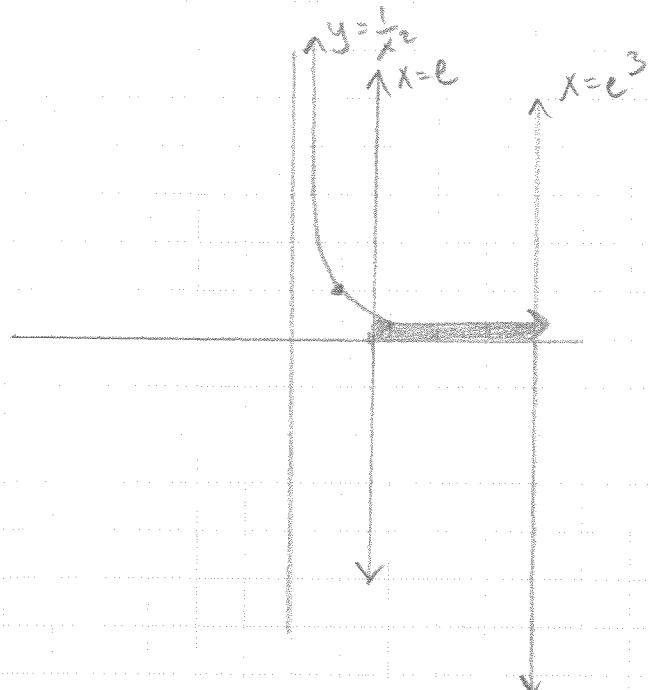
$$F'(x) = \frac{1}{x}$$

$$\textcircled{63} \quad y = \ln|\sin x|; \quad y' = \frac{\cos x}{\sin x} = \cot x$$

$$\textcircled{68} \quad y = \ln \sqrt{2 + \cos^2 x} = \frac{1}{2} \ln|2 + \cos^2 x|$$

$$y' = \frac{1}{2} \cdot \frac{-2\cos x \sin x}{2 + \cos^2 x} = \frac{-\cos x \sin x}{2 + \cos^2 x}$$

1136 $y = \frac{1}{x^2} \rightarrow x = e, x = e^3, y = 0$, Axis: x-axis



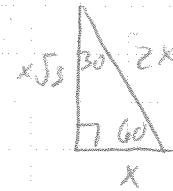
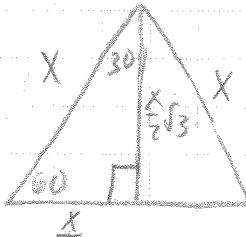
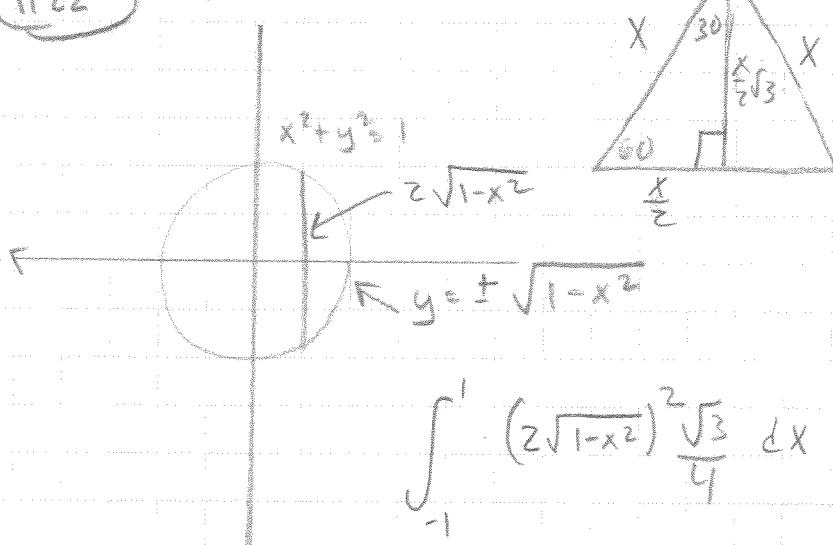
DISK METHOD

$$V = \pi \int_e^{e^3} \left(\frac{1}{x^2}\right)^2 dx$$

$$= \pi \left[-\frac{3}{x^3} \right]_e^{e^3} = \pi \left[-\frac{3}{(e^3)^3} + \frac{3}{e^3} \right]$$

$$\pi \left[-\frac{3}{e^9} + \frac{3}{e^3} \right]$$

1122



$$\frac{x}{2} \cdot \frac{x\sqrt{3}}{2}$$

$$A = \frac{x^2\sqrt{3}}{4}$$



ASSIGNMENT +9
CPB EXC 993, 996, 986, 989,
992, 1006, 1129, 1114

993 $\int \frac{\sec \theta \tan \theta}{\sec \theta - 1} d\theta = \ln |\sec \theta - 1| + C$

996 $\int e^x \sqrt{1-e^x} dx = -\frac{2}{3}(1-e^x)^{3/2} + C$

986 $\int \frac{(\ln x)^2}{x} dx = \frac{(\ln x)^3}{3} + C$

989 $\int \frac{2x}{(x-1)^2} dx$
 $U = x-1$
 $dU = dx$
 $x = U+1$

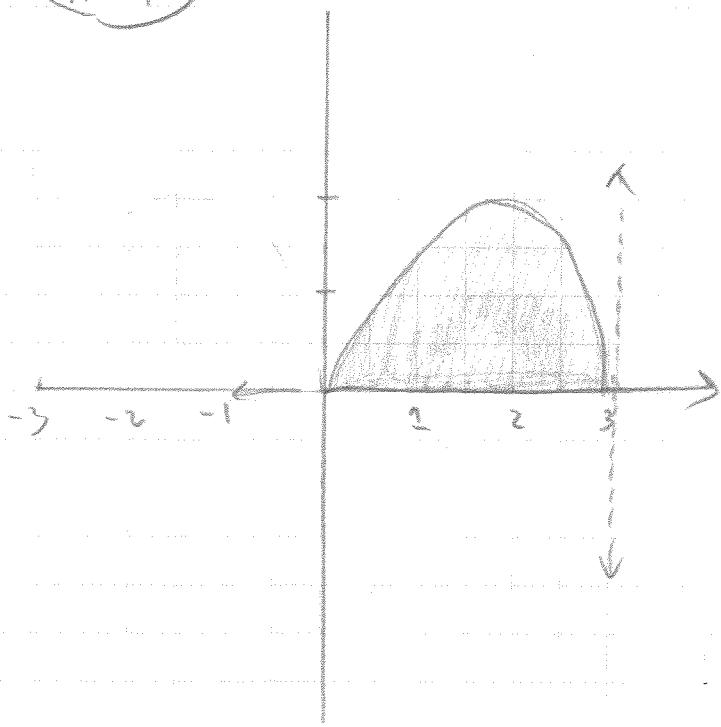
$$\int \frac{2(U+1)}{U^2} du = \int \left[\frac{2}{U} + \frac{2}{U^2} \right] du = 2\ln|U| - \frac{2}{U} + C$$
$$= 2\ln|x-1| - \frac{2}{x-1} + C$$

992 $\int \frac{\cos \theta}{1+\sin \theta} d\theta = \ln |1+\sin \theta| + C$

1006 $\int_e^{e^2} \frac{1}{x \ln x} dx = \int_e^{e^2} \left(\frac{1}{\ln x} \cdot \frac{1}{x} \right) dx = \left[\ln |\ln x| \right]_e^{e^2}$

$$\frac{\ln(\ln(e^2)) - \ln(\ln(e))}{\ln 2 - \ln 1} = \boxed{\ln 2}$$

1129



a) EQUILATERAL Δ AREA

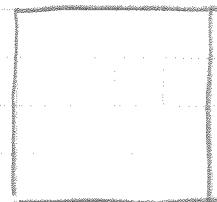
$$\frac{x^2 \sqrt{3}}{4}$$

$$A_{\Delta} = \frac{(2\sqrt{\sin x})^2 \sqrt{3}}{4} = \frac{4 \sin x \sqrt{3}}{4}$$

$$V = \sqrt{3} \int_0^{\pi} \sin x \, dx = -\sqrt{3} \cos x \Big|_0^{\pi}$$

$$= -\sqrt{3}(\cos \pi - \cos 0) \\ = -\sqrt{3}(-1 - 1) \\ = \boxed{2\sqrt{3}}$$

b)



$$2\sqrt{\sin x} \cdot A \square = 4 \sin x$$

$$2\sqrt{\sin x}$$

$$V = \int_0^{\pi} 4 \sin x \, dx = -4 \cos x \Big|_0^{\pi} = -4(\cos \pi - \cos 0) \\ = -4(-1 - 1) = \boxed{8}$$

1114

$$a(t) = 6t, t=0, v=1, d=3$$

$$v(t) = \int 6t \, dt = \frac{6t^2}{2} = 3t^2 + C$$

$$1 = 3(0)^2 + C \\ 1 = C$$

$$v(t) = 3t^2 + 1$$

$$s(t) = \int 3t^2 + 1 \, dt = \frac{3t^3}{3} + t = t^3 + t + C$$

$$3 = (0)^3 + (0) + C \\ 3 = C$$

$$s(t) = t^3 + t + 3$$



ASSIGNMENT 80

CPB EXC 458, 482, 510, 587, 578,
1111, 1118, 1126, 1138

458

$$g(z) = \frac{1}{\sqrt{36-z^2}} dz = (36-z^2)^{-\frac{1}{2}} dz$$

$$g'(z) = -\frac{1}{2} (36-z^2)^{-\frac{3}{2}} \cdot (-2z) = \frac{z}{\sqrt{(36-z^2)^3}}$$

482

$$y = \tan x \quad y = 2x$$

$$y' = \sec^2 x \quad y' = 2$$

$$\sec^2 x = 2$$

$$\sec x = \sqrt{2} \Rightarrow \cos x = \frac{\sqrt{2}}{2} \in [-\pi/2, \pi/2]$$

$$x = -\frac{\pi}{4} \text{ or } \frac{\pi}{4}$$

510

$$x^3 - xy + y^3 = 1$$

$$3x^2 - y + -xy' + 3y^2 y' = 1$$

$$-xy' + 3y^2 y' = y - 3x^2$$

$$y' = \frac{y - 3x^2}{3y^2 - x}$$

587

$$F(x) = x^2 e^{-4 \ln x} = x^2 e^{\ln x^{-4}} = x^2 \cdot x^{-4} = x^{-2}$$

$$F'(x) = x^{-2} dx = -2x^{-3} = \frac{-2}{x^3}$$

578

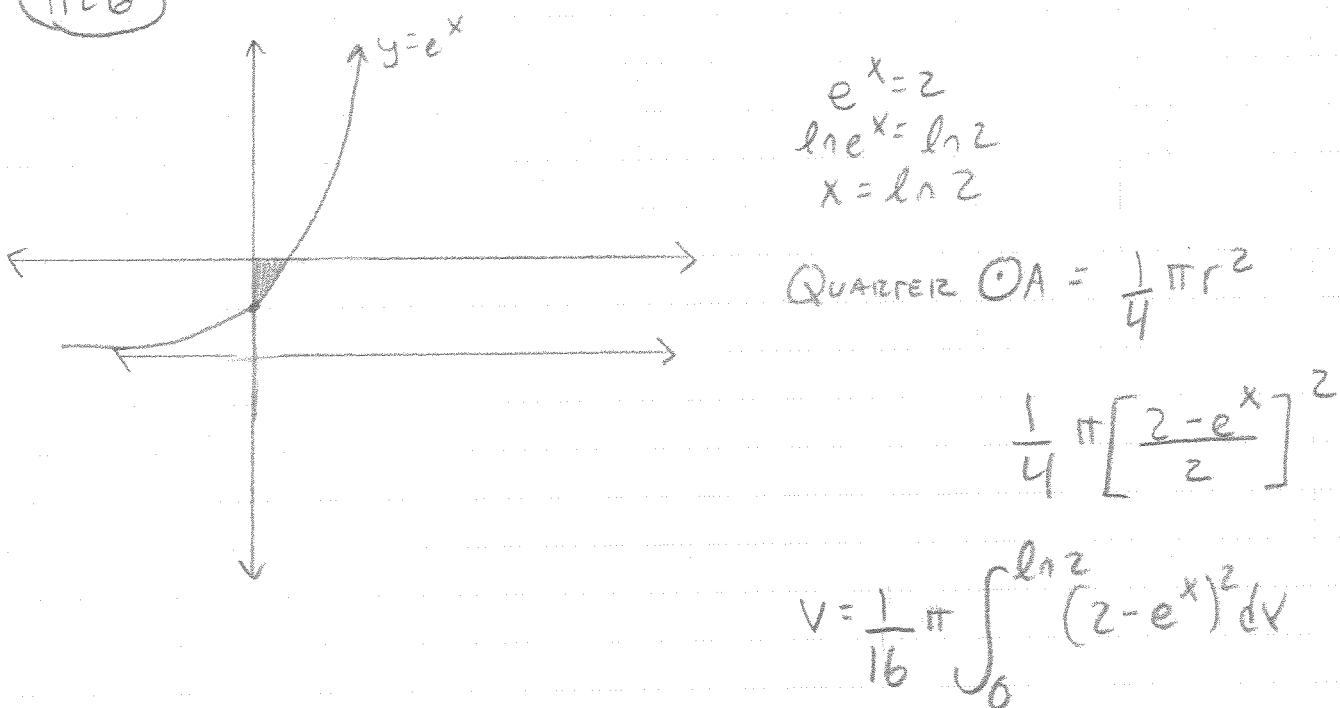
$$K(x) = \log_3 (x^2 + e^x) = \frac{\ln(x^2 + e^x)}{\ln 3}$$

$$K'(x) = \frac{1}{\ln 3} \cdot \frac{2x + e^x}{x^2 + e^x}$$

$$\text{III} \int_0^1 \frac{e^x}{(3-e^x)^2} dx = -\int \frac{-e^x}{(3-e^x)^2} = -\ln|3-e^x| + C$$

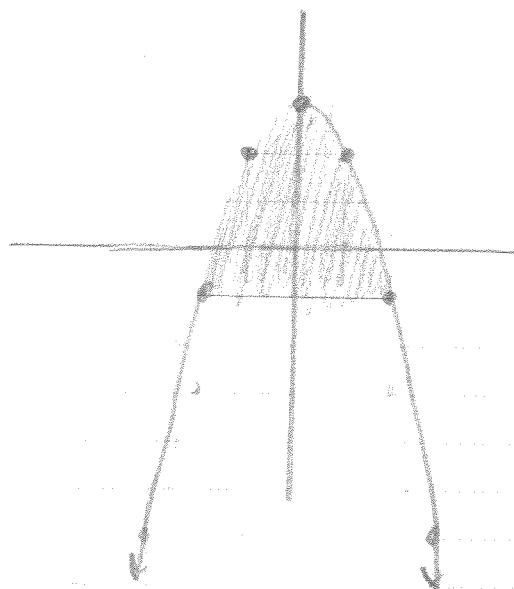
$$\text{III} 18 \int_1^e \left(x - \frac{1}{2x}\right) dx = \left[\frac{x^2}{2} - \frac{1}{2} \ln x\right]_1^e$$

1126



(1138)

$$y = 3 - x^2, y = -1, \text{ axis: } y = -1$$



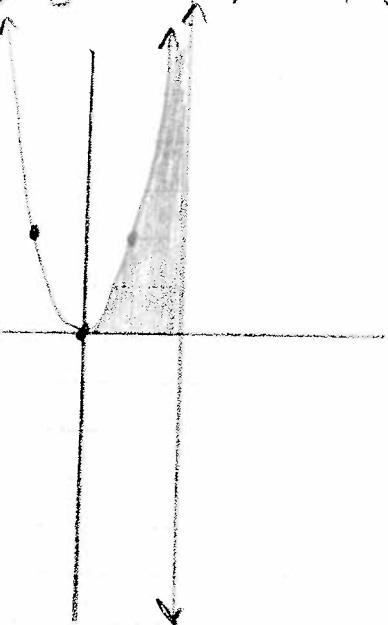
$$V = \pi \int_{-2}^2 [(3-x^2) - (-1)]^2 dx$$

$$= \pi \int_{-2}^2 (4-x^2)^2 dx = \frac{512\pi}{15}$$

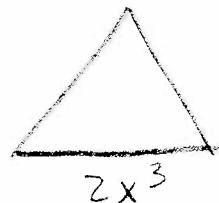


ASSIGNMENT 8/
CPB EX 1130, 1141, 1063, 1060

1130 $y = 2x^3$, $x=2$, $y=0$



$$\text{AREA} \triangle: \frac{x^2 \sqrt{3}}{4}$$

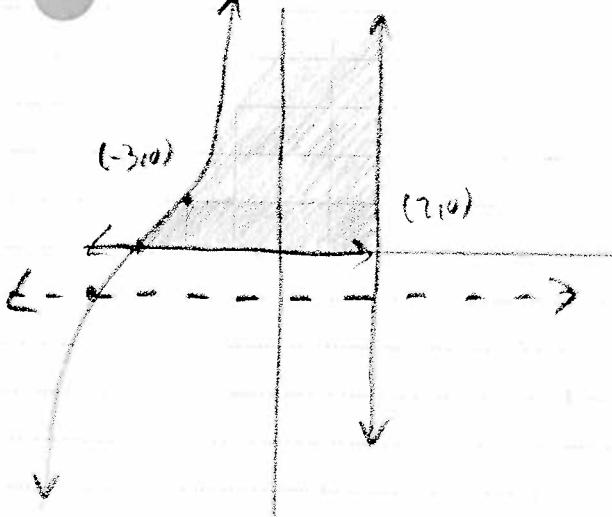


$$\begin{aligned}\text{AREA} &= \frac{(2x^3)^2 \sqrt{3}}{4} \\ &= \frac{4x^6 \sqrt{3}}{4} \\ &= x^6 \sqrt{3}\end{aligned}$$

$$V = \int_0^2 \sqrt{3} x^6 dx$$

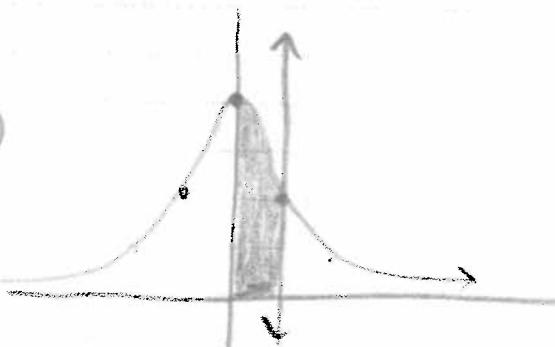
$$= \left[\sqrt{3} \frac{x^7}{7} \right]_0^2 = \frac{\sqrt{3} (2)^7}{7} = \frac{128\sqrt{3}}{7}$$

1141 $y = (x+3)^3$, $y=0$, $x=2$; Axis $y=-1$



$$\begin{aligned}V &= \pi \int_{-3}^2 \left([(x+3)^3 + 1]^2 - (0+1)^2 \right) dx \\ &= \frac{160625\pi}{14}\end{aligned}$$

1063 $y = \frac{4}{1+x^2}$, $y=0$, $x=0$, $x=1$



$$A = \int_0^1 \frac{4}{1+x^2} dx = 4 \tan^{-1} x \Big|_0^1$$

$$\begin{aligned}&4[\tan^{-1} 1 - \tan^{-1} 0] \\ &4[\pi/4 - 0]\end{aligned}$$

II

(1060) $y = x^3 + 4, y = 0, x = 0, x = 1$

$$A = \int_0^1 (x^3 + 4) dx = \left[\frac{x^4}{4} + 4x \right]_0^1 = \frac{1}{4} + 4 = \boxed{\frac{17}{4}}$$

ASSIGNMENT 82

SEC 7-2 EXC 38, 63b & c

SEC 7-1 EXC 48

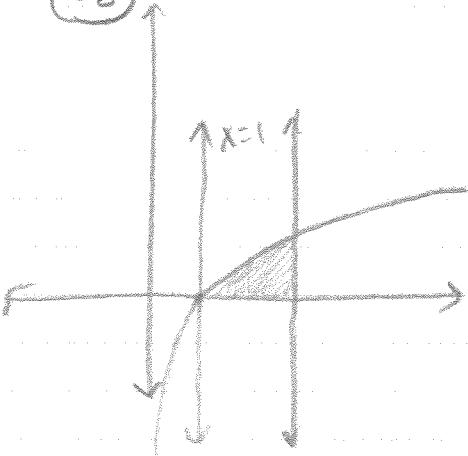
SEC 5-4 EXC 87, 93

SEC 5-3 EXC 6

SEC 5-2 EXC 14, 24

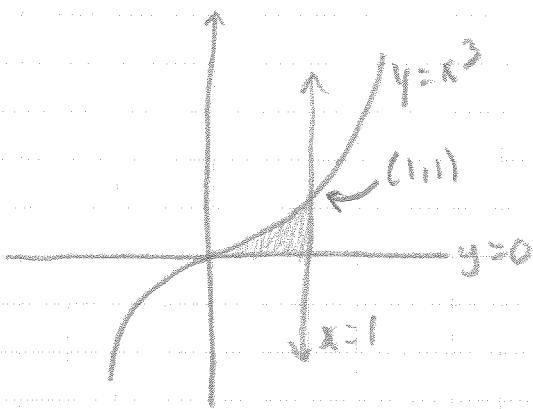
SEC 5-1 EXC 65, 66

(38)



$$V = \pi \int_1^3 (\ln x)^2 dx \approx 3.23$$

(63) (b) SEMICIRCLES



$$AO = \pi r^2$$

$$A\frac{1}{2} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left[\frac{1-\sqrt[3]{y}}{2}\right]^2$$

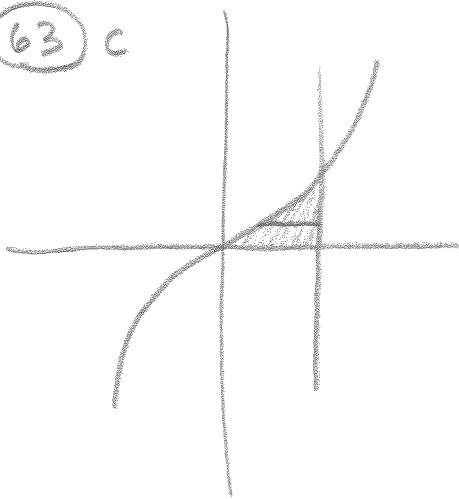
$$V = \frac{\pi}{8} \int_0^1 (1-\sqrt[3]{y})^2 dy$$

$$= \frac{\pi}{8} \int_0^1 (1-2\sqrt[3]{y} + y^{2/3}) dy = \boxed{\frac{\pi}{80}}$$

$$y = x^{2/3} \Rightarrow \sqrt[3]{y} = x$$

$$r(y) = 1 - \sqrt[3]{y}$$

(63) C



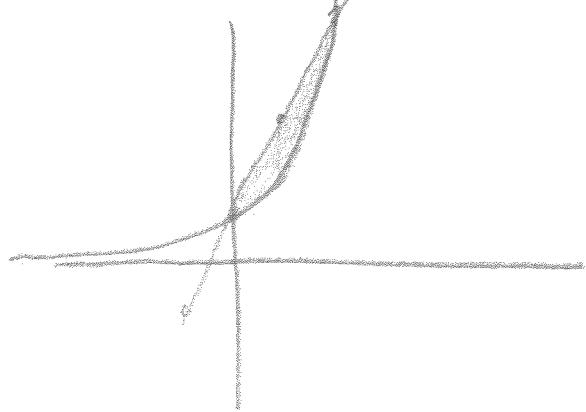
$$\text{AREA OF EQUILATERAL } \Delta = \frac{x^2\sqrt{3}}{4}$$

$$= (1 - 3\sqrt{y}) \frac{2\sqrt{3}}{4}$$



$$V = \frac{\sqrt{3}}{4} \int_0^1 (1 - 3\sqrt{y})^2 dy = \frac{\sqrt{3}}{40}$$

(42) $f(x) = 3^x, g(x) = 2x + 1$



$$\lim_{x \rightarrow 0} : 3^x = 2x + 1$$

$$x=0, k=1$$

$$A = \int_0^1 [(2x+1) - 3^x] dx$$

$$= \left[\frac{2x^2}{2} + x - \frac{3^x}{\ln 3} \right]_0^1 = 2 - \frac{2}{\ln 3}$$

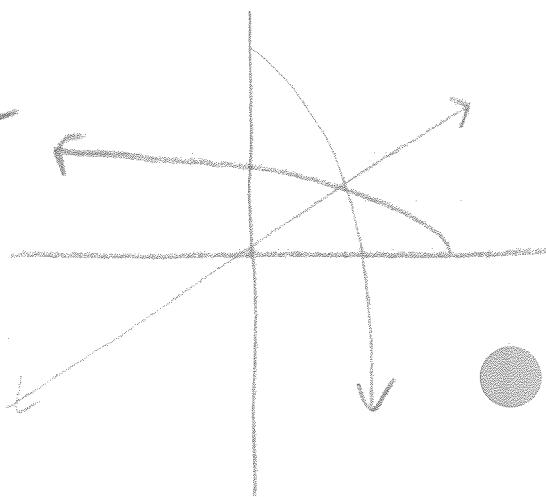
$$(87) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = 2e^{\sqrt{x}} + C$$

$$(93) \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \ln |e^x - e^{-x}| + C$$

$$(6) f(x) = 16 - x^2, x \geq 0 \quad g(x) = \sqrt{16-x}$$

$$f(g(x)) = 16 - (\sqrt{16-x})^2 = 16 - 16 + x = x \quad \checkmark$$

$$g(f(x)) = \sqrt{16 - (16 - x^2)} = \sqrt{x^2} = x \quad \checkmark$$



$$\begin{aligned}
 ⑭ \int \frac{2x^2+7x-3}{x-2} dx & \quad x-2 \quad \begin{array}{l} 2x+11 \\ 2x^2+7x-3 \\ -2x^2+4x \\ \hline 11x-3 \\ -11x+22 \\ \hline 19 \end{array} \\
 & = \int \left[2x+11 + \frac{19}{x-2} \right] dx \\
 & = \frac{2x^2+11x+19}{2} \ln|x-2| + C
 \end{aligned}$$

$$\begin{aligned}
 ⑯ \int \frac{x(x-2)}{(x-1)^3} dx & = \int \frac{x^2-2x+1-1}{(x-1)^3} dx \\
 & = \int \frac{(x-1)^2}{(x-1)^3} dx - \int \frac{1}{(x-1)^3} dx \\
 & = \int \frac{1}{x-1} dx - \int \frac{1}{(x-1)^3} dx \\
 & = \ln|x-1| + \frac{1}{2(x-1)^2} + C
 \end{aligned}$$

$$⑮ y = \ln \left| \frac{\cos x}{\cos x - 1} \right| = \ln |\cos x| - \ln |\cos x - 1|$$

$$y' = \frac{-\sin x}{\cos x} - \frac{(-\sin x)}{\cos x - 1}$$

$$y' = -\tan x + \frac{\sin x}{\cos x - 1}$$

$$⑯ y = \ln |\sec x + \tan x| =$$

$$y' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x (\tan x + \sec x)}{\sec x + \tan x} = \boxed{\sec x}$$



ASSIGNMENT 83

CPB EXC 454, 460, 480, 577,
 561, 587, 1127, 1133
 SEC 5.1 EXC 58, 59, 95

(454)

$$\sqrt[3]{(5d^2-1)^5} = (5d^2-1)^{\frac{5}{3}}$$

$$c'(d) = \frac{5}{3} (5d^2-1)^{\frac{2}{3}} \cdot 10d$$

$$(460) h(u) = \sqrt{u-1} \sqrt[3]{2u+3} = (u-1)^{\frac{1}{2}} (2u+3)^{\frac{1}{3}}$$

$$h'(u) = \frac{1}{2} (u-1)^{-\frac{1}{2}} (2u+3)^{\frac{1}{3}} + (u-1)^{\frac{1}{2}} \cdot \frac{1}{3} (2u+3)^{-\frac{2}{3}} (2)$$

$$= \frac{\sqrt[3]{2u+3}}{2\sqrt{u-1}} + \frac{2\sqrt{u-1}}{3\sqrt[3]{(2u+3)^2}}$$

(480)

$$y = x^2 \tan x$$

$$y' = (2x) \tan x + x^2 \sec^2 x$$

(577)

$$f(x) = \frac{e^x - 1}{e^x + 1} = \frac{(e^x - 1)'(e^x + 1) - (e^x - 1)(e^x + 1)'}{(e^x + 1)^2}$$

$$= \frac{e^x(e^x + 1) - e^x(e^x - 1)}{(e^x + 1)^2}$$

(561)

$$y = 3^{x^2}$$

$$y' = (\ln 3) 3^{x^2} \cdot (2x)$$

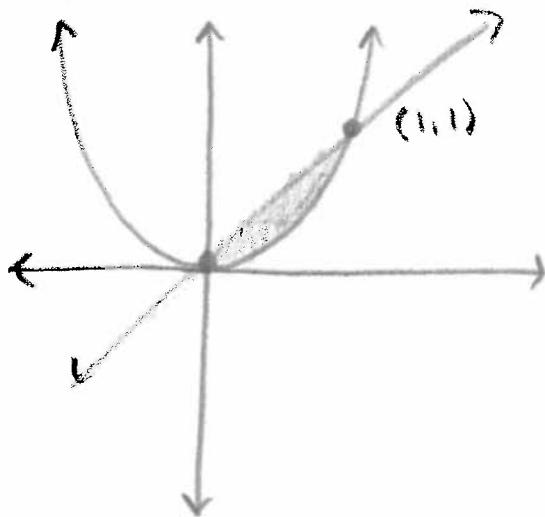
(587)

$$F(x) = x^2 e^{-4 \ln x}$$

$$F'(x) = (2x) e^{-4 \ln x} + (x^2) e^{-4 \ln x} \cdot -\frac{4}{x}$$

$$= 2x e^{-4 \ln x} [1 - 2x]$$

1127 $y = x^2$ & $y = x$



$$A \frac{1}{2} \odot = \frac{1}{2} \pi r^2$$

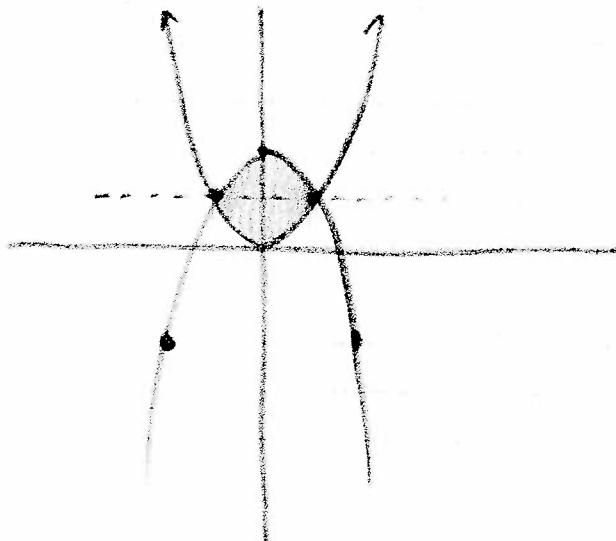
$$\frac{1}{2} \pi \left(\frac{x - x^2}{2} \right)^2$$

$$V = \frac{\pi}{8} \int_0^1 (x - x^2)^2 dx$$

$$= \frac{\pi}{8} \int_0^1 (x^2 - 2x^3 + x^4) dx$$

$$= \frac{\pi}{8} \left[\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right]_0^1 = \boxed{\frac{\pi}{240}}$$

1133 $y = x^2$, $y = 2 - x^2$; AXIS: y-AXIS



$$y = 2 - x^2$$

$$y - 2 = -x^2$$

$$2 - y = x^2$$

$$\sqrt{2-y} = x$$

$$x = \sqrt{y}$$

$$V = \pi \int_0^1 \sqrt{y}^2 dy + \pi \int_1^2 (\sqrt{2-y})^2 dy$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \boxed{\pi}$$

$$95) y = \frac{x^2 \sqrt{3x-2}}{(x-1)^2}$$

$$\ln y = \ln \left[\frac{x^2 \sqrt{3x-2}}{(x-1)^2} \right]$$

$$\ln y = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x-1)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{1}{2} \cdot \frac{3}{3x-2} - 2 \cdot \frac{1}{x-1} \quad \text{LCD: } 2(3x-2)(x-1)x$$

$$y' = y \left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{(x-1)} \right]$$

$$y' = \frac{x^2 \sqrt{3x-2}}{(x-1)^2} \left[\frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{(x-1)} \right]$$

SECTION 5.1

$$(58) \quad y = \ln 3\sqrt{\frac{x-1}{x+1}} = \frac{1}{3} [\ln(x-1) - \ln(x+1)]$$

$$y' = \frac{1}{3} \left[\frac{1}{x-1} - \frac{1}{x+1} \right] = \frac{1}{3} \left[\frac{(x+1)-(x-1)}{(x-1)(x+1)} \right] = \boxed{\frac{2}{3(x-1)(x+1)}}$$

$$(59) \quad f(x) = \ln \left(\frac{\sqrt{4+x^2}}{x} \right) = \frac{1}{2} \ln(4+x^2) - \ln x$$

$$f'(x) = \frac{1}{2} \cdot \frac{2x}{4+x^2} - \frac{1}{x} = \boxed{\frac{x}{4+x^2} - \frac{1}{x}}$$

$$(93) \quad y = x \sqrt{x^2-1}$$

$$\ln y = \ln [x \sqrt{x^2-1}]$$

$$\ln y = \ln x + \frac{1}{2} \ln(x^2-1)$$

$$\ln y = \ln x + \frac{1}{2} [\ln(x+1) + \ln(x-1)]$$

$$\frac{1}{y} \cdot y' = \frac{1}{x} + \frac{1}{2} \left[\frac{1}{x+1} + \frac{1}{x-1} \right]$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{2} \left[\frac{2x}{(x+1)(x-1)} \right]$$

$$y' = y \left[\frac{1}{x} + \frac{x}{(x+1)(x-1)} \right]$$

$$y' = x \sqrt{x^2-1} \left[\frac{1}{x} + \frac{x}{(x+1)(x-1)} \right]$$