# Techniques of Integration Answers



① $\int_0^{\frac{\pi}{4}} \sin x\,dx = \left[-\cos x\right]_0^{\pi/4} = -\left[\cos\frac{\pi}{4} - \cos 0\right] =$

(D) because $\int \sin x\,dx = -\cos x + C$ $\quad = -\left[\frac{\sqrt{2}}{2} - 1\right]$

$= \boxed{1 - \frac{\sqrt{2}}{2}}$ looks like D

it is D

② $\int_0^1 e^{-4x}\,dx = -\frac{1}{4}\int_0^{-4} e^u\,du = -\frac{1}{4}\left[e^u\right]_0^{-4} =$

(D) must fix that $-4x$ use u-sub.

$\boxed{u = -4x}$ if $x = 0 \Rightarrow u = 0$

$du = -4dx$ if $x = 1 \Rightarrow u = -4$

$-\frac{1}{4}du = dx$

$-\frac{1}{4}\left[e^{-4} - e^0\right] = -\frac{1}{4}\left[\frac{1}{e^4} - 1\right]$

$= \frac{1}{4} - \frac{1}{4e^4}$

$\frac{1}{4} - \frac{1}{4}e^{-4}$

③ $\frac{1}{2}\int e^{\frac{t}{2}}\,dt = \frac{1}{2}(2)\int e^u\,du = \int e^u\,du$

(C) u-sub again $u = \frac{t}{2}$ $\quad du = \frac{1}{2}dt$

$2du = dt$

$e^{\frac{t}{2}} + C$

④ $\int x^2\cos(x^3)\,dx = \frac{1}{3}\int \cos u\,du$

(B) u sub again. $\quad = \frac{1}{3}\left[\sin u\right] + C$

let $u = x^3$

$du = 3x^2dx$ $\quad = \frac{1}{3}\left[\sin(x^3)\right] + C$

$\frac{1}{3}du = x^2dx$

⑤ $\int_0^2 \sqrt{2x+1}\,dx$ if $\boxed{u = 2x+1}$ when $x=0$, $u=1$; when $x=2$, $u=5$

$du = 2dx$

$\frac{1}{2}du = dx$

(C) $\frac{1}{2}\int_1^5 u^{1/2}\,du = \frac{1}{2}\int_1^5 \sqrt{u}\,du$

⑥ $\int_1^e \frac{x^2-1}{x}\,dx$ whenever ONE term in denominator, simplify!!!

(E) $\int_1^e x\,dx - \int_1^e \frac{1}{x}\,dx$ → natural log.

$\left[\frac{x^2}{2}\right]_1^e - \left[\ln x\right]_1^e = \frac{e^2}{2} - \frac{1}{2} - \ln e + \overset{0}{\ln 1}$

$\frac{e^2}{2} - \frac{1}{2} - 1 = \boxed{\frac{e^2}{2} - \frac{3}{2}}$

⑦ $\int_{-3}^{k} x^2\,dx = 0$ Integrate

(A) $\left[\frac{x^3}{3}\right]_{-3}^{k} = \frac{k^3}{3} - \frac{(-3)^3}{3} = 0$

$\frac{k^3}{3} - \frac{-27}{3} = 0$

$\frac{k^3}{3} + 9 = 0 \Rightarrow \frac{k^3}{3} = -9$

$k^3 = -27$

$k = -3$

⑧ $\int_1^2 4x^3\,dx - \int_1^2 6x\,dx = \left[\frac{4x^4}{4} - \frac{6x^2}{2}\right]_1^2 =$

(C) $\left[x^4 - 3x^2\right]_1^2 = 16 - 12 - (1-3) =$

$4 - (-2) = \boxed{6}$

⑨ Ave value for integ. is $\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$

(A) $\frac{1}{2-0}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\left(\frac{1}{2}\right)\int_1^9 u^{1/2}\,du$

use u-sub.

$u = x^3+1$ when $x = 0$, $u = 1$

$du = 3x^2dx$ when $x = 2$, $u = 9$

$\frac{1}{3}du = x^2dx$

$\frac{1}{6}\left[\frac{2}{3}u^{3/2}\right]_1^9$

$\frac{1}{9}\left[9^{3/2} - 1^{3/2}\right] = \frac{1}{9}\left[27 - 1\right] = \frac{26}{9}$

⑩ antiderivative ~~means integration~~

(D) Fundamental theorem says

$\int_a^b f(x)\,dx = F(b) - F(a)$ so

$\int_1^4 \frac{x^2}{1+x^5}\,dx = F(4) - F(1)$ use calc.

put in calc $\quad F(4) = ?$

$F(1) = 0$

$.376 = F(4) - 0$

$\boxed{F(4) = .376}$

⑪ same things as #10

$F(1) = 0$, $F(9) = ?$ $\quad \int_1^9 \frac{(\ln x)^3}{x}\,dx = F(9) - F(1)$

(C)

$5.827 = F(9) - 0$

$5.827 = F(9)$

⑫ $\int_3^5 f(x)\,dx + \int_3^5 g(x)\,dx =$

(B) since $f(x) = g(x) + 7$

$\int_3^5 (g(x) + 7)\,dx + \int_3^5 g(x)\,dx$

$\int_3^5 g(x)\,dx + 7\int_3^5 dx + \int_3^5 g(x)\,dx$

$2\int_3^5 g(x)\,dx + 7\left[x\right]_3^5$

$7\left[5-3\right] = 7(2) = 14$

(13) $\int_a^b (f(x)+5)\,dx = \int_a^b f(x)\,dx + \int_a^b 5\,dx$ Since $\int_a^b f(x)\,dx = a+2b$ then

(C) $a+2b + \left[5x\right]_a^b = a+2b+5b-5a = \boxed{7b-4a}$

(14) $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{(\cos x)^2}\,dx = \int_0^1 e^u\,du = \left[e^u\right]_0^1 = e^1 - e^0 = \boxed{e-1}$

(C)

use u-sub

$u = \tan x$ let $x=0$, then $u=\tan 0 = 0$

$du = \sec^2 x\,dx$ let $x=\frac{\pi}{4}$ then $u=\tan\frac{\pi}{4}=1$

$\frac{1}{\cos^2 x}du = dx$

(15) $\int_a^b f''(x)\,dx = ?$ if $f$ is linear then $f'$ is constant and $f''$ is 0.

(A) $\int_a^b 0\,dx = 0$

(16) rewrite $\int_1^2 x^{-2}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^2 = -\left[\frac{1}{x}\right]_1^2 = -\left[\frac{1}{2}-1\right] = \frac{1}{2}$

(C)

(17) $\int_0^x \sin t\,dt = -\left[\cos x\right]_0^x = -\left[\cos x - \cos 0\right] = -\cos x + 1$

(E)

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

(18) $\int_{-3}^{3} (f(x)+1)\,dx = \int_{-3}^{3} f(x)\,dx + \int_{-3}^{3} 1\,dx$

(C)

$A = -\# \Rightarrow -2$ $\quad -2+2-2 \quad + \left[x\right]_{-3}^{3} = 3-(-3) = 6$

$B = +\# \Rightarrow +2$ $\quad -2+6 = 4$

$C = -\# \Rightarrow -2$