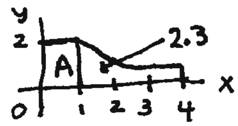
① By FTC $\int_1^3 f(x)\,dx = F(3) - F(1) = 2.3$. We need to find $F(3)-F(0)$

Ⓓ

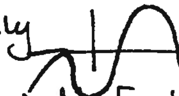


use geometry to find area A. $(1)(2)=2$

so $2+2.3=4.3$

② Since you are looking at graphs of derivatives and you want to find a relative max, you find the zero (on x axis) and then look for the graph to go from (+) to (−). Happens on f' only

Ⓐ



right here

③ By FTC $\int_{-1}^4 f(x)\,dx = F(4) - F(-1)$. Don't really need this for this problem, but good to know. On this problem, use geometry to find area of the 2 trapezoids.

Ⓑ

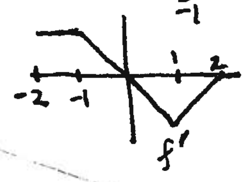
trap $A = \frac{1}{2}(2)(3+1) = 4$

trap $B = \frac{1}{2}(1)(2+1) = \frac{3}{2}$

$\int_{-1}^4 f(x)\,dx = 4 - \frac{3}{2} = \frac{5}{2}$

④

Ⓑ



a) f dec. $[-1,1]$ False, because f' is (+) from $[-2,0]$

b) f inc $[-2,0]$ TRUE because of what I said in a.

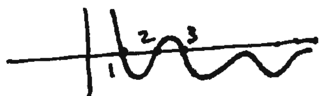
c) f inc $[1,2]$ False, because f' is (−) $[1,2]$, f will be decreasing

d) f has local min @ $x=0$. False, f' goes from + to −, so max.

e) f not diff @ $x=-1, x=1$. False, f' exists @ $x=-1, x=1$ so ∠BD.

⑤ put $\frac{(\cos x)^2}{x} - \frac{1}{5}$ in calc to graph - set window for x min = 0, x max = 10. Make y min −1 and y max 1 with y scl .1

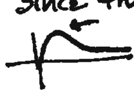
Ⓑ



3 times cross x axis.

⑥ put $\frac{\sqrt{x}}{1+x+x^3}$ in calc. Since this is a f' graph to find poi, look for a max (in this case) or min on graph.

Ⓑ



x part is .4725

⑦ The graph drawn is f', so looking at f' gives slope. so when $x=3$, $f'(3)$ is 2 (this is the slope)

Ⓒ so tangent line is $y-5=2(x-3)$

⑧ Since this is an f' graph, to find poi's look at max & mins on f'! There are 6.

Ⓔ

⑨ Absolute min will occur at either an endpoint, or a critical # (zero of the function) at $x=0$, to $x=6$, f would continue to increase (quite a bit). From $x=6$ to $x=8$, x decreases, but not that much, so the ab. min happens @ $x=0$.

Ⓐ

⑩ Look at this f'' graph and find where graph changes from + to − or − to +.

Ⓐ This happens @ $x=a$, $x=0$.

⑪ This is a graph of f. $F(1)=0$. F is inc, so f' is pos. F is concave down so f'' is neg.

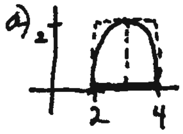
$f'' < F < f'$

Ⓓ

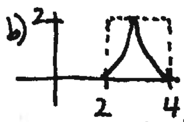
(2)

(12) Rewriting $\frac{1}{4-2}\int_2^4 f(t)\,dt = 1$ would be $\int_2^4 f(t)\,dt = 2$. Find graph where area under the curve is 2.

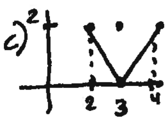
(C)

a) 

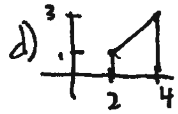
A = 2(2) = 4 greater than 2

b) 

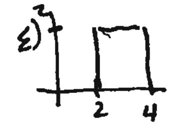
hard to tell. Looks less than 2

c) 

$\frac{1}{2}(1)(2)+\frac{1}{2}(1)(2)$
1+1 = (2)

d) 

$\frac{1}{2}(2)(1+3) =$
4

e) 

2×2 = 4

(13) Graph shown is f'. Told $f(0)=0$

(B)

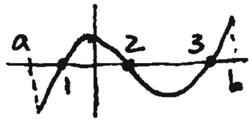
I. $f(0) > f(1)$ <u>False</u> $f(0)$ would be less than $f(1)$ and also $f(2)$ because f' is positive from $[0,2]$ so f is continuing to increase.

II. $f(2) > f(1)$ <u>True</u> For reason given in I. f' is + so f increases until $f(2)$.

III. $f(1) > f(3)$ False. F(2) is pretty big. F(1) not so big. at F(3), just took alittle off but not enough to make $f(1) > f(3)$.

(14) Graph shown is f'. When graph goes from − to +, a rel. min will occur on f, because f will decrease then increase. When f' goes from + to −, f will have a rel. max.

(A)

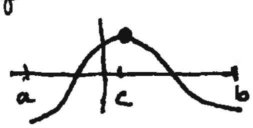


rel minimums at 1 and 3 because f' goes from − to +. <u>2</u> min

rel. max at 2 because f' goes from + to −. <u>1</u> max

(15) Graph shown is f. f is increasing from a to the point drawn (c) so f' would be above the x axis (+) from a to c

(A)



f is decreasing from c to b, so f' would be below x axis.

③

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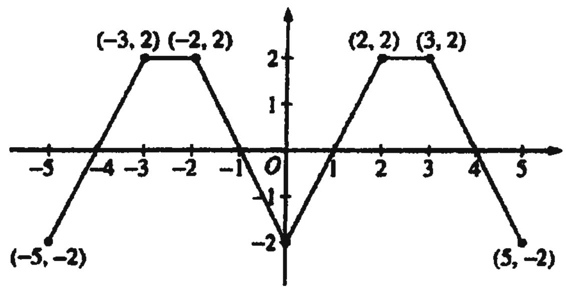
Question 3

The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



Graph of f

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

(a) $g(4) = \int_0^4 f(t)\,dt = 3$

$g'(4) = f(4) = 0$

$g''(4) = f'(4) = -2$

$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$

(b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x.

$g(10) = 2g(5) = 4$

$$g(108) = \int_0^{105} f(t)\,dt + \int_{105}^{108} f(t)\,dt$$
$$= 21g(5) + g(3) = 44$$

$g'(108) = f(108) = f(3) = 2$

An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

(4)

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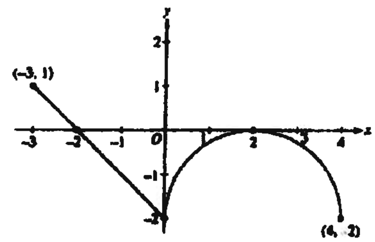
Question 4

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.



Graph of f'

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

(a) The function f is increasing on $[-3, -2]$ since $f' > 0$ for $-3 \le x < -2$.

$2: \begin{cases} 1: \text{interval} \\ 1: \text{reason} \end{cases}$

(b) $x = 0$ and $x = 2$

f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

$2: \begin{cases} 1: x = 0 \text{ and } x = 2 \text{ only} \\ 1: \text{justification} \end{cases}$

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

1 : equation

(d)

$$f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$$

$$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$$

$$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$$

$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$

$4: \begin{cases} 1: \pm\left(\frac{1}{2} - 2\right) \\ \quad \text{(difference of areas of triangles)} \\ 1: \text{answer for } f(-3) \text{ using FTC} \\ 1: \pm\left(8 - \frac{1}{2}(2)^2\pi\right) \\ \quad \text{(area of rectangle} \\ \quad - \text{area of semicircle)} \\ 1: \text{answer for } f(4) \text{ using FTC} \end{cases}$

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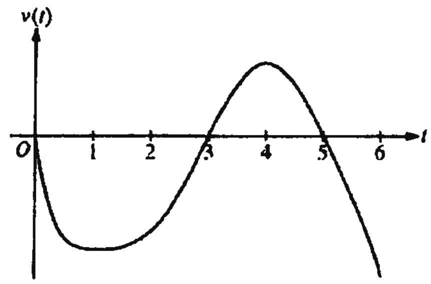
Question 4



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

(a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$ $x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$.	$3: \begin{cases} 1: \text{identifies } t = 3 \text{ as a candidate} \\ 1: \text{considers } \int_0^6 v(t)\,dt \\ 1: \text{conclusion} \end{cases}$
(b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of t for which the particle is at $x(t) = -8$.	$3: \begin{cases} 1: \text{positions at } t = 3,\ t = 5, \\ \quad \text{and } t = 6 \\ 1: \text{description of motion} \\ 1: \text{conclusion} \end{cases}$
(c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and v is increasing.	1 : answer with reason
(d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$