Integral-Defined Functions — Answers

① 2ND Fund Th of Calc (maybe 1st)

Ⓔ $\sin(x^2)^3(2x) = 2x\sin(x^6)$

② Fund Thm again (FTC)

Ⓔ $\ln((x^3)^2+1)(3x^2) = 3x^2\ln(x^6+1)$

③ FTC again because we need to find $F'(2)$

Ⓓ so $\frac{d}{dx}\int_0^x \sqrt{t^3+1}\,dt = \sqrt{x^3+1}$ then plug in 2 for x. $\sqrt{8+1} = \sqrt{9} = \boxed{3}$

④ FTC again because need to find $g'(3)$

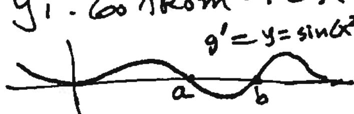
Ⓒ $\int_0^{2x} f(t)\,dt \Rightarrow f(2x)(2) = 2f(2x) = g'(x)$ so $g'(3) = 2f(6)$
and $f(6) = -1$ so $2(-1) = \boxed{-2}$

⑤ since $g(x) = \int_a^x f(t)\,dt$, f is the derivative of g. so on the

Ⓒ graph of f, to predict max on g, must look for sign change on f. Place where f changes from + to −.
This happens @ c.

⑥ Use calc. put $\sin(x^2)$ in for y_1. Go from $-1 \le x \le 3$

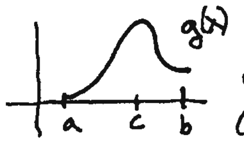
Ⓓ Graph looks like this (kind of)



to find where g is decreasing look at a & b.

a = 1.772
b = 2.507

⑦



Ⓒ g is original function.
Question wants you to choose a g'
I put in a c where the max on g is. From a to c, g is inc, so g' will be (+)
from c to b, g is dec, so g' will be (−). Graph Ⓒ shows this best.

⑧ The graph drawn is an f' graph even though it's not labeled directly that

Ⓔ way. $\int_a^x h(t)\,dt = f(x)$ says $h(t)\,dt$ is $\underline{f'(x)}$
analyzing f' from a to b, f' is (+) so f will be increasing.
At b, f will have a max. from b to c f' is (−) so f will be decreasing. E represents this the best.

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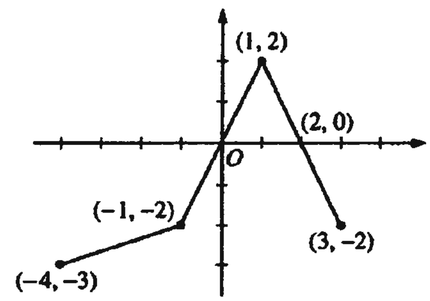
Question 4

The graph of the function f above consists of three line segments.



Graph of f

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

(a) $g(-1) = \int_{-4}^{-1} f(t)\,dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$ $g'(-1) = f(-1) = -2$ $g''(-1)$ does not exist because f is not differentiable at $x = -1$.	$3 : \begin{cases} 1 : g(-1) \\ 1 : g'(-1) \\ 1 : g''(-1) \end{cases}$
(b) $x = 1$ $g' = f$ changes from increasing to decreasing at $x = 1$.	$2 : \begin{cases} 1 : x = 1 \text{ (only)} \\ 1 : \text{reason} \end{cases}$
(c) $x = -1,\ 1,\ 3$	2 : correct values $\langle -1 \rangle$ each missing or extra value
(d) h is decreasing on $[0, 2]$ $h' = -f < 0$ when $f > 0$	$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

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Question 5

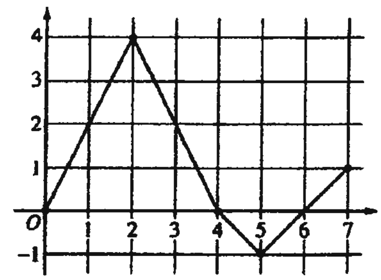
Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.



Graph of f

(a) $g(3) = \int_2^3 f(t)\,dt = \frac{1}{2}(4 + 2) = 3$

$g'(3) = f(3) = 2$

$g''(3) = f'(3) = \frac{0 - 4}{4 - 2} = -2$

$$3 : \begin{cases} 1 : g(3) \\ 1 : g'(3) \\ 1 : g''(3) \end{cases}$$

(b) $\frac{g(3) - g(0)}{3} = \frac{1}{3}\int_0^3 f(t)\,dt$

$= \frac{1}{3}\left(\frac{1}{2}(2)(4) + \frac{1}{2}(4 + 2)\right) = \frac{7}{3}$

$$2 : \begin{cases} 1 : g(3) - g(0) = \int_0^3 f(t)\,dt \\ 1 : \text{answer} \end{cases}$$

(c) There are two values of c.

We need $\frac{7}{3} = g'(c) = f(c)$

The graph of f intersects the line $y = \frac{7}{3}$ at two places between 0 and 3.

$$2 : \begin{cases} 1 : \text{answer of 2} \\ 1 : \text{reason} \end{cases}$$

Note: 1/2 if answer is 1 by MVT

(d) $x = 2$ and $x = 5$

because $g' = f$ changes from increasing to decreasing at $x = 2$, and from decreasing to increasing at $x = 5$.

$$2 : \begin{cases} 1 : x = 2 \text{ and } x = 5 \text{ only} \\ 1 : \text{justification} \\ \quad (\text{ignore discussion at } x = 4) \end{cases}$$

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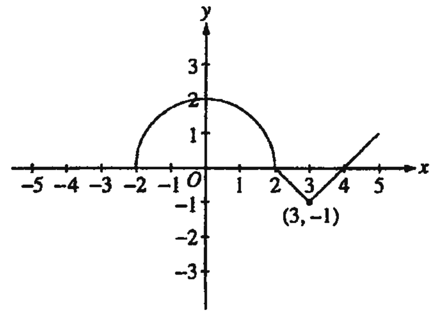
5. The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



(a) Find $g(3)$.

(b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

(a) $g(3) = \int_0^3 f(t)\,dt$

$$= \frac{1}{4}\cdot\pi\cdot 2^2 - \frac{1}{2} = \pi - \frac{1}{2}$$

2 { 2: answer
<−1> each incorrect area
<−1> error in summing }

(b) $g(x)$ has relative maximum at $x = 2$

because $g'(x) = f(x)$ changes from positive to negative at $x = 2$

3 { 1: relative maximum at $x = 2$ only
1: $g'(x) = f(x)$ or interprets $g(x)$ as area accumulator
1: justification (ignore discussion at $x = 5$) }

(c) $g(3) = \pi - \frac{1}{2}$

$g'(3) = f(3) = -1$

$$y - \left(\pi - \frac{1}{2}\right) = -1(x - 3)$$

2 { 1: $g'(3) = -1$
1: equation using $g(3)$ and $g'(3)$ }

(d) graph of g has points of inflection with x-coordinates $x = 0$ and $x = 3$

because $g''(x) = f'(x)$ changes from positive to negative at $x = 0$ and from negative to positive at $x = 3$

or

because $g'(x) = f(x)$ changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = 3$

2 { 1: points of inflection with x-coordinates 0 and 3 only
1: justification (ignore discussion at $x = 2$) }

1/2 if $x = 0, 3$ selected as candidates and $x = 3$ discarded because $g''(3)$ does not exist

1/2 if $x = 0, 2, 3$ selected as candidates and $x = 2$ and $x = 3$ discarded because $g''(2)$ and $g''(3)$ do not exist