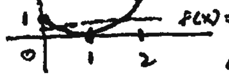
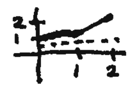
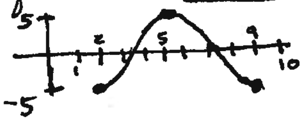
THE "BIG" THEOREMS

Answers ①

① Draw a sketch from table

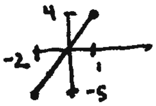
(A)  $f(c)=\frac{1}{2}$ if $k=0$ at least 2 sol.  if $k=\frac{1}{2}$ one sol.  if $K=1$ no sol. if $K=2$ or 3 no sol.

② if function is differentiable, then function is continuous. Draw a sketch. Since diff, no spikes.

(E) 

I. f has at least 2 zeros. True
II. f has at least one hor. tan. True
III. This is saying btw $x=2$ and $x=5$, f must have a value of 3. True

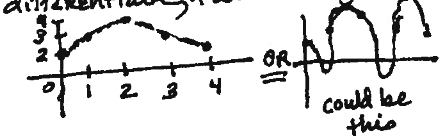
③ If f is cont AND diff, there are no breaks or spikes. Draw a sketch.

(B)  the graph of f could be a straight line — this means 'b' wouldn't happen. B could happen, but doesn't have to to meet all conditions.

'A' is true because of the Intermediate Value Th. IVT
'C' is true because of the IVT.
'D' is true because of the mean value th. MVT $\frac{f(1)-f(-2)}{1-(-2)} = \frac{4-(-5)}{3} = \frac{9}{3} = 3$
'E' is true because of the IVT.

④ f is cont and diff (again).
(B)
'A' is true because thats the MVT exactly.
'B' doesn't have to be true. Graph could look like 
'C' has to be true because of extreme value th. EVT
'D' " " " " " " " " EVT
'E' has to be true because if f is differentiable, f will be integrable

⑤ f is cont and diff so no breaks or sharp points. Plot the points.



(E)
'A' doesn't say anything about min or max. Graph could go <2.
'B' " " " " " " " " " go >4
'C' doesn't have to
'D' doesn't have to → actually Rolle's
'E' This is the MVT and because f is cont on [0,4] and diff, thats what's needed for MVT.

⑥ (calc) This is a tough one... but a GOOD one. This problem contains MVT idea and 2ND FTC. First,
(C) integrate $\int_0^{(x)^2}\sin t\,dt$. You should get $1-\cos(x^2)$. This is the original function. The ave. rate of change is $\frac{1-\cos(\sqrt{\pi})^2-(1-\cos(0)^2}{\sqrt{\pi}-0} = \frac{1-\cos\pi-1+\cos 0}{\sqrt{\pi}} = \frac{2}{\sqrt{\pi}} \approx 1.120$ This is the ave. rate. The der. of $\int_0^{x^2}\sin t\,dt$ is the 2ND (or 1st, depending on the calc book) FTC. You get $2x\sin(x^2)$. This is the instantaneous rate.
By MVT. Set $\frac{2}{\sqrt{\pi}} = 2x\sin(x^2)$ and solve for x. Problem # 7 is the same type.

⑦ (calc) ave rate of change of f is $\frac{4+\ln 4-(1+\ln 1)}{3} = \frac{3+\ln(4)}{3}$. Now find instant rate which is der. of f
(C) $f' = 1+\frac{1}{x}$. Set these = to each other because of MVT. $1+\frac{1}{x} = \frac{3+\ln 4}{3}$. Use calc. get 2.164

⑧ (calc) antiderivative means integrate the thing! $\int_1^4 \frac{x^2}{1+x^5}dx$. By FTC, $\int_1^4 \frac{x^2}{1+x^5}dx = F(4)-F(1)$.
(D) So, $F(4) = f(1) + \int_1^4 \frac{x^2}{1+x^5}dx$. ← use calc for this
$f(1)=0$
$f(4) = 0 + .376 =$ **.376**

⑨ This is the FTC (either one or two, whatever). Replace x where there's a t and then take
(D) the der. of x. You should get $\sqrt{x^3+1}\,(1)$. Now put in 2 for x $\sqrt{2^3+1} = \sqrt{9} = 3$

⑩ FTC one more time. Replace x^2 for t, then (don't forget this part) take derivative
(E) of x^2. You get $\sin(x^2)^3(2x)$. This simplifies to $2x\sin(x^6)$

(2)

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Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_0^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

(a) $\int_{30}^{60} |v(t)|\, dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)|\, dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

$2: \begin{cases} 1: \text{explanation} \\ 1: \text{value} \end{cases}$

(b) $\int_0^{30} a(t)\, dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\int_0^{30} a(t)\, dt = \int_0^{30} v'(t)\, dt = v(30) - v(0)$$
$$= -14 - (-20) = 6 \text{ ft/sec}$$

$2: \begin{cases} 1: \text{explanation} \\ 1: \text{value} \end{cases}$

(c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

$2: \begin{cases} 1: v(35) < -5 < v(50) \\ 1: \text{Yes; refers to IVT or hypotheses} \end{cases}$

(d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

$2: \begin{cases} 1: v(0) = v(25) \\ 1: \text{Yes; refers to MVT or hypotheses} \end{cases}$

Units of ft in (a) and ft/sec in (b)

1 : units in (a) and (b)

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3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.
   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

(a) $\int_0^{24} R(t)\,dt \approx 6[R(3) + R(9) + R(15) + R(21)]$

$= 6[10.4 + 11.2 + 11.3 + 10.2]$

$= 258.6$ gallons

This is an approximation to the total flow in gallons of water from the pipe in the 24-hour period.

3 { 1: $R(3) + R(9) + R(15) + R(21)$; 1: answer; 1: explanation }

(b) Yes;

Since $R(0) = R(24) = 9.6$, the Mean Value Theorem guarantees that there is a t, $0 < t < 24$, such that $R'(t) = 0$.

2 { 1: answer; 1: MVT or equivalent }

(c) Average rate of flow

$\approx$ average value of $Q(t)$

$= \frac{1}{24}\int_0^{24} \frac{1}{79}(768 + 23t - t^2)\,dt$

$= 10.785$ gal/hr or 10.784 gal/hr

3 { 1: limits and average value constant; 1: $Q(t)$ as integrand; 1: answer }

(units) Gallons in part (a) and gallons/hr in part (c), or equivalent.

1: units

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Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

(a) $\frac{T(8)-T(6)}{8-6} = \frac{55-62}{2} = -\frac{7}{2}$ °C/cm

1 : answer

(b) $\frac{1}{8}\int_0^8 T(x)\,dx$

Trapezoidal approximation for $\int_0^8 T(x)\,dx$:

$$A = \frac{100+93}{2}\cdot 1 + \frac{93+70}{2}\cdot 4 + \frac{70+62}{2}\cdot 1 + \frac{62+55}{2}\cdot 2$$

Average temperature $\approx \frac{1}{8}A = 75.6875$°C

3 : { 1 : $\frac{1}{8}\int_0^8 T(x)\,dx$; 1 : trapezoidal sum; 1 : answer }

(c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45$°C

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

2 : { 1 : value; 1 : meaning }

(d) Average rate of change of temperature on [1, 5] is $\frac{70-93}{5-1} = -5.75$.

Average rate of change of temperature on [5, 6] is $\frac{62-70}{6-5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval (1, 5) and $T'(c_2) = -8$ for some c_2 in the interval (5, 6). It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in [0, 8].

2 : { 1 : two slopes of secant lines; 1 : answer with explanation }

Units of °C/cm in (a), and °C in (b) and (c)

1 : units in (a), (b), and (c)

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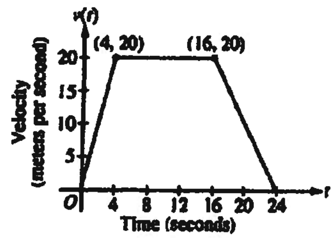
Question 5

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.



(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$

The car travels 360 meters in these 24 seconds.

$2: \begin{cases} 1: \text{value} \\ 1: \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$$\lim_{t\to 4^-}\left(\frac{v(t)-v(4)}{t-4}\right) = 5 \ne 0 = \lim_{t\to 4^+}\left(\frac{v(t)-v(4)}{t-4}\right).$$

$v'(20) = \frac{20-0}{16-24} = -\frac{5}{2}$ m/sec^2

$3: \begin{cases} 1: v'(4) \text{ does not exist, with explanation} \\ 1: v'(20) \\ 1: \text{units} \end{cases}$

(c) $$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2: \begin{cases} 1: \text{finds the values } 5, 0, -\frac{5}{2} \\ 1: \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is

$\frac{v(20)-v(8)}{20-8} = -\frac{5}{6}$ m/sec^2.

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2: \begin{cases} 1: \text{average rate of change of } v \text{ on } [8, 20] \\ 1: \text{answer with explanation} \end{cases}$