Average and Instantaneous Rates of Change Answers

① To find instantaneous Rate of change, take der. of $f(x)$

(D)

$$f'(x)=\frac{(x-1)(2x)-(x^2-2)(1)}{(x-1)^2}=\frac{2x^2-2x-x^2+2}{(x-1)^2}=\frac{2x^2-x^2-2x+2}{(x-1)^2}=\frac{x^2-2x+2}{(x-1)^2}$$

could put 2 in here ↑ OR simplify, then put in 2

$$f'(2)=\frac{(+1)(4)-(2)(1)}{(2-1)^2}=\boxed{2} \qquad =\frac{4-4+2}{1^2}=\boxed{2}$$

pretty easy

use calc

② ∥ lines mean find derivatives. $f(x)=3e^{2x}$ $f'(x)=3e^{2x}(2)=6e^{2x}$

(C)

f is transcendental function $g(x)=6x^3$ $g'(x)=18x^2$

g is polynomial function

set $f'=g'$ because parallel means slopes are =.

Use calculator. $6e^{2x}=18x^2$

$e^{2x}=3x^2$ takes awhile

$\boxed{x=-.391}$ (always 3 places)

use calc

③ $f(x)=x+\ln x$ $[1,4]$ Need to find what is 'c' when instant rate of change = ave. rate of change over $[1,4]$

(C)

$f'(x)=1+\frac{1}{x}$ [this is instant ROC]

$\frac{f(b)-f(a)}{b-a}$ is ave. ROC so $\frac{f(4)-f(1)}{4-1}=\frac{4+\ln 4-1-\ln 1}{3}=\frac{3+\ln 4}{3}$

$1+\frac{1}{x}=\frac{3+\ln 4}{3}$ ← set = to $f'(x)$

solve for x (with calc)

$\boxed{x=2.164}$ (pretty quick)

use calc

④ Word slope implies derivative

(A)

$f'(x)=2e^{4x^2}(8x)=16xe^{4x^2}$ set this equal to 3

$16xe^{4x^2}=3$ use calc $\boxed{x=.168}$ (took awhile)

⑤ Rate of change (ROC) means derivative

(A)

$f'(x)=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$ Sentence says so $f'(c)=2f'(1)$

$$\frac{1}{2\sqrt{c}}=2\left(\frac{1}{2\sqrt{1}}\right)$$

$$\frac{1}{2\sqrt{c}}=1$$

$\frac{1}{2}=\sqrt{c}$ square both sides

$\boxed{\frac{1}{4}=c}$

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Question 3

The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at 0.286 °F/mph.

$2: \begin{cases} 1: \text{value} \\ 1: \text{explanation} \end{cases}$

(b) The average rate of change of W over the interval $5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

$3: \begin{cases} 1: \text{average rate of change} \\ 1: W'(v) = \text{average rate of change} \\ 1: \text{value of } v \end{cases}$

(c) $\left.\dfrac{dW}{dt}\right|_{t=3} = \left.\left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\right|_{t=3} = W'(35) \cdot 5 = -0.892 \text{ °F/hr}$

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left.\dfrac{dW}{dt}\right|_{t=3} = -0.892 \text{ °F/hr}$

$3: \begin{cases} 1: \dfrac{dv}{dt} = 5 \\ 1: \text{uses } v(3) = 35, \\ \quad\quad \text{or} \\ \quad \text{uses } v(t) = 20 + 5t \\ 1: \text{answer} \end{cases}$

Units of °F/mph in (a) and °F/hr in (c)

1 : units in (a) and (c)

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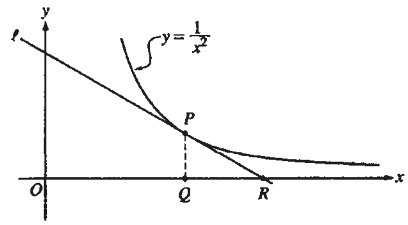
6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at the point R, with coordinates $(k, 0)$.



(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

(a) $\frac{dy}{dx} = -\frac{2}{x^3}$; $\left.\frac{dy}{dx}\right|_{x=3} = -\frac{2}{27}$

Line ℓ through $\left(3, \frac{1}{9}\right)$ and $(k, 0)$ has slope $-\frac{2}{27}$.

Therefore, $\frac{0 - \frac{1}{9}}{k - 3} = -\frac{2}{27}$ or $0 - \frac{1}{9} = -\frac{2}{27}(k - 3)$

$k = \frac{9}{2}$

2 $\begin{cases} 1\text{: } \left.\frac{dy}{dx}\right|_{x=3} \\ 1\text{: answer} \end{cases}$

(b) Line ℓ through $\left(w, \frac{1}{w^2}\right)$ and $(k, 0)$ has slope $-\frac{2}{w^3}$.

Therefore, $\frac{0 - \frac{1}{w^2}}{k - w} = -\frac{2}{w^3}$ or $0 - \frac{1}{w^2} = -\frac{2}{w^3}(k - w)$

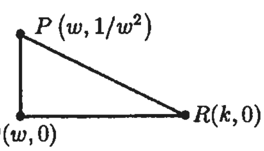
$k = \frac{3}{2}w$

2 $\begin{cases} 1\text{: equation relating } w \text{ and } k\text{, using slopes} \\ 1\text{: answer} \end{cases}$

(c) $\frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt} = \frac{3}{2} \cdot 7 = \frac{21}{2}$; $\left.\frac{dk}{dt}\right|_{w=5} = \frac{21}{2}$

1: answer using $\frac{dw}{dt} = 7$

(d)



$A = \frac{1}{2}(k - w)\frac{1}{w^2} = \frac{1}{2}\left(\frac{3}{2}w - w\right)\frac{1}{w^2} = \frac{1}{4w}$

$\frac{dA}{dt} = -\frac{1}{4w^2}\frac{dw}{dt}$

$\left.\frac{dA}{dt}\right|_{w=5} = -\frac{1}{100} \cdot 7 = -0.07$

Therefore, area is decreasing.

4 $\begin{cases} 1\text{: area in terms of } w \text{ and/or } k \\ 1\text{: } \frac{dA}{dt} \text{ implicitly} \\ 1\text{: } \left.\frac{dA}{dt}\right|_{w=5} \text{ using } \frac{dw}{dt} = 7 \\ 1\text{: conclusion} \end{cases}$

Note: 0/4 if A constant

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Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$
Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

2 : { 1 : considers $R'(6)$; 1 : answer with reason }

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$

To the nearest whole number, there are 964 mosquitoes.

2 : { 1 : integral; 1 : answer }

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$
$R(t) > 0$ on $0 < t < 2.5\pi$
$R(t) < 0$ on $2.5\pi < t < 7.5\pi$
$R(t) > 0$ on $7.5\pi < t < 31$
The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357,$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

4 : {
- 2 : absolute maximum value
  - 1 : integral
  - 1 : answer
- 2 : analysis
  - 1 : computes interior critical points
  - 1 : completes analysis

}

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Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\,dt = 2474$ cars	3 : { 1 : limits 1 : integrand 1 : answer
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	3 : { 1 : limits 1 : integrand 1 : answer
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)