31 ... 1
33-36 3
29-31 2

42 POINTS

AP Calculus AB Sections 4.1 – 4.3 Exam

Name: ______________________ Date: __________ Period: ________

1. Solve the differential equation. 5

$$\frac{df}{ds} = -12s^3 - 4, \qquad f(-2) = 2$$

$d(s) = \int(-12s^3 - 4)\,ds = \frac{-12s^4}{4} - 4s + C$

$d(s) = -3s^4 - 4s + C$

$2 = -3(-2)^4 - 4(-2) + C$

$2 = -48 + 8 + C$

$42 = -16 + C$

$42 = C$

$\boxed{d(s) = -3s^4 - 4s + 42}$

2. Find the limit of $s(n)$ as $n \to \infty$. 3

$$s(n) = \frac{1}{n^4}\left[\frac{n(n+1)(3n+1)}{4}\right] = \frac{1}{n^3}\left[\frac{3n^2+4n+1}{4}\right] = \frac{3n^2}{4n^3} + \frac{4n}{4n^3} + \frac{1}{4n^3}$$

$\lim_{n\to\infty} s(n) = \left(\frac{3}{4n} + \frac{4}{n^2} + \frac{1}{4n^3}\right) = \boxed{0}$

3. Find the general solution of the differential equation below and check the result by differentiation. 2

$$\frac{dQ}{dz} = \frac{5}{2}z^{\frac{3}{2}}$$

$d(z) = \int \frac{5}{2} z^{3/2}\,dz = \frac{5}{2}\cdot\frac{2}{5}\cdot z^{5/2} + C = \boxed{z^{5/2} + C}$

10

4. Find the indefinite integral and check the result by differentiation.

$$\int \frac{2x^2+14x-9}{x^4}\,dx$$

4

$$\int\left[\frac{2}{x^2}+\frac{14}{x^3}-\frac{9}{x^4}\right]dx = \int(2x^{-2}+14x^{-3}-9x^{-4})\,dx$$

$$= \frac{2\cdot x^{-1}}{-1}+\frac{14x^{-2}}{-2}-\frac{9x^{-3}}{-3}+C$$

$$= \boxed{\frac{-2}{x}-\frac{7}{x^2}+\frac{3}{x^3}+C}$$

5. Write the following limit as a definite integral on the interval [2 , 5], where c_i is any point in the ith subinterval.

2

$$\lim_{\|\Delta x\|\to 0}\sum_{i=1}^{n}\sqrt{2c_i^2+2c_i}\;\Delta x_i$$

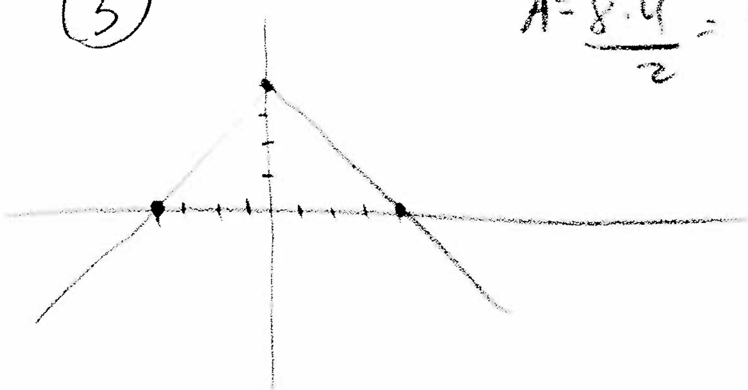
$$\int_2^5 \sqrt{2x^2+2x}\,dx$$

6. Sketch the region whose area is given by the definite integral and then use a geometric formula to evaluate the integral.

$$\int_{-4}^{4}(4-|x|)\,dx$$

3

$$A=\frac{8\cdot 4}{2}=4\cdot 4=16$$



9

$\Delta x = \frac{1+4}{n} = \frac{5}{n}$

7. Evaluate the following definite integral by the limit definition.

10

$$\int_{-4}^{1}(3u^2+4)\,du \qquad A=\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$$

$[-4, 1]$ $c_i = -4+\frac{5}{n}i$

$f(-4+\frac{5}{n}i)\Delta x$

$\left[3(-4+\frac{5}{n}i)^2+4\right]\left[\frac{5}{n}\right]$

$\left[3(16-\frac{40}{n}i+\frac{25}{n^2}i^2)+4\right](\frac{5}{n})$

$$\frac{15}{n}\sum_{i=1}^{n}\left(16-\frac{40i}{n}+\frac{25}{n^2}i^2\right)$$

$$\frac{15}{n}\left[16n-\frac{\overset{20}{\cancel{40}}}{\cancel{n}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}}+\frac{25}{n^2}\cdot\frac{\cancel{n}\overbrace{(n+1)(2n+1)}^{2n^2+3n+1}}{6}\right]$$

$$\frac{15}{n}\left[16n-20n-20+\frac{50n^2}{n}+\frac{75n}{n}+\frac{25}{n}\right]$$

$$240-300-\frac{300}{n}+\frac{750n^2}{n^2}+\frac{1125n}{n^2}+\frac{375}{n^2}$$

85

8. A ball is thrown vertically upwards from a height of 5 ft with an initial velocity of 70 ft per second.

10

How high will the ball go?

$a(t)$ = acceleration
$v(t)$ = velocity
$s(t)$ = position

$s(0)=5$
$v(0)=70$

$a(t)=-32$ ft/sec^2

$v(t)=\int -32\,dt = -32t+C$

$70=-32(0)+C \Rightarrow C=70 \Rightarrow v(t)=-32t+70$

$s(t)=\int(-32t+70)\,dt = \frac{-32t^2}{2}+70t+C$

$5=-16(0)^2+70(0)+C \Rightarrow C=5 \Rightarrow s(t)=-16t^2+70t+5$

1ST: WHEN $s'(t)=0$

$-32t+70=0$

$-32t=-70$

$t=\frac{70}{32}=\frac{35}{16}$

HOW HIGH?

$s(\frac{35}{16})=-16(\frac{35}{16})^2+70(\frac{35}{16})+5$
$=81.56$

20

9. Evaluate the integral 3

$$\int_2^3 \left(-24x^2 - 1\right) dx \;=\; -24\int_2^3 x^2 - \int_2^3 dx$$

given

$$-\cancel{24}^{\,8} \cdot \frac{19}{\cancel{3}} - 1$$

$$\int_2^3 x^3 \, dx = \frac{65}{4},$$

$$-152 - 1 = -153$$

$$\int_2^3 x^2 \, dx = \frac{19}{3},$$

$$\int_2^3 x \, dx = \frac{5}{2},$$

$$\int_2^3 dx = 1.$$

3

$$\left(48 - \frac{120i}{n} + \frac{75i^2}{n^2} + 4\right)\left(\frac{5}{n}\right)$$

$$\frac{240}{n} - \frac{600i}{n^2} + \frac{375i^2}{n^3} + \frac{20}{n}$$

$$240 - \frac{\overset{300}{\cancel{600}}}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}} + \frac{\overset{125}{\cancel{375}}}{n^{\cancel{3}2}}\cdot\frac{\overbrace{\cancel{n}(n+1)(2n+1)}^{(2n^2+3n+1)}}{\cancel{6}2} + 20$$

$$240 - 300 + \frac{300}{n} + \frac{250}{2} + 20 = 85$$

tt
1wt