Unit 0: Pre-Calculus Review Summer Assignment

Do all of your work on lined notebook paper.
Show all work.
Make an answer column.
Tittle all your work on the top line of the assignment
In the upper right hand corner write your name and period 3

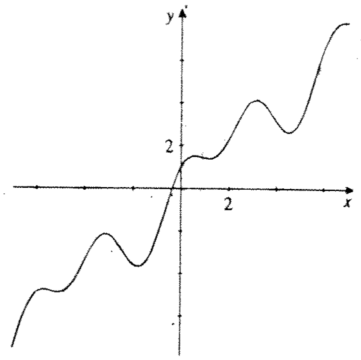
I. **Functions and their graphs**
   a. Label, Label, Label worksheet
   b. Jurupa Hills Calculus Book
      i. Page 27#1-36 all
      ii. Page 28# 39-63 odd
      iii. Page 29# 64-84 even
   c. Alternative calculus book (selected Photo copied pages)
      i. Page 59 #1-30 #31-52

II. **Trigonometry Review**
   a. Alternative Trig Book (selected Photo copied pages)
      i. Pg 223 #5-30 all (trig worksheet 1)
      ii. Page 232 #1-33 odd (trig worksheet 2)
      iii. Page 241 #1-12 all (trig worksheet 3)
      iv. Page 242 #13-49 odd #36 and #42 (trig worksheet 4)
      v. Page 251 #19-39 odd (trig worksheet 5)
      vi. Page 260 #1-24 all (trig worksheet 6)
      vii. Flashcards

III. **Logarithms**
   a. logarithms worksheet

GROUP WORK 1, SECTION 1.3
Label Label Label, I Made it Out of Clay

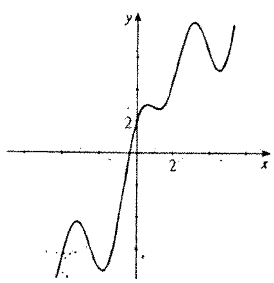
This is a graph of the function $f(x)$:



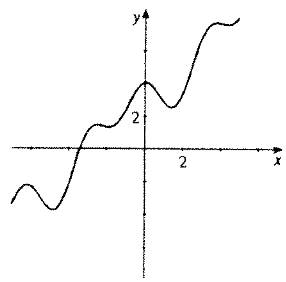
Give each graph below the correct label from the following:

(a) $f(x+3)$ (b) $f(x-3)$ (c) $f(2x)$ (d) $2f(x)$ (e) $|f(x)|$

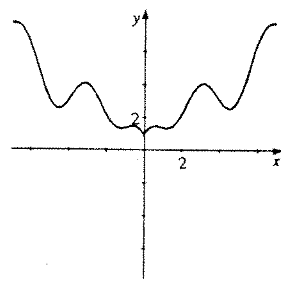
(f) $f(|x|)$ (g) $2f(x)-1$ (h) $f(2x)+2$ (i) $f(x)-x$ (j) $1/f(x)$



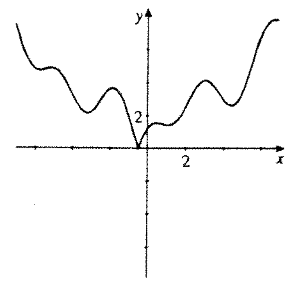
Graph 1



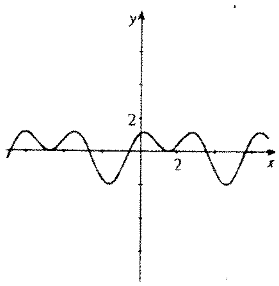
Graph 2



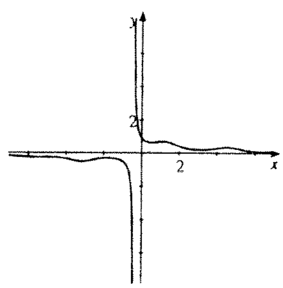
Graph 3



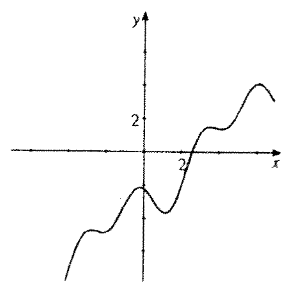
Graph 4



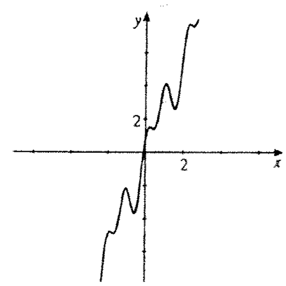
Graph 5



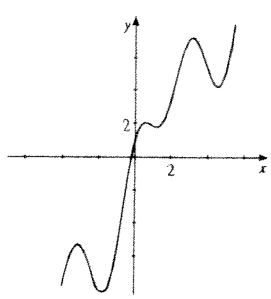
Graph 6



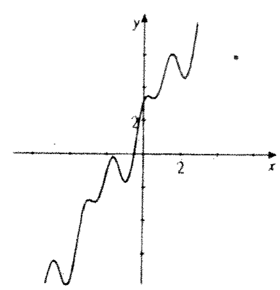
Graph 7



Graph 8



Graph 9



Graph 10

To graph this function, take the parabola $y = x^2$ (see Figure 0.80a) and translate the graph 2 units to the left and 1 unit down. (See Figure 0.80b.) ■

The following table summarizes our discoveries in this section.

Transformations of $f(x)$

Transformation	*Form*	*Effect on Graph*
Vertical translation	$f(x) + c$	$\lvert c \rvert$ units up ($c > 0$) or down ($c < 0$)
Horizontal translation	$f(x + c)$	$\lvert c \rvert$ units left ($c > 0$) or right ($c < 0$)
Vertical scale	$cf(x)$ ($c > 0$)	multiply vertical scale by c
Horizontal scale	$f(cx)$ ($c > 0$)	divide horizontal scale by c

You will explore additional transformations in the exercises.

Alternative Calculus book pg 59 #1-30 #31-52 is on the back

EXERCISES 0.6

WRITING EXERCISES

1. The restricted domain of example 6.2 may be puzzling. Consider the following analogy. Suppose you have an airplane flight from New York to Los Angeles with a stop for refueling in Minneapolis. If bad weather has closed the airport in Minneapolis, explain why your flight will be canceled (or at least rerouted) even if the weather is great in New York and Los Angeles.

2. Explain why the graphs of $y = 4(x^2 - 1)$ and $y = (4x)^2 - 1$ in Figures 0.77c and 0.78c appear "thinner" than the graph of $y = x^2 - 1$.

3. As illustrated in example 6.9, completing the square can be used to rewrite any quadratic function in the form $a(x - d)^2 + e$. Using the transformation rules in this section, explain why this means that all parabolas (with $a > 0$) will look essentially the same.

4. Explain why the graph of $y = f(x + 4)$ is obtained by moving the graph of $y = f(x)$ four units to the left, instead of to the right.

In exercises 1–6, find the compositions $f \circ g$ and $g \circ f$, and identify their respective domains.

1. $f(x) = x + 1, \quad g(x) = \sqrt{x - 3}$
2. $f(x) = x - 2, \quad g(x) = \sqrt{x + 1}$
3. $f(x) = e^x, \quad g(x) = \ln x$
4. $f(x) = \sqrt{1 - x}, \quad g(x) = \ln x$
5. $f(x) = x^2 + 1, \quad g(x) = \sin x$
6. $f(x) = \dfrac{1}{x^2 - 1}, \quad g(x) = x^2 - 2$

In exercises 7–16, identify functions $f(x)$ and $g(x)$ such that the given function equals $(f \circ g)(x)$.

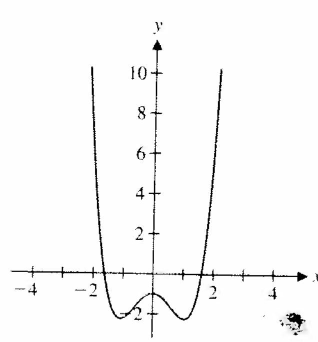
7. $\sqrt{x^4 + 1}$ 8. $\sqrt[3]{x + 3}$ 9. $\dfrac{1}{x^2 + 1}$
10. $\dfrac{1}{x^2} + 1$ 11. $(4x + 1)^2 + 3$ 12. $4(x + 1)^2 + 3$
13. $\sin^3 x$ 14. $\sin x^3$ 15. e^{x^2+1} 16. e^{4x-2}

In exercises 17–22, identify functions $f(x)$, $g(x)$ and $h(x)$ such that the given function equals $[f \circ (g \circ h)](x)$.

17. $\dfrac{3}{\sqrt{\sin x + 2}}$ 18. $\sqrt{e^{4x} + 1}$
19. $\cos^3(4x - 2)$ 20. $\ln \sqrt{x^2 + 1}$
21. $4e^{x^2} - 5$ 22. $\left[\tan^{-1}(3x + 1)\right]^2$

In exercises 23–30, use the graph of $y = f(x)$ given in the figure to graph the indicated function.

23. $f(x) - 3$ 24. $f(x + 2)$ 25. $f(x - 3)$
26. $f(x) + 2$ 27. $f(2x)$ 28. $3f(x)$
29. $-3f(x) + 2$ 30. $3f(x + 2)$



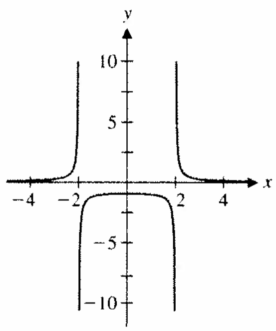
Graph for exercises 23–30

In exercises 31–38, use the graph of $y = f(x)$ given in the figure to graph the indicated function.

31. $f(x-4)$ **32.** $f(x+3)$ **33.** $f(2x)$

34. $f(2x-4)$ **35.** $f(3x+3)$ **36.** $3f(x)$

37. $2f(x)-4$ **38.** $3f(x)+3$



Graph for exercises 31–38

In exercises 39–44, complete the square and explain how to transform the graph of $y = x^2$ into the graph of the given function.

39. $f(x) = x^2 + 2x + 1$ **40.** $f(x) = x^2 - 4x + 4$

41. $f(x) = x^2 + 2x + 4$ **42.** $f(x) = x^2 - 4x + 2$

43. $f(x) = 2x^2 + 4x + 4$ **44.** $f(x) = 3x^2 - 6x + 2$

In exercises 45–48, graph the given function and compare to the graph of $y = x^2 - 1$.

45. $f(x) = -2(x^2 - 1)$

46. $f(x) = -3(x^2 - 1)$

47. $f(x) = -3(x^2 - 1) + 2$

48. $f(x) = -2(x^2 - 1) - 1$

In exercises 49–52, graph the given function and compare to the graph of $y = (x-1)^2 - 1 = x^2 - 2x$.

49. $f(x) = (-x)^2 - 2(-x)$

50. $f(x) = -(-x)^2 + 2(-x)$

51. $f(x) = (-x+1)^2 + 2(-x+1)$

52. $f(x) = (-3x)^2 - 2(-3x) - 3$

53. Based on exercises 45–48, state a rule for transforming the graph of $y = f(x)$ into the graph of $y = cf(x)$ for $c < 0$.

54. Based on exercises 49–52, state a rule for transforming the graph of $y = f(x)$ into the graph of $y = f(cx)$ for $c < 0$.

55. Sketch the graph of $y = |x|^3$. Explain why the graph of $y = |x|^3$ is identical to that of $y = x^3$ to the right of the y-axis. For $y = |x|^3$, describe how the graph to the left of the y-axis compares to the graph to the right of the y-axis. In general, describe how to draw the graph of $y = f(|x|)$ given the graph of $y = f(x)$.

56. For $y = x^3$, describe how the graph to the left of the y-axis compares to the graph to the right of the y-axis. Show that for $f(x) = x^3$, we have $f(-x) = -f(x)$. In general, if you have the graph of $y = f(x)$ to the right of the y-axis and $f(-x) = -f(x)$ for all x, describe how to graph $y = f(x)$ to the left of the y-axis.

57. Iterations of functions are important in a variety of applications. To iterate $f(x)$, start with an initial value x_0 and compute $x_1 = f(x_0)$, $x_2 = f(x_1)$, $x_3 = f(x_2)$ and so on. For example, with $f(x) = \cos x$ and $x_0 = 1$, the **iterates** are $x_1 = \cos 1 \approx 0.54$, $x_2 = \cos x_1 \approx \cos 0.54 \approx 0.86$, $x_3 \approx \cos 0.86 \approx 0.65$ and so on. Keep computing iterates and show that they get closer and closer to 0.739085. Then pick your own x_0 (any number you like) and show that the iterates with this new x_0 also converge to 0.739085.

58. Referring to exercise 57, show that the iterates of a function can be written as $x_1 = f(x_0)$, $x_2 = f(f(x_0))$, $x_3 = f(f(f(x_0)))$ and so on. Graph $y = \cos(\cos x)$, $y = \cos(\cos(\cos x))$ and $y = \cos(\cos(\cos(\cos x)))$. The graphs should look more and more like a horizontal line. Use the result of exercise 57 to identify the limiting line.

59. Compute several iterates of $f(x) = \sin x$ (see exercise 57) with a variety of starting values. What happens to the iterates in the long run?

60. Repeat exercise 59 for $f(x) = x^2$.

61. In cases where the iterates of a function (see exercise 57) repeat a single number, that number is called a **fixed point.** Explain why any fixed point must be a solution of the equation $f(x) = x$. Find all fixed points of $f(x) = \cos x$ by solving the equation $\cos x = x$. Compare your results to that of exercise 57.

62. Find all fixed points of $f(x) = \sin x$ (see exercise 61). Compare your results to those of exercise 59.

EXPLORATORY EXERCISES

1. You have explored how completing the square can transform any quadratic function into the form $y = a(x-d)^2 + e$. We concluded that all parabolas with $a > 0$ look alike. To see that the same statement is not true of cubic polynomials, graph $y = x^3$ and $y = x^3 - 3x$. In this exercise, you will use completing the cube to determine how many different cubic graphs there are. To see what "completing the cube" would look like, first show that $(x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3$. Use this result to transform the graph of $y = x^3$ into the graphs of (a) $y = x^3 - 3x^2 + 3x - 1$ and (b) $y = x^3 - 3x^2 + 3x + 2$. Show that you can't get a simple transformation to $y = x^3 - 3x^2 + 4x - 2$. However, show that $y = x^3 - 3x^2 + 4x - 2$ can be obtained from $y = x^3 + x$ by basic transformations. Show that the following statement is true: any cubic

Trig Worksheet # 1

\ **1.** List the reciprocal identities and identities for negatives without looking at the text.

2. List the quotient identities and Pythagorean identities without looking at the text.

3. One of the following equations is an identity and the other is a conditional equation. Identify each, and explain the difference between the two.

$$3(2x - 3) = 3(3 - 2x)$$

$$3(2x - 3) = 6x - 9$$

4. One of the following equations is an identity and the other is a conditional equation. Identify each, and explain the difference between the two.

$$\sin x + \cos x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

In Problems 5–10, use the fundamental identities to find the exact values of the remaining trigonometric functions of x, given the following:

5. $\sin x = 2/\sqrt{5}$ and $\cos x = 1/\sqrt{5}$

6. $\sin x = \sqrt{5}/3$ and $\tan x = -\sqrt{5}/2$

7. $\cos x = 1/\sqrt{10}$ and $\csc x = -\sqrt{10}/3$

8. $\cos x = \sqrt{7}/4$ and $\cot x = -\sqrt{7}/3$

9. $\tan x = 1/\sqrt{15}$ and $\sec x = -4/\sqrt{15}$

10. $\cot x = 2/\sqrt{21}$ and $\csc x = 5/\sqrt{21}$

In Problems 11–22, simplify each expression using the fundamental identities.

11. $\tan u \cot u$

12. $\sec x \cos x$

13. $\tan x \csc x$

14. $\sec \theta \cot \theta$

15. $\dfrac{\sec^2 x - 1}{\tan x}$

16. $\dfrac{\csc^2 y - 1}{\cot y}$

17. $\dfrac{\sin^2 \theta}{\cos \theta} + \cos \theta$

18. $\dfrac{1}{\csc^2 x} + \dfrac{1}{\sec^2 x}$

19. $\dfrac{1}{\sin^2 \beta} - 1$

20. $\dfrac{1 - \sin^2 u}{\cos u}$

21. $\dfrac{(1 - \cos x)^2 + \sin^2 x}{1 - \cos x}$

22. $\dfrac{\cos^2 x + (\sin x + 1)^2}{\sin x + 1}$

B **23.** If an equation has an infinite number of solutions, is it an identity? Explain.

24. Does an identity have an infinite number of solutions? Explain.

In Problems 25–30, use the fundamental identities to find the exact values of the remaining trigonometric functions of x, given the following:

25. $\sin x = 2/5$ and $\cos x < 0$

26. $\cos x = 3/4$ and $\tan x < 0$

27. $\tan x = -1/2$ and $\sin x > 0$

28. $\cot x = -3/2$ and $\csc x > 0$

29. $\sec x = 4$ and $\cot x > 0$

30. $\csc x = -3$ and $\sec x < 0$

In Problems 31 and 32, is it possible to use the given information to find the exact values of the remaining trigonometric functions? Explain.

31. $\sin x = 1/3$ and $\csc x > 0$

32. $\tan x = 2$ and $\cot x > 0$

33. For the following graphing calculator displays, find the value of the final expression without finding x or using a calculator:

(A)

```
sin(X)
                 .4350
sin(-X)
```

(B)

```
(sin(X))²
                 .1892
(cos(X))²
```

34. For the following graphing calculator displays, find the value of the final expression without finding x or using a calculator:

(A)

```
tan(X)
                 .4831
tan(-X)
```

(B)

```
sin(X)
                 .8660
cos(X)
                 .5000
tan(X)
```

Using fundamental identities, write the expressions in Problems 35–44 in terms of sines and cosines, and then simplify.

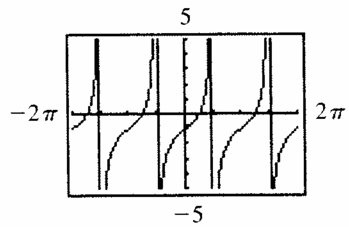
35. $\csc(-y)\cos(-y)$

36. $\sin(-\alpha)\sec(-\alpha)$

37. $\cot x \cos x + \sin x$

38. $\cos u + \sin u \tan u$

6. (B)



The equation appears to be an identity, which is verified as follows:

$$(\sec x)(\sin x - \cos x) = \frac{1}{\cos x}(\sin x - \cos x)$$
$$= \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}$$
$$= \tan x - 1$$

Trig Worksheet # 2

EXERCISE 4.2

A *In Problems 1–26, verify each identity.*

1. $\cos x \sec x = 1$

2. $\sin x \csc x = 1$

3. $\tan x \cos x = \sin x$

4. $\cot x \sin x = \cos x$

5. $\tan x = \sin x \sec x$

6. $\cot x = \cos x \csc x$

7. $\csc(-x) = -\csc x$

8. $\sec(-x) = \sec x$

9. $\dfrac{\sin \alpha}{\cos \alpha \tan \alpha} = 1$

10. $\dfrac{\cos \alpha}{\sin \alpha \cot \alpha} = 1$

11. $\dfrac{\cos \beta \sec \beta}{\tan \beta} = \cot \beta$

12. $\dfrac{\tan \beta \cot \beta}{\sin \beta} = \csc \beta$

13. $(\sec \theta)(\sin \theta + \cos \theta) = \tan \theta + 1$

14. $(\csc \theta)(\cos \theta + \sin \theta) = \cot \theta + 1$

15. $\dfrac{\cos^2 t - \sin^2 t}{\sin t \cos t} = \cot t - \tan t$

16. $\dfrac{\cos \alpha - \sin \alpha}{\sin \alpha \cos \alpha} = \csc \alpha - \sec \alpha$

17. $\dfrac{\cos \beta}{\cot \beta} + \dfrac{\sin \beta}{\tan \beta} = \sin \beta + \cos \beta$

18. $\dfrac{\tan u}{\sin u} - \dfrac{\cot u}{\cos u} = \sec u - \csc u$

19. $\sec^2 \theta - \tan^2 \theta = 1$

20. $\csc^2 \theta - \cot^2 \theta = 1$

21. $(\sin^2 x)(1 + \cot^2 x) = 1$

22. $(\cos^2 x)(\tan^2 x + 1) = 1$

23. $(\csc \alpha + 1)(\csc \alpha - 1) = \cot^2 \alpha$

24. $(\sec \beta - 1)(\sec \beta + 1) = \tan^2 \beta$

25. $\dfrac{\sin t}{\csc t} + \dfrac{\cos t}{\sec t} = 1$

26. $\dfrac{1}{\sec^2 m} + \dfrac{1}{\csc^2 m} = 1$

B **27.** How does solving a conditional equation differ from verifying an identity (both in one variable)?

28. Is $(1 - \cos^2 x)/(\sin x) = \sin x$ an identity for all real values of x? Explain.

In Problems 29–60, verify each identity.

29. $\dfrac{1 - (\cos \theta - \sin \theta)^2}{\cos \theta} = 2 \sin \theta$

30. $\dfrac{1 - (\sin \theta - \cos \theta)^2}{\sin \theta} = 2 \cos \theta$

31. $\dfrac{\tan w + 1}{\sec w} = \sin w + \cos w$

32. $\dfrac{\cot y + 1}{\csc y} = \cos y + \sin y$

33. $\dfrac{1}{1 - \cos^2 \theta} = 1 + \cot^2 \theta$

34. $\dfrac{1}{1 - \sin^2 \theta} = 1 + \tan^2 \theta$

EXAMPLE 5 **Verifying an Identity**

Verify the identity: $\cot y - \cot x = \dfrac{\sin(x - y)}{\sin x \sin y}$

Verification

$$\frac{\sin(x-y)}{\sin x \sin y} = \frac{\sin x \cos y - \cos x \sin y}{\sin x \sin y}$$
$$= \frac{\sin x \cos y}{\sin x \sin y} - \frac{\cos x \sin y}{\sin x \sin y}$$
$$= \cot y - \cot x$$

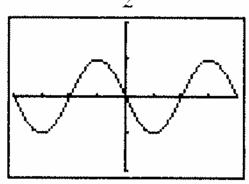
■

Matched Problem 5 Verify the identity: $\tan x + \cot y = \dfrac{\cos(x - y)}{\cos x \sin y}$ ■

Answers to Matched Problems

1. $\sin x$

2. $y_1 = \sin(x - \pi)$; $y_2 = -\sin x$



3. $2 + \sqrt{3}$ **4.** $\dfrac{-4\sqrt{5}}{9}$

5. $$\frac{\cos(x-y)}{\cos x \sin y} = \frac{\cos x \cos y + \sin x \sin y}{\cos x \sin y}$$
$$= \frac{\cos x \cos y}{\cos x \sin y} + \frac{\sin x \sin y}{\cos x \sin y}$$
$$= \cot y + \tan x = \tan x + \cot y$$

Trig Worksheet # 3

EXERCISE 4.3

A *We can use sum identities to verify periodic properties for the trigonometric functions. Verify the following identities using sum identities.*

1. $\cos(x + 2\pi) = \cos x$

2. $\sin(x + 2\pi) = \sin x$

3. $\cot(x + \pi) = \cot x$

4. $\tan(x + \pi) = \tan x$

5. $\sin(x + 2k\pi) = \sin x$, k an integer

6. $\cos(x + 2k\pi) = \cos x$, k an integer

7. $\tan(x + k\pi) = \tan x$, k an integer

8. $\cot(x + k\pi) = \cot x$, k an integer

Verify each identity using cofunction identities for sine and cosine and the fundamental identities discussed in Section 4.1.

9. $\tan\left(\dfrac{\pi}{2} - x\right) = \cot x$

10. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

11. $\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$

12. $\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$

In Problems 13–20, use the formulas developed in this section to convert the indicated expression to a form involving $\sin x$, $\cos x$, and/or $\tan x$.

Trig Worksheet #4

Check your results with a graphing utility.

13. $\sin(x + 30°)$ **14.** $\cos(x - 45°)$

15. $\cos(x + 60°)$ **16.** $\sin(x - 60°)$

17. $\tan\left(x + \frac{\pi}{4}\right)$ **18.** $\tan\left(\frac{\pi}{3} - x\right)$

19. $\cot\left(\frac{\pi}{6} - x\right)$ **20.** $\cot\left(x - \frac{\pi}{4}\right)$

B *In Problems 21–28, use appropriate identities to find the exact value of the indicated expression. Check your results with a calculator.*

21. $\sin 15°$ **22.** $\cos 15°$

23. $\sin 20° \cos 25° + \cos 20° \sin 25°$

24. $\sin 55° \cos 10° - \cos 55° \sin 10°$

25. $\cos 81° \cos 21° + \sin 81° \sin 21°$

26. $\cos 12° \cos 18° - \sin 12° \sin 18°$

27. $\dfrac{\tan 17° + \tan 28°}{1 - \tan 17° \tan 28°}$ **28.** $\dfrac{\tan 52° - \tan 22°}{1 + \tan 52° \tan 22°}$

In Problems 29–34, use the given information and appropriate identities to find the exact value of the indicated expression.

29. Find $\sin(x + y)$ if $\sin x = 1/3$, $\cos y = -3/4$, x is in quadrant II, and y is in quadrant III.

30. Find $\sin(x - y)$ if $\sin x = -2/5$, $\sin y = 2/3$, x is in quadrant IV, and y is in quadrant I.

31. Find $\cos(x - y)$ if $\tan x = -1/4$, $\tan y = -1/5$, x is in quadrant II, and y is in quadrant IV.

32. Find $\cos(x + y)$ if $\tan x = 2/3$, $\tan y = 1/3$, x is in quadrant I, and y is in quadrant III.

33. Find $\tan(x + y)$ if $\sin x = -1/4$, $\cos y = -1/3$, $\cos x < 0$, and $\sin y < 0$.

34. Find $\tan(x - y)$ if $\sin x = 1/5$, $\cos y = 2/5$, $\tan x < 0$, and $\tan y > 0$.

In Problems 35–48, verify each identity.

35. $\sin 2x = 2 \sin x \cos x$

36. $\cos 2x = \cos^2 x - \sin^2 x$

37. $\cot(x - y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

38. $\cot(x + y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

39. $\cot 2x = \dfrac{\cot^2 x - 1}{2 \cot x}$

40. $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

41. $\dfrac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta} = \dfrac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)}$

42. $\dfrac{\cot \alpha + \cot \beta}{\cot \alpha - \cot \beta} = \dfrac{\sin(\beta + \alpha)}{\sin(\beta - \alpha)}$

43. $\tan x - \tan y = \dfrac{\sin(x - y)}{\cos x \cos y}$

44. $\cot x - \tan y = \dfrac{\cos(x + y)}{\sin x \cos y}$

45. $\tan(x + y) = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$

46. $\tan(x - y) = \dfrac{\cot y - \cot x}{\cot x \cot y + 1}$

47. $\dfrac{\sin(x + h) - \sin x}{h} = (\sin x)\left(\dfrac{\cos h - 1}{h}\right) + (\cos x)\left(\dfrac{\sin h}{h}\right)$

48. $\dfrac{\cos(x + h) - \cos x}{h} = (\cos x)\left(\dfrac{\cos h - 1}{h}\right) - (\sin x)\left(\dfrac{\sin h}{h}\right)$

49. How would you show that $\csc(x - y) = \csc x - \csc y$ is not an identity?

50. How would you show that $\sec(x + y) = \sec x + \sec y$ is not an identity?

51. Use a graphing utility to show that $\sin(x - 2) = \sin x - \sin 2$ is not an identity. Then explain what you did.

52. Use a graphing utility to show that $\cos(x + 1) = \cos x + \cos 1$ is not an identity. Then explain what you did.

In Problems 53–56, use sum or difference identities to convert each equation to a form involving $\sin x$, $\cos x$, and/or $\tan x$. To check your result, enter the original equation in a graphing utility as y_1 and the converted form as y_2. Then graph y_1 and y_2 in the same viewing window. Use $\boxed{\text{TRACE}}$ *to compare the two graphs.*

53. $y = \cos(x + 5\pi/6)$ **54.** $y = \sin(x - \pi/3)$

55. $y = \tan(x - \pi/4)$ **56.** $y = \tan(x + 2\pi/3)$

In Problems 57–60, write each equation in terms of a single trigonometric function. Check the result by entering the original equation in a graphing utility as y_1 and the converted form as y_2. Then graph y_1 and y_2 in the same viewing window. Use $\boxed{\text{TRACE}}$ *to compare the two graphs.*

57. $y = \sin 3x \cos x - \cos 3x \sin x$

58. $y = \cos 3x \cos x - \sin 3x \sin x$

59. y

60. y

C Verify

[Hint:

61. si

62. c

63. P th

64.

65.

Trig Worksheet #5

19. $\cos^2 \frac{w}{2} = \frac{1 + \cos w}{2}$ **20.** $\sin^2 \frac{w}{2} = \frac{1 - \cos w}{2}$

21. $\cot \frac{\alpha}{2} = \frac{1 + \cos \alpha}{\sin \alpha}$ **22.** $\cot \frac{\alpha}{2} = \frac{\sin \alpha}{1 - \cos \alpha}$

23. $\frac{\cos 2t}{1 - \sin 2t} = \frac{1 + \tan t}{1 - \tan t}$

24. $\cos 2t = \frac{1 - \tan^2 t}{1 + \tan^2 t}$

25. $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ **26.** $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$

27. $\tan \frac{x}{2} = \frac{\sin x}{1 + \cos x}$ **28.** $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$

29. $\sec^2 x = (\sec 2x)(2 - \sec^2 x)$

30. $2 \csc 2x = \frac{1 + \tan^2 x}{\tan x}$

In Problems 31–34, use the given information to find the exact value of sin 2x, cos 2x, and tan 2x. Check your answer with a calculator.

31. $\sin x = \frac{7}{25}$, $\pi/2 < x < \pi$

32. $\cos x = -\frac{8}{17}$, $\pi/2 < x < \pi$

33. $\cot x = -\frac{12}{35}$, $-\pi/2 < x < 0$

34. $\tan x = -\frac{20}{21}$, $-\pi/2 < x < 0$

In Problems 35–40, use the given information to find the exact value of sin (x/2) and cos (x/2). Check your answer with a calculator.

35. $\cos x = \frac{1}{4}$, $0° < x < 90°$

36. $\sin x = \frac{\sqrt{21}}{5}$, $0° < x < 90°$

37. $\tan x = -\sqrt{8}$, $90° < x < 180°$

38. $\cot x = -\frac{3}{\sqrt{7}}$, $90° < x < 180°$

39. $\csc x = -\frac{5}{\sqrt{24}}$, $-90° < x < 0°$

40. $\sec x = \frac{3}{2}$, $-90° < x < 0°$

Your friend is having trouble finding exact values of sin θ and cos θ from the information given in Problems 41 and 42, and comes to you for help. Instead of just working the problems, you guide your friend through the solution process using the following questions (A)–(E). What is the correct response to each question for each problem?

(A) The angle 2θ is in which quadrant? How do you know?

(B) How can you find sin 2θ and cos 2θ? Find each.

(C) Which identities relate sin θ and cos θ with either sin 2θ or cos 2θ?

(D) How would you use the identities in part (C) to find sin θ and cos θ exactly, including the correct sign?

(E) What are the exact values for sin θ and cos θ?

41. Find the exact values of $\sin \theta$ and $\cos \theta$, given $\sec 2\theta = -\frac{5}{4}$, $0° < \theta < 90°$.

42. Find the exact values of $\sin \theta$ and $\cos \theta$, given $\tan 2\theta = -\frac{4}{3}$, $0° < \theta < 90°$.

43. In applied mathematics, approximate forms are often substituted for exact forms to simplify formulas or computations. Graph each side of each statement below in the same viewing window, $-\pi/2 \le x \le \pi/2$, to show that the approximation is valid for x close to 0. Use TRACE and describe what happens to the approximation as x gets closer to 0.

(A) $\sin 2x \approx 2 \sin x$ (B) $\sin \frac{x}{2} \approx \frac{1}{2} \sin x$

44. Repeat Problem 43 for:

(A) $\tan 2x \approx 2 \tan x$ (B) $\tan \frac{x}{2} \approx \frac{1}{2} \tan x$

In Problems 45–48, graph y_1 and y_2 in the same viewing window for $-2\pi \le x \le 2\pi$, and state the intervals for which y_1 and y_2 are identical.

45. $y_1 = \sin \frac{x}{2}$, $y_2 = \sqrt{\frac{1 - \cos x}{2}}$

46. $y_1 = \sin \frac{x}{2}$, $y_2 = -\sqrt{\frac{1 - \cos x}{2}}$

47. $y_1 = \cos \frac{x}{2}$, $y_2 = -\sqrt{\frac{1 + \cos x}{2}}$

48. $y_1 = \cos \frac{x}{2}$, $y_2 = \sqrt{\frac{1 + \cos x}{2}}$

In Problems 49–54, use the given information to find the exact value of sin x, cos x, and tan x. Check your answer with a calculator.

49. $\sin 2x = \frac{55}{73}$, $0 < x < \pi/4$

50. $\cos 2x = -\frac{28}{53}$, $\pi/4 < x < \pi/2$

51. $\tan 2x = -\frac{28}{45}$, $\pi/4 < x < \pi/2$

52. $\cot 2x = -\frac{55}{48}$, $-\pi/4 < x < 0$

53. $\sec 2x = \frac{65}{33}$, $-\pi/4 < x < 0$

54. $\csc 2x = \frac{65}{33}$, $0 < x < \pi/4$

Trig Worksheet # 6

EXAMPLE 5 **Music**

When certain keys on a piano are struck, a felt-covered hammer strikes two strings. If the piano is out of tune, the tones from the two strings create a beat, and the sound is sour. If a piano tuner counts 15 beats in 5 sec, how far apart are the frequencies of the two strings?

Solution

$$f_b = \tfrac{15}{5} = 3 \text{ beats/sec}$$

$$f_1 - f_2 = f_b = 3 \text{ Hz}$$

Thus, the two strings are out of tune by 3 cycles/sec. ■

Matched Problem 5 What is the beat frequency for the two tones in Figure 2? ■

Answers to Matched Problems

1. $\cos 5\theta \cos 2\theta = \frac{1}{2}\cos 7\theta + \frac{1}{2}\cos 3\theta$ 2. $(-\sqrt{3} - 2)/4$
3. $\cos 3t + \cos t = 2\cos 2t \cos t$ 4. $-\sqrt{6}/2$ 5. $f_b = 8$ Hz

EXERCISE 4.5

A *In Problems 1–8, write each product as a sum or difference involving sines and cosines.*

1. $\cos 4w \cos w$
2. $\sin 2t \sin t$
3. $\cos 2u \sin u$
4. $\sin 4B \cos B$
5. $\sin 2B \cos 5B$
6. $\cos 3\theta \cos 5\theta$
7. $\sin 3m \sin 4m$
8. $\cos 2A \sin 3A$

In Problems 9–16, write each sum or difference as a product involving sines and cosines.

9. $\cos 5\theta + \cos 3\theta$
10. $\sin 6A + \sin 4A$
11. $\sin 6u - \sin 2u$
12. $\cos 3t - \cos t$
13. $\sin 3B + \sin 5B$
14. $\cos 2m + \cos 4m$
15. $\cos w - \cos 5w$
16. $\sin 3C - \sin 7C$

B *Evaluate each of the following exactly using an appropriate identity.*

17. $\cos 75° \sin 15°$
18. $\sin 195° \cos 75°$
19. $\sin 105° \sin 165°$
20. $\cos 15° \cos 75°$

Evaluate each of the following exactly using an appropriate identity.

21. $\sin 195° + \sin 105°$
22. $\cos 285° + \cos 195°$
23. $\sin 75° - \sin 165°$
24. $\cos 15° - \cos 105°$

In Problems 25 and 26, use sum and difference identities from Section 4.3 to establish each of the following:

25. $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$
26. $\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$
27. Explain how you can transform the product–sum identity

$$\cos u \cos v = \tfrac{1}{2}[\cos(u + v) + \cos(u - v)]$$

into the sum–product identity

$$\cos x + \cos y = 2\cos\frac{x + y}{2}\cos\frac{x - y}{2}$$

by a suitable substitution.

28. Explain how you can transform the product–sum identity

$$\sin u \sin v = \tfrac{1}{2}[\cos(u - v) - \cos(u + v)]$$

into the sum–product identity

$$\cos x - \cos y = -2\sin\frac{x + y}{2}\sin\frac{x - y}{2}$$

by a suitable substitution.

In Problems 29–36, verify each identity.

29. $\dfrac{\cos t - \cos 3t}{\sin t + \sin 3t} = \tan t$

Flash Cards

Make and study flash cards of the following:

Yes you will be quizzed on these during the first week of school. Yes you must know them instantaneously. Yes they are that important. Finally, yes that quiz will count in your grade.

Trig Identities

Sin(0) = 0	Cos(0) = 1	Tan(0) = 0
$sin\frac{\pi}{2} = 1$	$cos\frac{\pi}{2} = \frac{\sqrt{2}}{2}$	$tan\frac{\pi}{2} = "\infty"$
$sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$tan\frac{\pi}{4} = 1$
$sin\frac{\pi}{6} = \frac{1}{2}$	$cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$
$sin\pi = 0$	$cos\pi = -1$	$\tan\pi = 0$
$sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos\frac{\pi}{3} = \frac{1}{2}$	$tan\frac{\pi}{3} = \sqrt{3}$

$$cscx = \frac{1}{sinx}$$

$$secx = \frac{1}{cosx}$$

$$cotx = \frac{1}{tanx} = \frac{cosx}{sinx}$$

$$tanx = \frac{sinx}{cosx}$$

$$sin^2x + cos^2x = 1$$

$$csc^2x - cot^2x = 1$$

$$\sin(u \pm v) = sinu\, cosv \pm cosu\, sinv$$

$$\cos(u \pm v) = cosu\, cosv \pm sinu\, sinv$$

$$sin2u = 2sinu\, cosu$$

$$cos2u = cos^2u - sin^2u = 2cos^2u - 1 = 1 - 2sin^2u$$

$$sin^2u = 1 - cos^2u = \frac{1 - cos2u}{2}$$

$$cos^2u = 1 - sin^2u = \frac{1 + cos2u}{2}$$

Logarithms Worksheet

Convert each exponential expression into fractional or root form

1) 2^{-3} 2) 4^{-2} 3) $3^{\frac{1}{2}}$ 4) $6^{\frac{2}{5}}$ 5) $5^{\frac{2}{3}}$ 6) $4^{-\frac{2}{3}}$

Convert each expression into exponential form

7) $\frac{1}{x^2}$ 8) $\sqrt[3]{x^2}$ 9) $\frac{2}{x^3}$ 10) $\frac{4}{x^2}$ 11) $\frac{1}{2\sqrt{x^3}}$ 12) $\frac{3}{2\sqrt{x}}$

Find the integer value of the given expression without using a calculator (Show all work)

13) $4^{\frac{3}{2}}$ 14) $8^{\frac{2}{3}}$ 15) $\frac{\sqrt{8}}{2^{\frac{1}{2}}}$ 16) $\frac{2}{(\frac{1}{3})^2}$

Solve the given equation for x

17) $e^{2x} = 2$ 18) $e^{4x} = 3$ 19) $e^x(x^2 - 1) = 0$ 20) $xe^{-2x} + 2e^{-2x} = 0$ 21) $4lnx = -8$

22) $x^2lnx - 9lnx = 0$ 23) $e^{2lnx} = 4$ 24) $\ln(e^{2x}) = 6$

25) $e^x = 1 + 6e^{-x}$ 26) $lnx + \ln(x-1) = ln2$

Without using a calculator use the definition of logarithm to determine the value

27) $\log_3 9$ 28) $\log_4 64$ 29) $\log_4 \frac{1}{16}$ 30) $\log_4 3$

Rewrite the expression as a single logarithm

31) ln3-ln4 32) 2ln4-ln3 33) $\frac{1}{2}ln4 - ln2$ 34) $3ln2 - ln\frac{1}{2}$ 35) $ln9 - 2ln3$

Find a function of the form $f(x) = ae^{bx}$ *with the given function values*

36) $f(0) = 2$ $f(2) = 6$ 37) $f(0) = 3$ $f(3) = 4$ 38) $f(0) = 4$ $f(2) = 2$

39) $f(0) = 5, f(1) = 2$