Sections 2.4 – 2.6 Practice Exam

1. Find the derivative of the function.

$$g(x) = \left(\frac{x+5}{x^2+6} \right)^7$$

2. Find *dy/dx* by implicit differentiation.

$$x^4 + 5x + x^3 y - y^8 = 16$$

3. Find the points at which the graph of the equation has a vertical or horizontal tangent line.

$$2x^2 + 10y^2 - 16x + 60y + 2 = 0$$

4. ***Depth*** A conical tank (with vertex down) is 10 ft across the top and 12 ft deep. If water is flowing into the tank at a rate of 9 cubic ft per minute, find the rate of change of depth of water when the water is 6 ft deep.

5. A point is moving along the graph of the function

$$y = \frac{1}{3x^2+2}$$

such that *dx/dt* = 2 centimeters per second.

Find *dy/dt* when $x = 3$.

6. Evaluate the derivative of the function at the given point.

$$f(t) = \frac{4t^2+2}{2t-1} \quad , \qquad \left(3, \frac{38}{5}\right)$$

7. Find *dy/dx* by implicit differentiation.

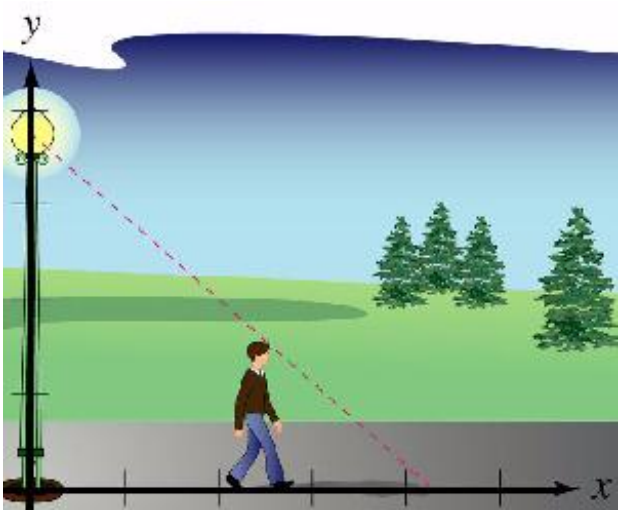
$$x^6 + 4x + 13xy - y^5 = 36$$

8. Find the derivative of the function.

$$f(t) = (3 - 4t^4)^6$$

9. ***Shadow Length*** A man 6 feet tall walks at a rate of 6 ft per second away from a light that is 16 ft above the ground (see figure). When he is 9 ft from the base of the light find the following.

(a) The rate the tip of the shadow is moving.
(b) The rate the length of his shadow is changing.



(a) (b)

10. Find *dy/dx* by implicit differentiation.

$$x^2 + y^2 = 16$$

11. A point is moving along the graph of the function

$$y = \sin 9x$$

such that *dx/dt* = 8 centimeters per second.

Find *dy/dt* when $x = \dfrac{\pi}{11}$.

12. Evaluate the derivative of the function at the given point.

$$y = \sqrt[3]{5x^4 + 5x} \quad , \qquad x = 4$$

13. Find an equation of the tangent line to the graph of the function given below at the given point.

$$(y-7)^2 = 5(x-2), \qquad (11.80, 0.00).$$

(The coefficients below are given to two decimal places.)

14. Find *dy/dx* by implicit differentiation.

$$\sin x + 5\cos 15y = 3$$

15. Find d^2y/dx^2 in terms of x and y.

$$5 - 5xy = 3x - 4y$$

16. Find the derivative of the function.

$$f(t) = 6\sec^2(7\pi t - 1)$$

17. Find *dy/dx* by implicit differentiation and evaluate it at the given point.

$$x^3 - 6y^2 = -2, \qquad \left(3, \sqrt{\frac{29}{6}}\right)$$

18. Find the derivative of the function.

$$f(t) = (3+6t)^{\frac{3}{7}}$$

19. Find an equation to the tangent line to the graph of *f* at the given point.

$$f(x) = \tan^4 x, \quad \left(\frac{8\pi}{9}, 0.018\right)$$

The coefficients below are given to two decimal places.

20. Find d^2y/dx^2 in terms of x and y.

$$x^2 + y^2 = 6$$

21. Find dy/dx by implicit differentiation.

$$x^2 + 5x + 2xy - y^2 = 25$$

22. A point is moving along the graph of the function

$$y = 7x^2 + 4$$

such that dx/dt = 5 centimeters per second.

Find dy/dt for the given values of x.

(a) $x = 3$ (b) $x = 8$

23. Find an equation of the tangent line to the graph of the function given below at the given point.

$$9x^2 - 5xy + 3y^2 - 17 = 0, \qquad (-1, 1)$$

(The coefficients below are given to two decimal places.)

24. Find the second derivative of the function.

$$f(x) = \sin 2x^4$$

25. Find the derivative of the function.

$$y = \cos\left(2x^6 - 3\right)$$

26. Find the derivative of the function.

$$f(\theta) = \frac{2}{5}\sin^2 2\theta$$

27. Find the derivative of the function.

$$y = 2\sin 5x$$

28. ***Area*** The radius, *r*, of a circle is decreasing at a rate of 5 centimeters per minute.

Find the rate of change of area, *A*, when the radius is 4

29. Find the derivative of the function.

$$y = -9\cos 5x$$

30. Find an equation to the tangent line for the graph of *f* at the given point.

$$f(x) = (4x^4 + 9)^2, \quad \left(-1, 169\right)$$

31. Find the derivative of the function.

$$f(x) = x^3(6 + 8x)^7$$

32. Find the derivative of the function.

$$f(x) = x^6\sqrt{4-7x}$$

33. Find the derivative of the function.

$$y = \frac{7}{5}\sec^2 x$$

34. Find the derivative of the function.

$$f(x) = x^3\sqrt{5-4x}$$

35. Find *dy/dx* by implicit differentiation.

$$x^{7/2} + y^{2/7} = 4$$

36. Find the second derivative of the function.

$$f(x) = (5x^5 + 9)^6$$

37. Evaluate the derivative of the function at the given point.

$$f(t) = \frac{4}{t-1}\ , \qquad \left(4, \frac{4}{3}\right)$$

38. Evaluate the derivative of the function at the given point.

$$f(t) = \frac{5t+3}{5t-1}\ , \qquad \left(2, \frac{13}{9}\right)$$

Answer Key

1. $g'(x) = \dfrac{7\left(6-10x-x^2\right)\left(5+x\right)^6}{\left(6+x^2\right)^8}$
2. $\dfrac{dy}{dx} = \dfrac{4x^3+5+3x^2y}{8y^7-x^3}$
3. There is a horizontal tangent at $x=4$ and a vertical tangent at $y=-3$.
4. $\dfrac{36}{25\pi}$ ft per minute
5. $\dfrac{dy}{dt} = -\dfrac{36}{841}$
6. $f'(3) = \dfrac{44}{25}$
7. $\dfrac{dy}{dx} = \dfrac{6x^5+4+13y}{5y^4-13x}$
8. $f'(t) = -96t^3(3-4t^4)^5$
9. $\dfrac{48}{5}$ ft per minute $\qquad$ $\dfrac{18}{5}$ ft per minute
10. $\dfrac{dy}{dx} = -\dfrac{x}{y}$
11. $\dfrac{dy}{dt} = 72\cos\left(\dfrac{9\pi}{11}\right)$
12. $y'(4) = \dfrac{1285}{3\left(1300\right)^{\frac{2}{3}}}$
13. $y = -0.36x + 4.21$
14. $\dfrac{dy}{dx} = \dfrac{\cos x}{75\sin 15y}$
15. $\dfrac{d^2y}{dx^2} = \dfrac{-130}{(4-5x)^3}$
16. $f'(t) = 84\pi\sec^2\left(7\pi t-1\right)\tan\left(7\pi t-1\right)$
17. $\left.\dfrac{dy}{dx}\right|_{x=3} = \dfrac{9}{4\left(\sqrt{\dfrac{29}{6}}\right)}$
18. $f'(t) = \dfrac{18}{7}(3+6t)^{\frac{-4}{7}}$
19. $y = -0.22x + 0.63$

20. Both A and C

21. $\dfrac{dy}{dx} = \dfrac{2x+5+2y}{2y-2x}$

22. $\dfrac{dy}{dt} = 210$ $\qquad$ $\dfrac{dy}{dt} = 560$

23. $y = 2.09x + 3.09$

24. $f'(x) = 24x^2 \cos 2x^4 - 64x^6 \sin 2x^4$

25. $y' = -12x^5 \sin\left(2x^6 - 3\right)$

26. $f'(\theta) = \dfrac{8 \sin 2\theta \cos 2\theta}{5}$

27. $y' = 10\cos 5x$

28. $\dfrac{dA}{dt} = -40\pi$

29. $y' = 45\sin 5x$

30. $y = -416x - 247$

31. $f'(x) = x^2(6+8x)^6\left(18+80x\right)$

32. $f'(x) = \dfrac{x^5\left(48-91x\right)}{2\sqrt{4-7x}}$

33. $y' = \dfrac{14}{5}\sec^2 x \tan x$

34. $f'(x) = \dfrac{x^2\left(30-28x\right)}{2\sqrt{5-4x}}$

35. $\dfrac{dy}{dx} = -\dfrac{49x^{5/2}}{4y^{-5/7}}$

36. $f''(x) = 150x^3\left(9+5x^5\right)^4\left(36+145x^5\right)$

37. $f'(4) = -\dfrac{4}{9}$

38. $f'(2) = -\dfrac{20}{81}$